A Consideration of Noise Cancellation by using PCA-ICA Method with Delay Estimation

Toshiaki Yokoda¹, Mohammad Reza Alsharif³, Mahdi Khosravy¹
¹Department of Information Engineering, University of the Ryukyus
Senbaru 1, Nishihara, Okinawa, Japan
E-mail: k078521@eve.u-ryukyu.ac.jp, alsharif@ie.u-ryukyu.ac.jp, mahdikhosravy@yahoo.com

Abstract: The noise cancellation technologies are useful for speech recognition and other applications. There are some kind of methods for cancellation of background noise. In this paper, the desired signals are separated from background noise by using proposed PCA-ICA method (Principal Component Analysis, Independent Component Analysis). The proposed PCA-ICA method requires several number of observed signals that is, the same as the number of sources. The noise signal can be removed in the same way as BSS (Blind Source Separation). We have done experiments (two-observation one-source one-noise, with delay) and evaluated the results.

1. Introduction

Usually, some different speech signals that are recorded with PCM are mutually independent. Independent component analysis (ICA) uses only the fact that "Source signals are mutually independent". On the other hand, Principal Component analysis (PCA) calculates the correlation between two sources and searching the decorrelation transform. These two methods perform different result for blind source separation because of differences in statistics. Figure 1 shows the histogram of Gaussian noise and audio signal. Audio signals are mostly super-gaussian distribution, and there simultaneous distribution is shown in figure 2 for two audio signals. We can recognize the two orthogonal axes in this scatter plot. PCA can not recognize these two orthogonal axes, but it has important function for decorrelation process. Our research uses PCA-ICA method for separation of the audio source signal from background noises.

2. Problem Definition

We have two source signals that are statistically independent, and their number is the same as the number of observation signals. There are different attenuation when signals transmit to each microphones from each sources. Usually, we call the transfer matrix as "Mixing matrix" \( A = (a_{11}, a_{21})^T \). Then, observation signals \( x = (x_1, x_2)^T \) are represented as

\[
x = As.
\]

(1)

Where \( s = (s_1, s_2)^T \) is source signals. We can find "Unmixing matrix" \( W = ([w_{11}, w_{21}]^T, [w_{12}, w_{22}]^T) \) easily if we have the information about mixing matrix \( A \).

\[
y = Wx = WAs = A^{-1}As = s
\]

(2)

Where \( y = (y_1, y_2)^T \). However, BSS problem has no information about mixing matrix \( A \) and source signals. And the problem assumes only that source signals are mutually independent. The method are already established to solving the BSS problem [1].

In our research, we defined the new problem as the observation signals include delay. The position of microphones and sources are symmetric as shown in Figure 3. Now, we define \( d_1 \) as distance between desired signal source and microphone 1, and also between noise source and microphone 2. \( d_2 \) is distance between desired signal source and microphone 2, and also between noise source and microphone 1.
Figure 3. The problem of Blind Source Separation with delay.

The difference of samples between coming signals from two sources to each microphone is \( D \). For example, at sampling frequency = 11025Hz, \( d_1 = 2m \) and \( d_2 = 3.5m \), the difference of samples \( D \) should be about 50-samples. In this problem, the observation signals \( x(t) = (x_1(t), x_2(t))^T \) are represented as below:

\[
\begin{align*}
x_1(t) &= a_{11}s_1(t) + a_{12}s_2(t - D) \\
x_2(t) &= a_{21}s_1(t - D) + a_{22}s_2(t)
\end{align*}
\]  

Also this formula can be represented as follows:

\[
x = As = \begin{bmatrix} a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}
\]  

Figure 4 and Figure 5 show an example of the source signals and observation signals.

\[\text{Figure 4. Desired signal and noise signal}\]

\[\text{Figure 5. Two observed signals.}\]

3. Proposed Method

The proposed method is constructed by three steps for solving the defined problem.

