Circular Antenna Arrays for Near-Field Focused or Multi-Focused Beams

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Abstract—The generation of focalized beams in the near-field (NF) with a circular antenna array are presented. The phase distribution for phase array launchers are first studied and explained. Gaussian excitation producing the same focalization effect in the NF are also discussed. Perfectly collimated beams in the NF and also generated with circular antenna arrays are discussed and theoretical examples are presented. Those collimated beams are Bessel ones of order 0 and higher.

I. INTRODUCTION

Electromagnetic waves in the radiofrequency range are an excellent support for telecommunication or radar applications where distances between transmitters and receivers are important. Naturally, antennas or array antennas required for those applications were and are still thoroughly studied. Currently, the increasing development of millimeter and terahertz (THz) waves opens the way for new and interesting applications for which propagation distances are much shorter, such as RFID, high-resolution hyperthermia for biological tissues, confined-beam THz spectroscopy, non-ionizing imaging, surface inspection, secure short-range high-speed point-to-point communication or even quasi-optical millimeter-wave laboratory experimental benches [1, 2]. These applications require the production of focused or collimated beams in the near-field region of the transmitters. For the former, focalization can be achieved, like in optics, with RF lens placed in front of a horn antenna or directly with large apertures or reflectors antennas [3, 4]. Antenna arrays operating at different wavelengths were also proposed for different applications [5-10]. Antenna arrays have the advantage of being low volume and lightweight but they are also well adapted to produce collimated beams [1]. The latter are multi-focused beams: instead of being focalized in a point, electromagnetic waves are focused along a straight line. Those beams can be produced with conical lens [11], cylindrical waveguides [12] or leaky-wave modes generated by a circular cavity [13]. However, antenna arrays produce much longer beams than the latter and can also generate several kind of collimated beams.

The paper presents some possibilities offered by antenna arrays to produce focalized or collimated beams in the near-field of an array. In section II, two different approaches for the focalization on a single point are presented: phase arrays and Gaussian excited arrays. In section III, the generation of Bessel beams of order 0 and higher order are presented and illustrated with theoretical examples.

II. CIRCULAR ANTENNA ARRAYS CONFIGURATION AND MODELING

Circular arrays are composed of radiating elements such as dipole or patch antennas arranged along a specific lattice comprised inside a circular region of radius $R_{array}$. The highest filling ratio lattice type to fit the circular geometry is the hexagonal one but other choices like the square lattice or the polar lattice are also possible. These circular array configurations are illustrated in Fig. 1. Because of its circular symmetry, the polar lattice probably leads to the simplest feeding network design.

For a mono-focusing beam or a multi-focusing beam, the focalization point or region (case of a multi-focusing beam) is located in the near-field region of the array. Therefore, a good estimate of the greatest admissible distance from the array to that region is given by the classical far-field limit, $z_{ff}$ [14]:

$$z_{ff} = \frac{2D_{array}^2}{\lambda_0} = \frac{8R_{array}^2}{\lambda_0}$$

where $D_{array}$ is the array diameter and $\lambda_0$ is the free-space wavelength. Calling the dimension of the antenna elements of the array $D$, where $D$ is approximately $\lambda/2$, according to the first relation of (1) with $D$ replacing $D_{array}$, any point at a distance greater than $\lambda/2$ to an antenna element is considered to be in its far-field region. Thus, the distances from the array within the region

$$\frac{\lambda}{2} < z < \frac{8R_{array}^2}{\lambda}$$

are in the far-field of the individual antenna elements and in the near-field of the array antenna.

Consequently, at any point $P(x, y, z)$ inside the region given by (2), the electric field is simply the superposition of the
fields produced by all the individual antenna elements of the array:
\[
\vec{E} = \sum_{i=1}^{L} s_i \vec{E}_i(x, y, z)
\]  
(3)
where \( L \) is the number of antenna elements of the array, \( s_i \) is the complex amplitude excitation of the element \( i \) and \( \vec{E}_i(x, y, z) \) is the field produced by the antenna \( i \) at the point \((x, y, z)\). For dipole or patch antennas, analytical models of the radiated fields are found in antenna textbooks [14].

