Mitigate the Impact of CFO Phenomena for an MC-CDMA System over the Short-term Fading by Applying Multi-dimension Combining Receiver

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Abstract
Based on the scenario assumed that the CFO (carrier frequency offset), which is caused by the ICI (inter-carrier interference), exists in the environments of short-term fading, a multi-dimension combining (M-D combining) receiver for an MC-CDMA (multi-carrier coded-division multiple-access) system is designed and proposed considerably to overcome the system performance degradation. It is worthwhile claiming that is not only the fading parameter of the correlated-fading model dominates the system performance of an MC-CDMA system, but the number of antenna with the M-D combining receiver also definitely affects the system performance.

Index Terms—MC-CDMA system, multi-dimension combining (M-D combining), MIP, antenna diversity

1. Introduction

Based on the motivation, the 4G (fourth generation) wireless cellular system, MC-CDMA (multi-carrier CDMA), which based on the OFDM (orthogonal frequency division multiplexing) signaling techniques, is now engaged in exploring [1]. Yang and Hanzo evaluated the performance of an MC-DS-CDMA to consider the correlation presents in the fading of the various subcarriers [4]. Recently, the present author, Chen, evaluated the performance of an MC-CDMA system with MRC diversity working in Nakagami-\(m\) fading channels [6]. Moreover, the CFO (carrier frequency offset) phenomenon, which is caused by the mismatch in frequency generated from the oscillator between the transmitter and the receiver, i.e. the estimation of the receiver goes wrong, induces the ICI (inter-carrier interference) which will abolish the orthogonality of the transmitted data over an MC-CDMA systems [7]. Recently, Xiang and Hanzo proposed the exact closed-form for the average BER calculation of OFDM system in the presence of both CFO and phase estimation error in frequency-selective fading channels.

In this paper we propose the investigation takes account of the CFO phenomena into the performance evaluation for an MC-CDMA system with multi-dimension combining receiver, called as M-D combining receiver, and which is considered working in frequency selective fading environments. Besides, under the assumption of both of the characteristics with independent- and correlated-branch between the paths of an M-D combining receiver is evaluated. At last, the proposed M-D combining receiver is definitely proved can mitigate the system performance of an MC-CDMA system of an MC-CDMA system degraded by the effect of CFO.

The paper is organized as follows. In section II the system model of an asynchronous MC-CDMA is described. A closed-form expression for the pdf (probability density function) of the decision static is shown in section III. In section IV illustrates the closed-form expression for the average BER with the impact of correlated and uncorrelated fading among spatially separated receiver fingers on system performance the numerical results are presented in section V. Finally, there is a simple conclusion drawn in section VI.

2. System Models

2.1. Transmitter Model

Assume that there exist \(K\) simultaneous users with \(N\) subcarriers within a single cell in the system model. The transmitted signal can be expressed as

\[
S_i(t) = \frac{P}{N} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} c_i^n h_m^P (t - nT_c) \text{Re}\{e^{-j2\pi f_c(t-nT_c)}\} \tag{1}
\]

where both \(c_i^n\) and \(h_m^P\) belong to \((-1, 1)\), \(P\) is the power of the data bit, \(M\) denotes the number of the data bit, \(N\) expresses the number of subcarriers, and the sequences \(c_0,...,c_5\) and \(h_0,...,h_4\) represent the signature sequence and the data bit of the \(k\)-th user, respectively. The term \(P_i(t)\) is defined as the unit amplitude pulse that is non-zero in the interval of \([0, T_c]\), \(\phi_n = 2\pi f_c n / T_s\) is the angular frequency of the \(n\)th subcarrier, where \(f_c\) indicates the carrier frequency, \(T_c\) is the symbol duration.

