Optimizing small particles for strong interactions with electromagnetic fields

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Abstract—In this presentation we will discuss the concept of electrically small “optimal metaparticles”. For the optimal particles the particle shape and material are chosen so that the particles gain the maximum possible reactive energy in the field of a given plane electromagnetic wave propagating or decaying in space where the particle is located. It appears that for maximizing response to propagating or evanescent waves of arbitrary polarization we need to make use of all four fundamental classes of bi-anisotropic particles: omega, chiral, moving, and Tellegen particles.

I. INTRODUCTION

Recently, it was noticed that the shape of small chiral particles (for example, helices) can be optimized in such a way that these optimal helices radiate waves of only one circular polarization, whatever is the polarization and orientation of the exciting field [1]. Later, it was shown that the optimal spiral shape corresponds to the maximum (or minimum) energy of the particle in a given plane-wave field [2], interacting with the wave in the most effective way. Furthermore, mixtures of optimal spirals realize effective media with the index of refraction \( n = -1 \) for one of the circular polarizations, while the same medium is transparent for the orthogonal circular polarization [3]. It has been therefore established that there exist particles, whose properties are optimal for interactions with circularly polarized waves. In the conference presentation [4] and in paper [5] we introduced the concept of optimal particles optimized for interactions with linearly polarized plane waves and noticed a possibility to generalize this concept for all fundamental classes of bi-anisotropic particles. In particular, it has been shown that the shape of omega-particles can be optimized so that they are most effectively excited by the fields of evanescent linearly propagating waves.

In this talk we will discuss the optimal particle types for interactions with propagating and evanescent waves of different polarizations (especially linear and circular). It will be shown that for each of the four main excitation types (propagating/evanescent, linearly/circularly polarized waves) there exist corresponding classes of bi-anisotropic particles (chiral/omega, moving/Tellegen) which can be optimized for the corresponding excitation. This study is restricted to composites formed by electrically small inclusions, modelled as electric and magnetic dipoles.

II. THEORY

It is obvious that in order to enhance the field-material interaction with an electrically and magnetically polarizable particle we need to make use of magnetoelectric interactions in the particle. That is, the particle should be bi-anisotropic, and not just a set of electric and magnetic dipole moments excited by respective external fields. Indeed, in the known optimal spiral [1], [2] the electromotive forces induced by electric and magnetic fields sum up in phase, this way enhancing the overall current induced in the spiral. It is important that for the orthogonal polarization these forces cancel out and the particle is not excited at all (invisible particle), which shows that the shape can be optimized not only to maximize the interaction effect but also to minimize it. With this in view, we assume that the optimal particle is a bi-anisotropic particle whose electric and magnetic moments are induced by both fields and write

\[
\begin{bmatrix}
  \mathbf{p}
  \\
  \mathbf{m}
\end{bmatrix} =
\begin{bmatrix}
  \pi_{ee} & \pi_{em} \\
  \pi_{me} & \pi_{mm}
\end{bmatrix}
\cdot
\begin{bmatrix}
  \mathbf{E}
  \\
  \mathbf{H}
\end{bmatrix}
\]  

(1)

Here \( \mathbf{p} \) and \( \mathbf{m} \) are the induced electric and magnetic dipole moments, respectively, and \( \pi_{ij} \) are the polarizability coefficients.

In view of potential applications in planar arrays and for simplicity we assume that the particle does not respond to external fields along a certain direction in space (along axis \( z \)). This means that the particle polarizabilities are two-dimensional dyadics defined in the plane transverse to the unit vector \( \mathbf{z}_0 \) along \( z \). Furthermore, we assume that the particle is uniaxial, so that its response is isotropic in the transverse plane, so that all the polarizability dyadics are linear combinations of the transverse unit dyadic \( \mathbf{T}_I = \mathbf{x}_0 \times \mathbf{y}_0 + \mathbf{y}_0 \times \mathbf{x}_0 \) and the vector-product operator \( \mathbf{T}_t = \mathbf{z}_0 \times \mathbf{T}_I \). Next, we write the general expression for the energy of the particle in a given external field \( \mathbf{E}, \mathbf{H} \)

\[
W = -\frac{1}{2} \text{Re}\{\mathbf{p} \cdot \mathbf{E}^* + \mathbf{m} \cdot \mathbf{H}^*\}
\]  