Fig. 1. Schematic representation of three lattices of the circular antenna array using patch antenna elements. The antennas are confined within a circle of radius \( R_{\text{array}} \). (a) Hexagonal lattice. (b) Square lattice. (c) Polar lattice.

III. FOCUSED BEAMS

The concept of NF focusing is very common in conventional optics. In contrast, it is rather uncommon in microwave antenna systems, where beams are generally formed in the far-field, such beams being referred to by optics practitioners as collimated or quasi-collimated beams. In the NF, fields launched by the transmitter first converge up to a small region, called the focal spot, and then diverge to infinity. The size of the launcher is an important parameter. The launcher needs to be few tens of wavelengths or even bigger to produce a small focusing spot. Smaller diameters also work but at the expense of greater focal spot. Moreover, the number of antennas will also impact the beam quality. A trade-off has to be done between the latter and the complexity of the feeding network. For practical applications, the concept of an electrically large array is thus more suited to shorter wavelength waves, such as millimeter-waves or THz waves.

A. Phased Arrays

A first possibility to produce a focal spot with an antenna array is to excite all the antenna elements with the same amplitude but with phases following a radial distribution, \( \rho \), in the plane of the array. The corresponding complex amplitude excitation distribution in (3) is:
\[
s_i = A e^{-ik_0\left(\rho^2 + f^2 - f\right)}
\]  
(4)
where \( A \) is the amplitude, \( k_0 \) is the wave number and \( f \) is the distance between the array and the center of the focal spot. The phase variation in (4) is easily established by geometrical optics, as shown in Fig. 2. A converging wave has spherical wave fronts. So, a wave radiated at point \( P_0 \) of the antenna array towards the focal point travels an extra-distance (indicated by a thick line in Fig. 2) that is compensated by the phase correction in (4). Another way of interpreting (4) is that all the waves radiated by the antenna elements combine in phase at the focal point [10].

Fig. 2. Schematic representation of the geometry to calculate the phase of (4) and coordinates axis used.

An advantage of using the polar lattice is the fact that all the antenna elements located at similar radial distance from the center (i.e. on a circle) have the same phase. So, for \( n \) circles, only \( n-1 \) phase shifters are required in the feeding network, like it is schematically shown in Fig. 3. Tunable focal length launchers are thus easily made with such a configuration. An example of the focalization achieved by a 128-element array (8 circles, 16 antennas per circle) is shown in Fig. 4.

Fig. 3. Schematic representation of the feeding network. All the discrete elements (phase shifters, power dividers, antenna elements) are connected with coaxial cables.

Fig. 4. Normalized power in the \( xOz \) plane produced by a circular phased antenna array made with 8 circular rows and 16 antennas per row, \( R_{\text{array}} = 300 \) mm (20\( \lambda_0 \)), \( f = 20 \) GHz and the focal length is 0.9 m.

B. Gaussian Arrays

One problem caused by a constant amplitude excitation is the unavoidable diffraction pattern that it produces and that results in a focal point surrounded by unwanted secondary lobes. This is particularly true for small array diameters. An illustrative example of this effect may be found in Fig. 2 of [2].

A first remedy to this issue is to taper the amplitude distribution of the antenna elements in such a manner that it decays to zero at the array edge, like it is often done in far-field arrays [15]. However, for near-field focusing beams, a more appropriate way to solve the problem is to use a
A gaussian amplitude excitation distribution. Then, both the phases and amplitudes are functions of $\rho$.