2.2. Channel Model
The $k$-th user’s $a_{k}$-th receiver antenna has the low-pass channel impulse response expressed as

$$h_{k}(t)=\sum_{l=0}^{L_{k}-1}a'_{k,l}e^{j2\pi f_{sc}l}e^{-\delta(t-t'_{k,l})}$$

where $L_{k}$ is the number of resolvable propagation paths that reach the receive antenna. Each path is characterized by $a'_{k,l}$, and its propagation delay $t'_{k,l}$. Under the assumption that the total time average channel gain per antenna for each user is normalized to one, that is able to be written an $\sum_{k=1}^{K}\sum_{l=1}^{L_{k}}\gamma_{k,l}e^{-\delta(t-t'_{k,l})}$ in the real world, general, the effect of MIP phenomena is an exponential type and is given as the channel impulse response expressed as Nakagami-$m_{k}$, where the pdf of the desired signal; the second term, 0, can be obtained as an approximate expression and illustrated to be calculated as an approximate expression and illustrated in some of the publications. Thereafter, by using the changing variable, let $y_{k}^{'}=m_{k}'/2N\gamma_{k}$, where $a_{i}=1,2,...,A$, and $l=0,1,...,L_{k}-1$. By substituting (10) into the MGF formula, and the MGF of the SINR, $\gamma_{k}^{'}$, can be expressed as

$$\varphi_{\gamma_{k}'}(t)=\frac{\gamma_{k}'}{\Gamma(m_{k}')}\int_{-\infty}^{\infty}e^{-\gamma_{k}'}\exp(-y_{k}'x)dx$$

By applying the closed form definition shown in [3], the formula (10) can be computed by some of the steps as follows. Firstly, the exponential function may be expressed as a contour integral $\exp(-x) = \int_{-\infty}^{\infty}\Gamma(-i\gamma_{k}x)\exp(-y_{k}'x)dx$ [2, p. 43], where $\gamma_{k}^{'}$ is the instantaneous SINR of the $n$-th finger of the $a_{i}$-th antenna finger, the number of the combining receiver is indicated by $L_{k}$. The pdf of $\gamma_{k}$ is able to be calculated as an approximate expression and illustrated in some of the publications. Thereafter, by using the changing variable, let $y_{k}^{'}=m_{k}'/2N\gamma_{k}$, where $a_{i}=1,2,...,A$, and $l=0,1,...,L_{k}-1$. By substituting (10) into the MGF formula, and the MGF of the SINR, $\gamma_{k}^{'}$, can be expressed as

$$\varphi_{\gamma_{k}'}(t)=\frac{\gamma_{k}'}{\Gamma(m_{k}')\int_{-\infty}^{\infty}\exp(-y_{k}'x)dx}$$

where the correlation between the branches has been included in the term $\Theta(\sum_{l=0}^{L_{k}-1}\beta_{k,l})^{2}$.

### 2.4. The CFO Consideration

The CFO coefficient caused by the $n$th subcarrier for $n=2,\ldots,N$ subchannel is given as

$$m_{n}=[M_{n}^{\prime}]_{n} = \Lambda(c)e^{-j\frac{2\pi c}{N}n}$$

where $\Lambda(c)=\sin[\pi(a-1+c)]/\sin[\pi(a-1+c)/N]$ , and $c$ indicates the CFO magnitude. Hence, the SNR at the combining receiver output of the MC-CDMA system can be determined by putting (5) and (6) together and obtained as

$$\sum_{n=1}^{N}\left[\sum_{k=1}^{K}\sum_{l=1}^{L_{k}}\gamma_{k,l}e^{-\delta(t-t'_{k,l})}\right]$$

where $SNR$ is the ratio of bit energy and the noise result in each antenna.

### 3. Statistical Analysis

The SINR (Signal-to-interference-plus noise ratio) at the output of an M-D combining receiver can be expressed as

$$\gamma = \sum_{k=1}^{K} \sum_{l=1}^{L_{k}} \gamma_{k,l}$$

where $\gamma_{k}^{'}=\sum(a_{k,l})^{2}$ is the instantaneous SNR of the $n$-th finger of the $a_{i}$-th antenna finger, the number of the combining receiver is indicated by $L_{k}$. The pdf of $\gamma$ is able to be calculated as an approximate expression and illustrated in some of the publications. Thereafter, by using the changing variable, let $y_{k}^{'}=m_{k}'/2N\gamma_{k}$, where $a_{i}=1,2,...,A$, and $l=0,1,...,L_{k}-1$. By substituting (10) into the MGF formula, and the MGF of the SINR, $\gamma_{k}^{'}$, can be expressed as

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where the correlation between the branches has been included in the term $\Theta(\sum_{l=0}^{L_{k}-1}\beta_{k,l})^{2}$.