(2)

assuming that the exciting field is an arbitrary polarized plane wave travelling (propagating) or decaying (evanescent) in an arbitrary direction. Using the Cartesian coordinate system, we write for the transverse field components

\[
\mathbf{H}_t = H_s \mathbf{x} + H_y \mathbf{y}, \quad \mathbf{E}_t = -\mathbf{Z} \cdot \mathbf{z}_0 \times \mathbf{H}_t
\]  

(3)
The wave impedance \( Z \) is a two-dimensional diagonal dyadic, whose two eigenvalues we denote as \( Z_{TE} \) and \( Z_{TM} \) for the TE- and TM-polarized (with respect to \( \phi_0 \)) field components. The stored reactive energy of the particle takes the general form

\[
W = -\frac{1}{2} \text{Re} \left\{ \alpha_{em}^2 |E|^2 \right\} + \frac{1}{2} \text{Re} \left\{ \alpha_{co}^2 |E|^2 \right\} \text{Im} \left\{ E_x E_y \right\}
\]

This expression allows us to understand what particle class can be optimized for exciting waves of different types. Clearly, in order to have room for optimization of particle excitation due to its bi-anisotropy, we need such particles for which the terms proportional to the field coupling coefficients are not zero. We note that for excitation by propagating waves the wave impedances \( Z_{TE} \) and \( Z_{TM} \) are real while for evanescent waves they are purely imaginary. Furthermore, the magneto-electric and electromagnetic coupling coefficients for bi-anisotropic particles of different classes are related as \( \alpha_{em}^m = \pm \alpha_{em}^c \), \( \alpha_{em}^m = \pm \alpha_{em}^t \) (see Table I).

### TABLE I: Classification of uniaxial bi-anisotropic particles

<table>
<thead>
<tr>
<th>Omega</th>
<th>Chiral</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{taw} = \alpha_{co}^c ) I _1</td>
<td>( \pi_{taw} = \alpha_{co}^c ) I _1</td>
</tr>
<tr>
<td>( \pi_{mm} = \alpha_{mm} ) I _1</td>
<td>( \pi_{mm} = \alpha_{mm} ) I _1</td>
</tr>
<tr>
<td>( \pi_{em} = \alpha_{em} ) I _2</td>
<td>( \pi_{em} = \alpha_{em} ) I _2</td>
</tr>
<tr>
<td>( \alpha_{em}^m ) is imaginary</td>
<td>( \alpha_{em}^m ) is imaginary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moving</th>
<th>Tellegen</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{mm} = \alpha_{mm} ) I _1</td>
<td>( \pi_{mm} = \alpha_{mm} ) I _1</td>
</tr>
<tr>
<td>( \pi_{em} = \alpha_{em} ) I _2</td>
<td>( \pi_{em} = \alpha_{em} ) I _2</td>
</tr>
</tbody>
</table>

This consideration leads to conclusion that for each type of the exiting wave there is an appropriate class of bi-anisotropic particles which can be optimized for strong (or negligible) interactions with this incident field. A simplified version (where some of the cells refer only to the axial propagation) of this classification is shown in Table II. In the talk we will present the optimal particles for various excitations and discuss the extreme properties of optimal particles and their arrays in scattering and absorption of electromagnetic waves.

### III. Conclusion

In summary, we have introduced the concept of optimal particles and composite materials for interactions with propagating or evanescent plane waves of different polarizations. The optimal particles extract maximum power from electromagnetic waves (for a given concentration and the resonant frequency of inclusions). Apparently, these particles are also the optimal radiators of power. It is expected that the use of the optimal optical particles in the design of artificial electromagnetic surfaces and materials will allow optimization of electromagnetic performance of various devices (antennas, absorbers, sensors, lenses, etc.)

### REFERENCES


