Performance Improvement of an Inversion Algorithm for the Reconstruction of Lossy Dielectric Cylinder

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Abstract—This paper considers the performance improvement of an iterative inversion algorithm of reconstructing the relative permittivity of a lossy dielectric cylinder using the modified frequency-hopping technique and the adaptive regularization method. A cost functional is defined as the sum of a residual error term in the scattered electric field and an additional regularization term. Then the electromagnetic inverse scattering problem can be treated as an optimization problem, where the relative permittivity of the object is determined by minimizing the cost functional. Numerical results for lossy dielectric circular cylinders show the performance comparison between the proposed method and the conventional one.

I. INTRODUCTION

In the fields of engineering and medicine, it is very important to characterize material properties, shape, size, and location of an unknown object from the measurements of the scattered electromagnetic wave. In recent years, a variety of reconstruction methods in frequency domain have been developed to solve the electromagnetic inverse scattering problem [1]-[16]. In most of these techniques, a regularization method has been utilized to circumvent the ill-posedness of the problem.

It is the purpose of this paper to accelerate the rate of convergence of an iterative inversion algorithm of estimating the relative permittivity of a lossy dielectric cylinder. The object situated in a homogeneous background medium is assumed to be illuminated with multi-frequency cylindrical electromagnetic waves in microwave region. As is well known [1], the inverse scattering problem can be formulated as the solution to a nonlinear integral equation for a contrast function, which is expressed by the relative permittivity of the object. We define a cost functional as the sum of a residual error term in the scattered electric field and an additional regularization term. Thus the inverse scattering problem is reduced to an optimization problem where the contrast function is determined by minimizing the cost functional. The optimization problem can be solved by using the modified frequency-hopping technique [15] and the adaptive regularization method. In the iteration process of reconstruction, the relaxation calculation is performed by using the conjugate gradient method [1],[19],[20]. Then one can obtain an iterative formula for getting the contrast function.

Computer simulations are made for lossy dielectric circular cylinders to confirm the effectiveness of the proposed method.

II. FORMULATION OF THE PROBLEM

Consider a lossy dielectric cylinder with relative permittivity \( \varepsilon_\alpha(\rho) \) and the cross section \( D \) located in a homogeneous background medium of relative permittivity \( \varepsilon_b \), as shown in Fig. 1. The object is illuminated with E-polarized cylindrical wave with electric field \( \mathbf{E}_p^o (= \mathbf{u}_x E_p^o(\theta; \rho)) \) corresponding to the \( p \)-th frequency of \( f_p \), where \( \mathbf{u}_x \) is the unit vector in the z-direction and \( p = 1, 2, \ldots, P \). Line sources generating the incident waves are placed at points with polar coordinates \( (\rho, \theta + \pi) \). Measurements of the scattered electric field \( \mathbf{E}_p^s (= \mathbf{u}_x E_p^s(\theta; \rho)) \) for each illumination are made at the observation points with polar coordinates \( (\rho, \phi) \). A contrast function, which characterizes the material property of the object, is expressed as

![Fig. 1. Geometry of the problem.](image-url)
\( c(\rho) = \varepsilon_\alpha(\rho) - \varepsilon_b. \)

The z-component of the total electric field \( E_p^z(\mathbf{c}; \theta; \rho) \) inside the object, which is written as the sum of the incident electric field and the resultant scattered electric field, is represented as a solution to the linear integral equation,

\[
E_p^z(\mathbf{c}; \theta; \rho) = E_p^i(\theta; \rho) + k_p^2 \iint_D c(\rho') E_p^z(\mathbf{c}; \theta; \rho') \cdot G_p(\rho; \rho') \, d\rho', \quad \rho \in D, \tag{2}
\]

where \( k_p \) indicates the free-space wavenumber for the p-th frequency of \( f_p \), and \( G_p(\rho; \rho') \) is the two-dimensional Green’s function for the background medium given by

\[
G_p(\rho; \rho') = -\frac{i}{4} H_0^{(2)}(\sqrt{k_p} |\rho - \rho'|), \tag{3}
\]

where \( H_0^{(2)}(\cdot) \) is the zeroth-order Hankel function of the second kind.

The z-component of the scattered electric field outside the object may be written as

\[
E_p^s(\mathbf{c}; \theta; \rho) = k_p^2 \iint_D c(\rho') E_p^s(\mathbf{c}; \theta; \rho') \cdot G_p(\rho; \rho') \, d\rho', \quad \rho \notin D. \tag{4}
\]

The inverse scattering problem discussed here can be treated as the solution to the nonlinear integral equation for the contrast function by replacing \( E_p^s(\mathbf{c}; \theta; \rho) \) in the left-hand side of (4) with the scattered electric field measured.

