Abstract—The properties of the combinatorial frequency generation and wave scattering by periodic stacks of nonlinear passive semiconductor layers are explored. It is demonstrated that the nonlinearity in passive weakly nonlinear semiconductor medium has the resistive nature associated with the dynamics of carriers. The features of the combinatorial frequency generation and the effects of the pump wave scattering and parameters of the constituent semiconductor layers on the efficiency of the frequency mixing are discussed and illustrated by the examples.

I. INTRODUCTION

Enhancement of nonlinear activity has been a long-standing goal in nonlinear optics. The recent theoretical and experimental investigations of nonlinear photonic crystals (PhC) have indicated significant potential for advances toward this goal in various applications such as waveform control, signal amplification and frequency conversion between different bands in nonlinear parametric processes [1-6]. The main distinctive features of nonlinear PhC are associated with the opportunity of engineering the dispersion and nonlinearity by altering the geometry and arrangements of constituent particles and their ensembles. Dispersion has profound impact on the nonlinear response of the medium despite being an essentially linear characteristic. It has crucial impact on the phase matching, which determines the efficiency of distributed nonlinear wave interactions. As the result, the frequency conversion efficiency may be increased by several orders of magnitude in the specially designed PhCs. For example, it was demonstrated in [7] that in the case of harmonic generation in semiconductor photonic crystal the intensity of the transmitted second harmonic (SH) reaches its maximum when the wavelengths of both the pump and SH waves are near the electromagnetic bandgap edges. The high density of modes at the band edge of PhC provides more favourable quasi-phase matching conditions for harmonic generation. However, it is necessary to note that the SH generation enhancement at the band edges is considerably impaired by the increased losses.

Combinatorial frequency generation by mixing pump waves of two different frequencies provides alternative means for frequency conversion. The earlier studies of the layered structures have demonstrated that its efficiency significantly varies with thickness of individual nonlinear layers and can be enhanced at the higher order Wolf–Bragg resonances [8]. The parametric analysis of the periodic stacks of binary anisotropic nonlinear dielectric layers has shown that in contrast to SH and third harmonic generation in the PhCs, the spectral band edges do not increase the combinatorial frequency generation efficiency for the refracted waves. However, it was shown that the frequency conversion efficiency can be significantly boosted at Wolf–Bragg resonances occurring only at special combinations of the pump wave frequencies, incidence angles and the layers’ constitutive parameters [9].

The aim of this paper is to explore the mechanisms of the combinatorial frequency generation in the periodic stacks of nonlinear semiconductor layers illuminated by plane waves of two tones incident at dissimilar oblique angles. The solution of the problem is obtained in the approximation of three-wave mixing process in weakly nonlinear layers taking into account the dynamics of charges in the semiconductor layers. The main features of the frequency mixing in the stacks of nonlinear passive semiconductor layers are examined and the fundamental role of carrier collisions in the combinatorial frequency generation is discussed.

II. NONLINEAR SCATTERING BY SEMICONDUCTOR STRUCTURE

We consider a PhC structure composed of the periodic binary semiconductor layers of thicknesses $d_1$ and $d_2$, and infinite extent in the $x$- and $y$- directions. The total thickness of the periodic stack is $L=N(d_1+d_2)$, where $N$ is the number of unit cells. The stack is surrounded by linear homogeneous medium with the dielectric permittivity $\varepsilon_0$ located at $z \leq 0$ and $z \geq L$. It is illuminated by two plane waves of frequencies $\omega_1$ and $\omega_2$ incident at angles $\Theta_1$ and $\Theta_2$, respectively, as shown in Fig. 1.