A gaussian beam, after passing through a converging lens, has curved wavefronts that focus the beam after the lens [16]. The idea is thus to excite the array with a magnitude function that realizes the same gaussian beam as the one in the plane tangent to the output face of the converging lens. The complex amplitude excitation is then:

$$s_i = A \frac{W}{\rho} \exp \left[ -\rho_j^2 \frac{1}{W^2} \right] \exp \left[ -ik_x \left( f + \rho_j^2 \frac{1}{2R} \right) + i \arctan \left( \frac{f}{\rho_j^2} \right) \right]$$

where $W_0$ is the radius of the desired focal spot, and $W$ and $R$ are given by:

$$W = W_0 \left[ 1 + \left( \frac{f}{z_0} \right)^2 \right]^\frac{1}{2}; \quad R = f \left[ 1 + \left( \frac{z_0}{f} \right)^2 \right]$$

where $z_0$ is the half-length of the focal spot and is related to $W_0$ by:

$$z_0 = \frac{W_0^2}{\lambda_0}$$

where $\lambda_0$ is the wavelength. The values of $f$ and of $W_0$ cannot be chosen in an entirely arbitrary fashion. Indeed, $s_i$ has to be small enough to prevent diffraction on the array edge for $\rho = R_{array}$, which implies that $R_{array}$ increases with increasing values of $f$ or decreasing values of $W_0$.

The same array as the one used in the previous section was designed at the same frequency and for the same focal length, but excited with a gaussian distribution, (5). Results are shown in Fig. 5. Differences with the results of Fig. 4 are almost undistinguishable but the size of the localization spot is slightly bigger and also closer to the array. In fact gaussian excitation does not seemed to present a real advantage when the array diameter is big enough. Its interest concerns mainly smaller arrays.

First described in their scalar form [19], Bessel beams have several vectorial representations depending on their polarization [20]. In the case of a linear polarization, it is:

$$E_x = -i c k \rho \alpha \left( k, \rho, \varphi \right) e^{i \left( \delta - \omega \right)}; \quad E_y = 0$$

where $\alpha$ is a proportionality constant, $k_\rho$ and $\beta$ are the radial and the longitudinal components of the wave vector $(k, \rho, \varphi)$, $\omega$ is the angular frequency, and $\rho$ and $\varphi$ are the polar coordinates in the $xy$-plane. The angle between $k_\rho$ and $\beta$ is the axicon angle, $\delta$ and is given by:

$$\delta = \arctan \left( \frac{k_\rho}{\beta} \right)$$

For small values of $\delta$, the longitudinal component of $E$ is small compared to the transverse one. The beam can thus be considerer as a scalar one where $E_x$ is equal to the scalar quantity.

The complex amplitude excitation required to generate a Bessel beam greatly depends on the element density of the antenna array. If it is dense enough, the complex amplitude excitation is simply the truncated function $J_\alpha(k, \rho, \delta)$. However, this lead to a very large number of antenna elements. With a sub-sampled lattice, the complex amplitude excitation geometry greatly departs from $J_\alpha(k, \rho, \delta)$ and becomes a function of the geometry of the lattice. Numerical method must be used to find it, as it is explained in [1].

An example of beam produced by a polar lattice array is shown in Fig. 7 while Fig. 8 shows the amplitude and the phase of the complex amplitude excitation used. The main lobe of the beam propagates up to 2.4 m without any diffraction.

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**Fig. 6.** Schematic representation of the multi-focusing property of Bessel beams.

**IV. MULTI-FOCUSED BEAMS**

**A. Bessel Beams of Order Zero**

Localized waves [17], whose simplest forms are Bessel beams, can also be launched in the near-field region of antenna array [1]. Bessel beams where first experimentally observed in the visible range at the output of a conical lens [18] but were first clearly described by Durand [19]. Conical lenses have the property of focusing light at several points along the central axis. Each point of the axis is the focal point of a ring section of the conical element, as can be seen in Fig. 6. The beam is thus multi-focused.

The E-field of a Bessel beam is described by a Bessel function of the first kind, $J_\alpha$, of order $\alpha$ often equal to 0. Such beam propagates without diffraction but have infinite energy, like a plane-wave. Truncated Bessel beams, however, have finite energy and still exhibit the non-diffracting property of their central lobe over the Rayleigh distance. Also, thanks to the multi-focused property, they have the capability of self-reconstructing after being diffracted by a shadowing object.
array were presented. For single point focusing, the excitation distribution is found with analytical formulas. In the case of multi-focusing, however, numerical methods must be used. Good quality focal spot or good quality beam can be obtained with under-sampled spherical antenna arrays.

REFERENCES