The discrete polar angles of incidence are \( \theta = \theta_l \) for one frequency, where \( l = 1, 2, \ldots, L \). For each angle of incidence, the scattered electric fields are measured at observation points with polar angles \( \phi = \phi_m \) along a circle of radius \( \rho \), where \( m = 1, 2, \ldots, M \). The square investigation domain \( S \) containing the object and the background medium is subdivided into \( N \times N \) elementary square cells for numerical computations. The method of moments with pulse-basis functions and point matching [17] is employed to discretize (2) and (4).

Let us define a residual error in the scattered field at the p-th frequency of \( f_p \) as follows:

\[
F_p(c) = \sum_{q=1}^{p} \sum_{l=1}^{L} \sum_{m=1}^{M} \left[ E_p^s(\mathbf{c}; \theta_l; \phi_m) - \tilde{E}_p^s(\theta_l; \phi_m) \right]^2, \tag{5}
\]

where \( \tilde{E}_p^s(\theta_l; \phi_m) \) and \( E_p^s(\mathbf{c}; \theta_l; \phi_m) \) denote the scattered electric fields measured and calculated for an estimated contrast function, respectively. Note that the measured data are simulated by solving the direct scattering problem for the true contrast function based on the CG-FFT method [18]. When (5) is used in the frequency-hopping process, the approach is called the modified frequency-hopping technique [15]. Note that the conventional frequency-hopping technique employs

\[
F_p(c) = \sum_{l=1}^{L} \sum_{m=1}^{M} \left[ E_p^s(\mathbf{c}; \theta_l; \phi_m) - \tilde{E}_p^s(\theta_l; \phi_m) \right]^2 \tag{6}
\]

as the residual error in the scattered field.

Next, we define a cost functional as the sum of \( F_p(c) \) and an additional regularization term,

\[
\hat{F}_p(c) = F_p(c) + \alpha_p \iint_D |f_p(\rho)|^2 \, d\rho, \tag{7}
\]

where \( \alpha_p \) is a regularization parameter, which is determined by minimizing the absolute value of the radius of curvature of the GCV function [8]. In the conventional regularization method, \( f_p(\rho) \) may be written as

\[
f_p(\rho) = c_{p,n}(\rho) - c_{p,n-1}(\rho), \tag{8}
\]

where \( c_{p,n} \) is the n-th estimate of the contrast function for the p-th frequency. On the other hand, \( f_p(\rho) \) is expressed as

\[
f_p(\rho) = c_n(\rho) - \tilde{c}_n(\rho) \tag{9}
\]

in the adaptive regularization method. Note that \( \tilde{c}_n(\rho) \) is adaptively changed depending on the value of

\[
c_{p,n+1}(\rho) = c_{p,n}(\rho) + \lambda_p n d_{p,n}(\rho). \tag{11}
\]

Introducing the cost functional \( \hat{F}_p(c) \), the inverse scattering problem is cast into an optimization problem where \( c(\rho) \) is found by minimizing the functional. Application of the conjugate gradient method [1],[19],[20] and the modified frequency-hopping technique [15] to the minimization of \( \hat{F}_p(c) \) provides a formula for estimating \( c(\rho) \) at the p-th frequency of \( f_p \). Then the contrast function is iteratively obtained from the equation

\[
\delta_p = \sum_{q=1}^{p} \sum_{l=1}^{L} \sum_{m=1}^{M} \left[ \tilde{E}_p^s(\theta_l; \phi_m) \right]^2 \tag{12}
\]

The direction \( d_{p,n}(\rho) \) and the step size \( \lambda_p n \) may be obtained from the Polak-Ribière-Polyak method and a univariate search technique, respectively [19],[20].