The nonlinearity of semiconductor layers, considered here, is assumed to be associated with microscopic motion of charges. A self-consistent formulation of the scattering problem for the combinatorial frequency generation involves simultaneous
solution of Maxwell’s equations and the current continuity equations, describing the nonlinear dynamics of charges
\[
\begin{align*}
\text{rot}\mathbf{H}_j & = \frac{\varepsilon_{ij}}{c} \frac{\partial \mathbf{E}_j}{\partial t} + 4\pi \mathbf{J}_j, \\
\text{rot}\mathbf{E}_j &= -\frac{1}{c} \frac{\partial \mathbf{H}_j}{\partial t}, \\
\frac{\partial \mathbf{j}_j}{\partial t} + \text{div} \mathbf{j}_j &= 0,
\end{align*}
\]
\[
\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial z}\right) H_{ij}(\omega_i) + k_i^2 \varepsilon_j(\omega_j) H_{ij}(\omega_i) = -\frac{e}{mc} \times
\]
\[
\times \frac{\varepsilon_{ij} \varepsilon_j(\omega_j)}{\left(\varepsilon_i(\omega_i) + iv_j(\omega_j)\right) \left(\varepsilon_i(\omega_i) + iv_j(\omega_j)\right)} \left[\frac{1}{\varepsilon_j(\omega_j)} - \frac{1}{\varepsilon_i(\omega_i)}\right] \times
\]
\[
\times \left(\frac{\partial H_{ij}(\omega_i)}{\partial x} \frac{\partial H_{ij}(\omega_i)}{\partial x} - \frac{\partial H_{ij}(\omega_j)}{\partial x} \frac{\partial H_{ij}(\omega_j)}{\partial x}\right),
\]
\[
\text{where } k_i = \omega_i/c, \quad l = 1, 2, 3 \quad \text{and} \quad \varepsilon_j(\omega) = \varepsilon_{ij} \left[1 - \frac{\varepsilon_{ij}^2}{\omega(\omega + iv_j)}\right],
\]
\[
\varepsilon_{ij} \text{ is the permittivity associated with the semiconductor lattice and the plasma frequency } \omega_{pj} = \sqrt{4\pi^2 \varepsilon_{0} j / (\varepsilon_{ij} m)}.
\]

Inspection of equation (2) reveals the fundamental property of the frequency mixing process in weakly nonlinear passive semiconductor medium. Namely, it is driven by the resistive nonlinearity dependent on collision frequency \(v_j\), cf. (2).

The solution of inhomogeneous equation (2) is composed of the partial and general solutions which can be represented in the form
\[
\begin{align*}
H_{ij}^{(0)}(\omega_i, x, z) &= \left(A_{ij}^{e} e^{i\omega_2 t} + A_{ij}^{d} e^{-i\omega_2 t} + D_{ij}^{e} e^{i\omega_3 t} + D_{ij}^{d} e^{-i\omega_3 t}\right) \\
+ &D_{ij}^{e} e^{i\omega_2 t} + D_{ij}^{d} e^{i\omega_3 t}) e^{i\omega_1 t + i\mathbf{k} \cdot \mathbf{r}},
\end{align*}
\]
\[
\text{where } k_i^{(1)} = k_i^{(1)} \pm k_i^{(2)}, k_i^{(1)} = \sqrt{(k_i^2 - k_i^2 + \varepsilon_0 \omega_i)}, \quad j = 1, 2; \quad l = 1, 2, 3; \quad \text{and } k_i = k_i \sqrt{\varepsilon_0} \sin \Theta_i \text{ are related by the phase synchronism condition for the three-wave mixing process}
\]
\[
k_i^{(1)} = k_i^{(1)} + k_i^{(2)}.
\]

The amplitude coefficients \(A_i^{e, d}\) are associated with the general solution of (2) and are determined by the continuity of the tangential field components at the layer interfaces. The coefficients \(D_i^{e, d}\) in the partial solution of (2) are expressed in terms of the refracted field amplitudes in each layer at the pump wave frequencies \(\omega_1\) and \(\omega_2\)
\[
\begin{align*}
D_{ij}^{e, d} &= \pm \alpha_{e, d} \beta_j \frac{B_{ij}^{e, d}(\omega_1)}{\left(k_i^{(1)} - k_i^{(2)}\right)},
\end{align*}
\]
\[
\text{where } \alpha_{e, d} = \frac{v}{mc} \left(\frac{1}{\varepsilon_1(\omega_1) - \varepsilon_1(\omega_2)\omega_2}\right) + 
\]
\[
\pm \gamma_j = -\left(k_i^{(1)} k_i^{(2)} + k_i^{(2)} k_i^{(3)}\right).
\]

Superscript \(q\) identifies the unit cell number in the stack. The coefficients \(B_{ij}^{e, d}(\omega_{q, l})\) are the field amplitudes in \(j\)th layer of the \(q\)th period at the incident wave frequencies \(\omega_{q, l}\).