### 2.3. Receiver Model

The received signal at the output of the referenced user can be obtained as

$$r(t) = \sum_{k=1}^{K} \sum_{l=1}^{L_{k}} a'_{k,l} e^{j2\pi f_{sc}l} e^{-\delta(t-t'_{k,l})}$$

where $a'_{k,l}$ denotes the channel fading intensity modeled as Nakagami-$m$-distributed, $\phi_{l} = \phi_{l} - \theta_{l}$ represents the phase difference between the transmitter and receiver and $N(t)$ is the AWGN (additive white Gaussian noise). The decision variable $\xi_{0}$ of the $n$th data bit is given by

$$\xi_{0} = \int_{0}^{T_{s}} r(t) \phi_{l} \cos (\omega_{l} t + \phi_{l}) dt$$

where $r(t)$ is the received signal shown in (3) for a single cell, and $T_{s}$ denotes the chip duration. The first term represents the desired signal; the second term, $I_{s,n}$, in this study the second term of (4) is assumed to be ignorable; $I_{s,n}$ is the MAI (multiple-access interference); and the last term, $I_{n,AWGN}$, is the AWGN with zero mean and $N_{s}T_{s}/4$ variance.

Since $h_{k}$ belongs to $[-1, 1]$, hence the average value of the desired signal, the first term shown in (4), for the referenced user, 0-th user, in a single-cell environment can be obtained by using the method of expectation operating and determined as

$$E(\xi_{0}) = \frac{\sum_{k=1}^{K} \sum_{l=1}^{L_{k}} a'_{k,l}^{2}}{2N}$$

where the correlation between the branches has been included in the term $\Theta(\sum_{l=0}^{L_{k}-1}\beta_{k,l})^{2}$.

The variance of the total interference of the referenced user can be obtained as [9]

$$\sigma_{\xi}^{2} = \frac{\sum_{k=1}^{K} \sum_{l=1}^{L_{k}} (K-1)\Omega_{k} E\left[\left(a'_{k,l}^{2}\right)^{2}\right] + N_{s}T_{s} L_{k} \sum_{l=0}^{L_{k}-1} \Omega'_{l}}{4T_{s}}$$

where $\Omega'_{l}$ is the number of resolvable propagation paths that reach the receive antenna. Each path is characterized by $a'_{k,l}$, and its propagation delay $t'_{k,l}$. Under the assumption that the total time average channel gain per antenna for each user is normalized to one, that is able to be written an $\sum_{k=1}^{K}\sum_{l=1}^{L_{k}}\gamma_{k,l}e^{-\delta(t-t'_{k,l})}$ in the real world, general, the effect of MIP phenomena is an exponential type and is given as the channel impulse response expressed as Nakagami-$m_{k}$, where the pdf of the desired signal; the second term, 0, can be obtained as an approximate expression and illustrated in some of the publications. Thereafter, by using the changing variable, let $y_{k}^{'}=m_{k}'/2N\gamma_{k}$, where $a_{i}=1,2,...,A$, and $l=0,1,...,L_{k}-1$. By substituting (10) into the MGF formula, and the MGF of the SINR, $\gamma_{k}^{'}$, can be expressed as

$$\varphi_{\gamma_{k}'}(t)=\frac{\gamma_{k}'}{\Gamma(m_{k}')\int_{-\infty}^{\infty}\exp(-y_{k}'x)dx}$$

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$$\varphi_{\gamma_{k}'}(t)=\frac{\gamma_{k}'}{\Gamma(m_{k}')\int_{-\infty}^{\infty}\exp(-y_{k}'x)dx}$$

where the correlation between the branches has been included in the term $\Theta(\sum_{l=0}^{L_{k}-1}\beta_{k,l})^{2}$.
the SINR can be calculated in terms of the confluent form of the multivariate Lauricella hyper-geometric function and obtained as

\[
f_\gamma(\gamma) = \frac{1}{\Gamma(\sum_{n=1}^{N_\alpha} \sum_{l=0}^{L_\alpha-1} m_i^l)} \left[ \prod_{n=1}^{N_\alpha} \prod_{l=0}^{L_\alpha-1} (y_{\alpha}^n)^{m_i^l} \right] \times \gamma^\left[\sum_{n=1}^{N_\alpha} \sum_{l=0}^{L_\alpha-1} m_i^l \right] \Phi_{\alpha}^{(N_\alpha,L_\alpha)}(m_0^0, m_1^1, ..., m_{L_\alpha-1}^{L_\alpha-1}),
\]