3.1 Delay Estimation and Synchronization

The first step measures the number of samples of delay between two observed signals by using cross-correlation measure. The cross-correlation is defined as

\[
R_{x_1 x_2}(\tau) = \sum x_1(t)x_2(t - \tau)
\]

In this problem, \( R_{x_1 x_2} \) has two peaks as shown in Figure 6. These peaks imply samples of delay. Assume these peaks can be measured is \( \tau_1, \tau_2 \). Then, we generate delayed observed signals \( x_1^\tau(t) = x_2(t - \tau_1) \) and \( x_2^\tau(t) = x_2(t - \tau_2) \). Finally, \( x_{\tau_1} = (x_1, x_1^\tau)^T \) and \( x_{\tau_2} = (x_1, x_2^\tau)^T \) will be used for unmixing procedures.

\[\text{Figure 6. Cross-correlation between } x_1 \text{ and } x_2\]

3.2 Decorrelation with PCA

Second step makes whitening matrix for \( x_{\tau_1} \) and \( x_{\tau_2} \) using PCA. Here eigenvector \( e = (e_1, e_2) \) of covariance matrix \( C_x^-1 = E[x_{\tau_1} x_{\tau_1}^T] \), and \( \Lambda = \text{diag}(\lambda_1, \lambda_2) \) is diagonal matrix of eigenvalue of \( C_x^-1 \). The matrix \( V \):

\[
V = \Lambda^{-\frac{1}{2}} e^T
\]
is "Whitening matrix" that can make uncorrelated signal. \( V \)
is used for the determining of initial value of \( W \).

### 3.3 Searching for \( W \) with ICA

Third step searches unmixing matrix \( W \) classically with
Kullback-Leibler divergence [2] is defined below.

\[
KL(W) = -H(Y; W) + \sum_i H(Y_i; W)
\]

Here, \( H(Y; W) \) is entropy of joint probability, \( H(Y_i; W) \) is
entropy of the marginal probability. \( y_1 \) and \( y_2 \) are mutually
independent means that \( KL(W) \) is reaching to zero. We used
steepest descent method for searching optimum \( W \). \( \phi \) is par-
tial derivatives of a approximated probability density function,

\[
\phi(y) = -\left( \frac{\partial \log p(y_1)}{\partial y_1}, \frac{\partial \log p(y_2)}{\partial y_2} \right)^T
\]

and the \( W \) will be converged by update rule below.

\[
W_{t+1} = W_t + \mu(I - \phi(y)y^T)W_t
\]

If \( W \) is unmixing matrix, unmixed signals \( y_1, y_2 \) should be
reached into source signals. Because if coefficients of \( a_{11}w_{11} \)
and \( -a_{21}w_{12} \) is almost same, and \( x_1, x_2 \) are exactly match-
ing to \( s_1 \). Then \( s_1 \) of unmixed signals \( y \) is canceled. The
remaining output \( y_1 \) or \( y_2 \) reaches to the one of source sig-
als. The same procedure also will be performed for \( x_2 \).

### 3.4 Block Diagram

Figure 7 shows the block diagram of proposed method.

![Block diagram of proposed method](image)

We must pay attention to the permutation problem. The two
signals are permuted by step2 and step3.

### 4. Experiment and Evaluation

We compared our proposal method and the classical method
for blind source separation with delay. A music source and
Gaussian-noise source are used for this experiment (See Figure 4).
In this experiment, we define the delay \( D = 50 \)
samples. The separation by our proposal method with delay
estimation exhibits better result than the classical method
without delay estimation.

![Result: BSS by our proposal method](image)

![Result: BSS by classical method](image)

Figure 8 shows the result of unmixing by the proposed
method and source signal. Figure 9 is the result of the clas-
sical method. There are obviously differences between two
methods results.

### 4.1 Asymmetric Position

On the another situation that the positions of microphones and
sources are asymmetric, It will also work well. Now we de-
fine the asymmetric transfer function,

\[ x_1(t) = a_{11}s_1(t - D_1) + a_{12}s_2(t - D_2) \]  
\[ x_2(t) = a_{21}s_1(t - D_3) + a_{22}s_2(t - D_4) \]  

(11)  
(12)

Here, \( D_1 \neq D_2 \neq D_3 \neq D_4 \). In this situation, we can find two peaks from cross-correlation (Figure 10) as same as symmetrical situation. The noise cancellation also has been performed in this situation, and a satisfactory result obtained the same as symmetric one.

4.2 Function of PCA

In this research, we discovered that whitening process by PCA has very important function in the case of mixture with delay makes high correlation between the observation signals. Figure 11 shows scatter plot of real situation. Assume the \( D_i = 0 \), the observation signals have only one peaks in cross-correlation. we can not find sources without ICA in this way.

Figure 12 shows scatter plot without delay. However, the delay should be included in real situation. Then the decorrelation process becomes useful method.

5. Conclusion

Blind source separation is difficult for convoluted observation signals [3] [4], and also for our proposed method. The classical method can not separate the signals that are convoluted mixture and or delay. On the other hand, our method that can estimate delay is more useful for the signals include delay.

References