Transmission Line Approach for Transformation Electromagnetics

Atsushi Sanada, Tsutomu Nagayama
Graduate School of Science and Engineering, Yamaguchi University
2-16-1 Tokiwadai, Ube, Yamaguchi 755-8611, Japan
1 sanada@ieee.org
2 r021vj@yamaguchi-u.ac.jp

Abstract—We present a physical equivalent circuit model for 2D full-tensor anisotropic materials to realize transmission-line-based invisibility cloaks. With the equivalent circuit, it is shown that the equivalent circuit expresses well with full-tensor anisotropic material and diagonal and off-diagonal components of the permeability tensor physically correspond to the self- and mutual inductances per unit length in the unit cell, respectively. A cylindrical cloak is designed and its inherent broadband operation is confirmed by SPICE circuit simulations.

I. INTRODUCTION

The concept of transformation optics or transformation electromagnetics has been presented based on the material interpretation of transformation of the coordinate system to realize highly flexible electromagnetic wave control [1,2]. Based on the concept, a 2D cylindrical cloak has been readily implemented with circularly arranged split-ring resonators (SRRs) and its invisibility cloaking operation of reducing reflected and refracted waves has been successfully confirmed experimentally [3]. However, the cloak exhibits an inherent limitation of narrow bandwidth and significant losses due to its resonant operation.

On the other hand, a transmission line approach is a potential route to realizing artificial materials with wideband and low loss operation. Several transmission-line-based implementations for invisible cloaks have been presented and demonstrated so far [4,5], however, the design and implementations are complex and not very intuitive.

Here, we present a simple physical equivalent circuit model for 2D full-tensor anisotropic materials to realize transmission-line-based invisibility cloaks. With the equivalent circuit, a direct connection between the permeability tensor components and circuit parameters can be derived and one can easily design and implement the 2D full-tensor anisotropic materials with an insight of the operation. As a cloaking implementation example, a cylindrical cloak is designed and its operation is confirmed by SPICE circuit simulations.

II. EQUIVALENT CIRCUIT FOR FULL-TENSOR ANISOTROPIC METAMATERIALS

Let us express an equivalent circuit for a square region $\Delta d \times \Delta d$ of a 2D full-tensor anisotropic metamaterial as in Fig. 1. Here, Ports $X_1$ and $Y_2$ are magnetically coupled with a mutual inductance $M/2$ with self-inductances $L_x/2$ and $L_y/2$ in the $x$- and $y$-branches, respectively. For symmetry, Ports $X_2$ and $Y_1$ are also magnetically coupled with the same coupling scheme. By apply Kirchhoff’s current and voltage laws to the equivalent circuit with an infinitesimal limit and comparing the equations to Maxwell’s equations for a 2D full-tensor anisotropic material, we can derive the effective permeability tensor of the equivalent circuit model as

$$\mu_{\text{eff}} = \begin{pmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{pmatrix} = \begin{pmatrix} L'_{xx} & M' \\ M' & L'_{yy} \end{pmatrix}, \tag{1}$$

where $L'_{xx}$ and $L'_{yy}$ are self inductances per unit length in the $x$- and $y$-branches in the unit cell and $M'$ is a mutual inductance per unit length, i.e., $L'_{xx} = L_x/\Delta d$, $L'_{yy} = L_y/\Delta d$, and $M' = M/\Delta d$. It is worth mentioning from Eq. (1) that the equivalent circuit expresses well with full-tensor anisotropic material and, more importantly, the diagonal components of the permeability tensor physically correspond to the self-inductance per unit length and the off-diagonal components also physically correspond to the mutual inductance per unit length.
III. INVISIBILITY CLOAK DESIGN EXAMPLE

In order to confirm the validity and operation of the equivalent circuit, a 2D cylindrical cloak [3] is designed and tested. Here, we consider a coordinate transformation in which a circular region of a radius \( r \) \((0 < r < b)\) is transformed to a cylindrical region of a radius \( r' \)(\(a < r' < b\)):

\[
r' = a + \frac{b - a}{b} r.
\]

First, permittivity tensor components in the Cartesian coordinate system are calculated by the material interpretation of transformation [1,2] assuming the TE wave propagation, and circuit parameters in a discretized cloak region of \( a = 10 \Delta d < r' < b = 20 \Delta d \) by the unit cell of Fig. 1 are obtained based on Eq. (1). The outer region of \( r' > b \) are assumed to be vacuum and the parameters \( L_x = L_y = \mu_0 \) and \( M = 0 \) are given. A voltage source is connected to a node near the cloak and the outer boundaries are terminated with the Bloch impedance of the periodic structure.

Then, complex voltage distributions in the computational region are calculated by a SPICE simulator. The results are given in Fig. 2 (a). As a comparison, complex voltage distributions without the cloak are also shown in Fig. 2 (b). In these calculations, the wavelength of the source is chosen as \( \lambda = 50 \Delta d \). It can be seen from these figures that the reflected and refracted waves are reduced significantly by the cloak. The residual reflected and refracted waves are considered to be mainly due to discretization errors in the staircase approximation of the cylindrical cloak in the model. It can be shown by similar simulations that the lower the frequency, the less the reflected and refracted waves.

Figure 3 shows similar calculation results for the case with \( \lambda = 20 \Delta d \). It can also be seen from the figure that, although the amount of reflected and refracted waves is increased by comparing the results in Fig. 2, the cloak still operates well and the broadband operation of the cloak is confirmed.

IV. CONCLUSION

A simple physical equivalent circuit model for 2D full-tensor anisotropic materials has been presented to realize transmission-line-based invisibility cloaks. It has been shown that the equivalent circuit expresses well with full-tensor anisotropic material and diagonal and off-diagonal components of the permeability tensor physically correspond to the self- and mutual-inductances per unit length in the unit cell, respectively. A cylindrical cloak has been designed and its inherent broadband operation has been confirmed by SPICE circuit simulations.

REFERENCES