where \( \Phi_{\alpha}^{(N_\alpha,L_\alpha)}(m_0^0, m_1^1, ..., m_{L_\alpha-1}^{L_\alpha-1}) \) is a known formula called as confluent Lauricella hyper-geometric function, which was define in [5]. The parameters \( y_{\alpha}^n \) shown in (12) are defined as equal to the ratio of the amount of fading \( m_i^l \) to the corresponding average SINR, \( \Sigma_{\alpha} \alpha_{\alpha}, \) of the \( l-th \) combining receiver finger at the \( \alpha \)-th antenna, i.e., \( y_{\alpha}^n = m_i^l / \Sigma_{\alpha} \alpha_{\alpha} \). For the negative exponential MIP with power decay factor, \( \delta \), from the mentioned equations shown in channel model section, the average power of the \( l-th \) path can be written as shown in [4].

4. BER Analysis

4.1. System BER with Un-correlated Channels

The coherent BPSK (binary phase shift keying) is applied to the modulator at the post-detector. It is known that the conditional BER of that in AWGN channel is given in [3]. By means of the random process means the average BER, \( \mathbb{P}_{\text{BER}} \), of the M-D combining receiver with un-correlated branch for an MC-CDMA system working in Nakagami-\( m \) fading channel is given as

\[
\mathbb{P}_{\text{BER}} = \int_{0}^{\infty} \mathbb{P}_{\text{BER}}(\gamma) \cdot f_\gamma(\gamma) d\gamma
\]

(13)

Therefore, the system BER of an MC-CDMA system with un-correlated combining branch and CFO, \( \mathbb{P}_{\text{BER}} \), in (13) is to be determined as an equation function of Lauricella multivariate hyper-geometric and expressed as

\[
\mathbb{P}_{\text{BER}} = \frac{1}{\Gamma(\frac{1}{2} + \sum_{n=1}^{N_\alpha} \sum_{l=0}^{L_\alpha-1} m_i^l)} \left[ \prod_{n=1}^{N_\alpha} \prod_{l=0}^{L_\alpha-1} (y_{\alpha}^n)^{m_i^l} \right] \times \gamma^\left[-\sum_{n=1}^{N_\alpha} \sum_{l=0}^{L_\alpha-1} m_i^l \right] \Phi_{\alpha}^{(N_\alpha,L_\alpha)}(m_0^0, m_1^1, ..., m_{L_\alpha-1}^{L_\alpha-1}).
\]

(14)

4.2. System BER with Correlated Channels

Consider the correlation coefficient, \( \rho_{\alpha \beta} \), between \( y_{\alpha}^n \) and \( y_{\beta}^n \) can be presented as

\[
\rho_{\alpha \beta} = \frac{\text{Cov}(y_{\alpha}^n, y_{\beta}^n)}{\sqrt{\text{var}(y_{\alpha}^n) \cdot \text{var}(y_{\beta}^n)}}, \quad 0 \leq \rho_{\alpha \beta} \leq 1
\]

(15)

where \( i, \alpha = 1, 2, ..., A \), and \( n = 0, 1, ..., L_\alpha - 1 \). From the results shown in [8], it is found that the CHF (characteristic function) of the instantaneous SINR, \( \phi_\delta(t) \), is given as

\[
\phi_\delta(t) = \prod_{i=0}^{L_\alpha-1} [I_{\alpha} + t(A - \Sigma_{\alpha} \alpha') + \sum_{l=0}^{L_\alpha-1} \prod_{i=0}^{L_\alpha-1} [I_{\alpha} + t(y_{\alpha}^n)^{-1} \lambda_i^l]]^m
\]

(16)

where \( I_{\alpha} \) is the \( A_{\alpha} \times A_{\alpha} \) identity matrix, \( \Sigma_{\alpha} \alpha' \) the determinant operator, and the matrices \( A' \) and \( c' \), \( l = 0, 1, ..., L_\alpha - 1 \), are \( A_{\alpha} \times A_{\alpha} \) diagonal matrices with entries \( \Sigma_{\alpha} \alpha' / m_i = (y_{\alpha}^n)^{-1} \) and \( A_{\alpha} \times A_{\alpha} \) positive definite matrices, respectively. Now, we can check with the case of the values of the eigenvalues with \( \lambda_i^l = 1, \alpha = 1, 2, ..., A \), \( \gamma_{\alpha}^l = 0, 1, ..., L_\alpha - 1 \), which represents the independent fading among the receive antennas. By means of the known equivalent (1-\( z \)-*\( z \)) = \( F_{\gamma}([c; -1; z]) \) [8], and (16) can be written as

\[
\phi_\delta(t) = \prod_{i=0}^{L_\alpha-1} F_{\gamma}(m_i, z; \lambda_i^l - \frac{\lambda_i^l}{y_{\alpha}^n})
\]