The amplitudes \(F_{r,l}\) of the waves of the combinatorial frequency \(\omega_{r}\) scattered by the stack of semiconductor layers into the surrounding linear media \(F_{r, l}\) at \(z < 0\) and \(F_{r, l}\) at \(z > L\) are obtained by enforcing the boundary conditions of the field continuity at the layer interfaces \(z = 0, L\). The modified transfer matrix method [9] is finally used to find the closed-form expressions for the nonlinear scattering coefficients of the finite periodic stack of the binary nonlinear semiconductor layers. The resulting analytical solutions for the coefficients \(F_{r, l}\) enable us to examine the mechanisms of nonlinear scattering by the periodic semiconductor stacks and gain insight in the features of the associated three-wave mixing process.

III. RESULTS AND DISCUSSIONS

The numerical simulations based upon the analytical solution of (2) illustrate here the properties of TM waves of combinatorial frequencies generated by the periodic stacks of
alternating semiconductor layers. The constituent layers of InSb and GaAs with thicknesses \(d_1=0.07\) mm and \(d_2=0.03\) mm and the following parameters have been used for simulations: \(\varepsilon_{11}=16.8\), \(\alpha_1=10^{12}\) s\(^{-1}\), \(\varepsilon_{12}=12.9\), \(\alpha_2=2\times10^{13}\) s\(^{-1}\). The media surrounding the stack have permittivity \(\varepsilon_0=1\).

Comparison of Fig. 2 and Fig. 3 shows good correlation between \(|F_{s\perp}(\alpha_1)|\) and \(|R(\alpha)|\). Indeed, \(|F_{s\perp}^2|\) reach their peaks at the frequencies corresponding to the minima of \(|R(\alpha)|\) while the peak magnitudes vary in accord with \(|R(\alpha)|\). Although such a relation between \(|F_{s\perp}(\alpha_1)|\) and \(|R(\alpha)|\) is similar to that in the stacks of nonlinear dielectric layers, their mechanisms of frequency mixing fundamentally differ. Namely, the nonlinearity of semiconductor layers is directly related to the losses inflicted by collisions of carriers. Therefore in contrast to the dielectrics where losses reduce the efficiency of nonlinear mixing, the peak magnitudes of \(|F_{s\perp}^2|\) grow in semiconductor layers with the collision frequency \(\nu_{1,2}\) as illustrated by Fig. 3.

The intensity of combinatorial frequency \(\omega_\perp=\omega_1+\omega_2\) directly depends on the pump wave magnitudes as follows from (5). Therefore it is instructive to analyse first the pump wave linear reflectance, \(R(\alpha)\), from the respective periodic stacks. Fig. 2 displays \(|R(\alpha)|\) of a TM wave incident at angle \(\Theta_1=30^\circ\) on the stack containing \(N=8\) unit cells.

At low frequencies \(\omega<\omega_{1,2}\) both semiconductors are opaque, and the stack exhibits a bandgap where \(|R(\alpha)|\) approaches unity that is especially evident at \(\nu_{1,2}=0\). In the intermediate band \(\omega_{1,2}<\omega<\omega_{1,2}\) (grey shaded area in Fig. 2), one layer is opaque and the other is translucent. The two narrow spikes of the resonance transparency appear here and they can be attributed to the plasmons of so-called “acoustic” and “optical” types in periodic stacks of semiconductor layers. In the short wavelength limit, their frequencies asymptotically converge to \(\omega_{ps}=\sqrt{\omega_{11}^2+\omega_{12}^2}\), where \(\omega_{11,2}=\omega_{11,2}\sqrt{\varepsilon_{11,2}}\). As \(\nu_{1,2}\) increases, the respective plasmonic resonance dips of \(|R(\alpha)|\) are smudged and become shallower, as seen in Fig. 2.

At \(\omega<\omega_{1}\), both semiconductor layers are translucent and \(|R(\alpha)|\) varies almost periodically as in the dispersive dielectric layers and \(|R(\alpha)|\) has minima (maxima of \(|\mathcal{T}(\alpha)|\)) at the frequencies corresponding to Wolf-Bragg resonances of the whole stack. It is noteworthy here that the reflectance peaks are stronger affected by \(\nu_{1,2}\) than \(|R(\alpha)|\) minima, cf. Fig. 2.

The intensities \(|F_{s\perp}(\alpha_1)|^2\) of the combinatorial frequency generated by the stack with the same parameters as above are shown in Fig. 3 for the translucent semiconductor layers at variable frequency \(\omega>\omega_{1}\) of a pump wave incident at \(\Theta_1=30^\circ\). The other pump wave, incident at \(\Theta_2=60^\circ\), has a fixed frequency \(\omega=1.775\times10^{13}\) s\(^{-1}\) such that \(|R(\omega)|=0\).