The reconstruction scheme terminates if the convergence criterion for the relative residual error in the scattered electric field is finally less than a prescribed value at the highest frequency of \( f_P \). The relative residual error is now defined as

\[
\delta_\varepsilon^{(\alpha)} = \sqrt{\int_S |\varepsilon_\alpha(\rho) - \varepsilon_\alpha(\rho)|^2 \, d\rho} \tag{13}
\]

The relative mean squared errors in the real and imaginary parts of the relative permittivity are also written as

\[
\delta_\varepsilon^{(\alpha)} = \sqrt{\int_S \frac{|\varepsilon_\alpha(\rho) - \varepsilon_\alpha(\rho)|^2 \, d\rho}{\int_S \varepsilon_\alpha(\rho) \, d\rho}} \quad \alpha = R, I.
\]
where $\varepsilon_1^{(R)}$ and $\varepsilon_1^{(I)}$, respectively, indicate the real and imaginary parts of the estimated relative permittivity, and $\tilde{\varepsilon}_a^{(R)}$ and $\tilde{\varepsilon}_a^{(I)}$ denote the real and imaginary parts of the true value.

For convenience, the reconstruction scheme with the adaptive or the conventional regularization method combined with the conventional frequency-hopping technique is called the method I or the method II. Furthermore, the scheme with the adaptive regularization method and the modified frequency-hopping technique is called the method III.

### III. Numerical Examples

Computer simulations are made for lossy dielectric circular cylinders located in free space. We employ six frequencies of $f_1=1\text{GHz}$, $f_2=2\text{GHz}$, $f_3=3\text{GHz}$, $f_4=4\text{GHz}$, $f_5=5\text{GHz}$, and $f_6=6\text{GHz}$. The number of illuminations is $L = 18$ for one frequency, and the number of measurements of the scattered electric field is $M = 18$ for each illumination. The line sources and the observation points are equally spaced along a circle of radius $2\lambda$, where $\lambda$ is the free-space wavelength at the highest frequency of $f_6$. The current frequency hops to the next higher-frequency when $\xi_{F_0}$ takes the values less than $10^{-2}$ two times in succession. The investigation domain $S$ is the $2\lambda \times 2\lambda$ square region. In the iteration process of reconstruction, the initial guess of the contrast function is zero, i.e., the relative permittivity of the object is the same as the value of the background medium.

Figures 2 and 3 illustrate the reconstructed results of the real part and the imaginary part of the relative permittivity of the object with the radius of $0.8\lambda$ and relative permittivity of $5.0 - j0.25$. Now the investigation domain $S$ is uniformly subdivided into $48 \times 48$ elementary square cells. The solid and the dotted lines present the results for the method I and the method II, respectively. The final convergent solutions for $\delta_0 < 5 \times 10^{-4}$ can be obtained after 109 and 142 iterations for the method I and the method II, respectively. For reference, the true profiles of the real and the imaginary parts of the relative permittivity are also shown by the thin solid lines in Figs. 2 and 3. The relative mean squared errors in the relative permittivity are $\delta_1^{(R)} = 3.47 \times 10^{-2}$, $\delta_1^{(I)} = 0.26$ and $\delta_2^{(R)} = 3.47 \times 10^{-2}$, $\delta_2^{(I)} = 0.38$ for the method I and the method II, respectively.

Figures 4 and 5 show the reconstructions of the real part and the imaginary part of the relative permittivity of a stratified circular cylinder consisting of two concentric homogeneous layers based on the method I and the method III. The radii and the relative permittivities of the two layers are, respectively, 0.4$\lambda$, 0.8$\lambda$ and 7.0$\lambda$, 5.0$\lambda$. The investigation domain $S$ is now uniformly subdivided into $56 \times 56$ small square cells. The final convergent solutions for $\delta_0 < 5 \times 10^{-4}$ are the results after 155 and 181 iterations for the method I and the method III, respectively.

In this case, the relative mean squared errors in the relative permittivity are $\delta_1^{(R)} = 3.35 \times 10^{-2}$, $\delta_1^{(I)} = 0.43$ and $\delta_2^{(R)} = 3.07 \times 10^{-2}$, $\delta_2^{(I)} = 0.15$ for the method I and the method III, respectively.

It is seen from Figs. 2–5 that the rate of convergence obtained for the modified frequency-hopping technique and the adaptive regularization method is faster than that for the conventional method to achieve the same accuracy in the reconstructed profile.

### IV. Conclusion

Performance improvement of an iterative inversion algorithm based on the modified frequency-hopping technique and the adaptive regularization method has been investigated. In the iteration process of reconstruction, the relaxation calculation is performed by using the conjugate gradient method. We present an iterative inversion algorithm of estimating the relative permittivity of the
Simulated results are given for lossy dielectric circular cylinders using the multifrequency scattering data. The results confirm that the proposed method provides high-quality reconstructions with the property of faster convergence than the conventional method. Study on the reconstruction in noisy case remains a topic for further study.

REFERENCES