(17)

By the same way, the previous equation is adopted to obtain the average BER in the case of un-correlated branch, for obtaining the determination of average BER for the case of correlated. By substituting (17) into (16), the CHF then becomes as

\[
\phi_\delta(t) = \left( \frac{1}{1 + \gamma_{\alpha}^l} \right)^{L_\alpha} \prod_{i=0}^{L_\alpha-1} \left[ \gamma_{\alpha}^l \right] \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right] \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right]
\]

(18)

\[
\times \left( \frac{1}{1 + \gamma_{\alpha}^l} \right) \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right] \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right]
\]

where the Barnes-Mellin contour-type integral [5, p. 43],

\[
F_{\gamma}(g_z; -h; t) = \frac{F_{\gamma}(g_z)}{1 + \gamma_{\alpha}^l} \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \Gamma(g_z + s) ds
\]

(19)

, has been applied in calculation of (18). It then follows that the pdf of \( \gamma \), which is in the case of correlated fading among combining fingers with the same path delay in spatially separated antennas, is given by

\[
f_\gamma(\gamma) = \frac{1}{\Gamma(A_{\alpha} \cdot \sum_{l=0}^{L_\alpha-1} m_i^l)} \left[ \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right] \right] \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right]
\]

(20)

\[
\times \left( \frac{1}{1 + \gamma_{\alpha}^l} \right) \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right] \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right] \prod_{i=0}^{L_\alpha-1} \prod_{l=0}^{L_\alpha-1} \left[ \frac{1}{1 + \gamma_{\alpha}^l} \right]
\]

where the restriction is considered with \( m_i^l = m_i \) for \( \alpha = 1, 2, ..., A \). The average BER in the spatially correlated
Nakagami-\textit{m} fading channel, $P_{\text{BER}}$, for an MC-CDMA system with combining receiver accompanied by CFO becomes as

$$
P_{\text{BER}} = \frac{\Gamma\left(\frac{1}{2} + A \cdot \sum_{l=0}^{L} m_l \right)}{2\sqrt{\pi} \Gamma(1 + A \cdot \sum_{l=0}^{L} m_l) \prod_{l=0}^{L} \left(\frac{y_l}{A \cdot \sum_{l=0}^{L} m_l + \gamma}\right)^{y_l}}
\times F_{\text{pdf}}(\frac{1}{2} + m_1, m_2, \ldots, m_L; 1 + A \cdot \sum_{l=0}^{L} m_l)
$$

where $A = \frac{\gamma}{\gamma_0}$ and $\gamma_0$ is the local oscillator frequency deviation.

5. Numerical Results and Discussion

The system performance influenced by the phenomenon of CFO is shown in Fig. 1, in which the applied parameters are set as, $N = 128$, $K = 25$, $L = L_0 = 4$, and $\delta = 1$, respectively. It is obviously understood that the more CFO values the worse system performance of the MC-CDMA system is, nevertheless how many the antenna number is. Moreover, the reason for the influence of CFO becomes constant after SNR is greater than about 10 dB, which is claimed that the interference (including CFO) will not dominate the behavior of system performance after the SNR increase an beyond a fixed level. On contrast the results in Fig. 1 to that of in Fig. 2 where the results of average BER function of user’s number, $K$, are presented. By adopting the same parameters in Fig. 1, but the SNR=15dB is fixed now. Same as to the results shown in Fig. 1, the curves shown in Fig. 2 can be observed that it is not only the better system performance can be obtained by reducing the CFO, but the addition of a second antenna can offer considerable improvement on the average BER performance.

6. Conclusion

The effect of CFO phenomenon is inspected in the paper for an MC-CDMA system based on the OFDM techniques. It is known that the CFO occurs in the environments where the estimation of the frequency in oscillator is not complete between the transmitter and the receiver. We try to adopt much more antenna number for promoting the capability of frequency estimation so as to try to reduce the effect of CFO.

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References