Fig. 2. Reflectance of plane TM wave incident at \(\Theta_1=30^\circ\) on the periodic stack of \(N=8\) binary semiconductor layers at different collision frequencies: \(\nu_{1,2}=0\) — black dashed line; \(\nu=0.1\times10^9\) s\(^{-1}\), \(\nu=0.2\times10^9\) s\(^{-1}\) — red dash-dot line; \(\nu=1\times10^{10}\) s\(^{-1}\), \(\nu=0.2\times10^{11}\) s\(^{-1}\) — blue solid line

Fig. 3. The field intensity at frequency \(\omega_\perp=\omega_1+\omega_2\) radiated in the reverse \((F_{s\perp})^2\) and forward \((F_{s\perp})^2\) directions of the \(z\)-axis at \(\Theta_1=30^\circ\); \(\Theta_2=60^\circ\); \(N=8\); \(d_1=0.07\) mm, \(d_2=0.03\) mm and \(\alpha_1=2\times10^{13}\) s\(^{-1}\)
that the peak magnitudes of $|F_{r,1}(\omega)|$ increase with the collision frequencies $\nu_{1,2}$ in the layers at all resonances, except the one at $\omega = \omega_{b1}$, as shown in Fig. 4.

![Fig. 4](image-url)

**Fig. 4.** The field intensity at frequency $\omega = \omega_1 + \omega_b$ radiated in the reverse $(|F_{r1}|)$ and forward $(|F_{f1}|)$ directions of the $z$-axis at $\Theta_0 = 30^\circ$; $\Theta_2 = 60^\circ$; $N=8$; $d_1=0.07 \text{ mm}$, $d_2=0.03 \text{ mm}$ and $\omega_b = 1.775 \times 10^{12} \text{ s}^{-1}$.

When both pump wave frequencies are in the band $\omega_{b2} < \omega < \omega_{b1}$, the semiconductor layer of type 1 is opaque. Nevertheless, a resonance of the combinatorial frequency occurs at $\omega = \omega_{b1}$. Fig. 5 shows that $|F_{r2}|$ has a maximum at $\omega = 0.75 \times 10^{12} \text{ s}^{-1}$ that corresponds to $\omega = \omega_{b1}$. Moreover, the peak intensities here are about two orders of magnitude higher than those in Fig. 4. Even a stronger resonance of $|F_{r,1}(\omega)|$ occurs at $\omega \approx 1 \times 10^{12} \text{ s}^{-1}$ that is well correlated with the plasmon resonance of the pump wave shown in Fig. 2. The peak intensities $|F_{r,1}(\omega)|$ at these resonances are enhanced by the plasmon absorption and may increase the intensity of the combinatorial frequency generation for several orders of magnitude as compared with that attainable at frequencies $\omega_{b2} < \omega < \omega_{b1}$ where both the semiconductor layers are translucent.

**IV. CONCLUSION**

The properties of nonlinear scattering and combinatorial frequency generation by the periodic stack of binary semiconductor layers illuminated by a pair of TM waves of frequencies $\omega_{b1,2}$ incident at dissimilar oblique angles have been investigated. The closed form solution of the respective boundary value problem including the nonlinear dynamics of charges has been obtained and analysed numerically. It has been shown that the mixing processes in such structures are driven by the collision of charges and depend on the dynamics of carriers. The effects of the pump wave dissipation and scattering, and parameters of the constituent semiconductor layers on the efficiency of the combinatorial frequency generation are discussed and illustrated by the examples.

![Fig. 5](image-url)

**Fig. 5.** The field intensity at frequency $\omega = \omega_1 + \omega_b$ radiated in the reverse $(|F_{r1}|)$ and forward $(|F_{f1}|)$ directions of the $z$-axis at $\Theta_0 = 30^\circ$; $\Theta_2 = 60^\circ$; $\nu_1 = 1 \times 10^{11} \text{ s}^{-1}$; $\nu_2 = 0.1 \times 10^{11} \text{ s}^{-1}$; $\nu_1 = 0.1 \times 10^{12} \text{ s}^{-1}$; $\nu_2 = 0.2 \times 10^{12} \text{ s}^{-1}$; $N=8$ and $\omega_b = 0.25 \times 10^{12} \text{ s}^{-1}$.

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**REFERENCES**


