ANALYSIS OF LIGHTING DISCHARGE MODELING IN THE
WAVELET DOMAIN

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Abstract: The aim of this paper is to develop a model for describing the interaction of lightning discharge phenomena with electrical systems and electronic devices, with special focus on the high frequency spectrum, which is thought to depend on kinks and branching of the lighting discharge. Through modeling and simulation of a tortuous path discharge we obtain the typical jagged-shaped waveform of the electric field at ground level. Thereafter, by applying wavelet analysis to the electric field signal we extract, through compression techniques, the high-frequency spectral content of the electric field, providing enhanced time-frequency localization. This work is the starting point for modeling the induced electromagnetic interference on a victim system, in the time-frequency domain.

Key words: Lightning Return Stroke Electric Field, Wavelet Analysis, Multi-resolution Analysis.

1. Introduction

In the last years we have observed an increasing interest in issues of electromagnetic compatibility, i.e. the study of induced disturbances on electric and electronic systems and devices [1-3].

In the attempt to better describe the typical jagged-shaped waveforms of electromagnetic disturbances, we transform the signal through wavelet orthogonal bases [4], which are more properly correlated with the electromagnetic signal, than the sinusoidal Fourier basis, thereby reducing the number of significant coefficients for signal reconstruction. In fact, wavelets analysis shows advantages over the traditional Fourier basis representation, especially for detecting signal discontinuities, sharp changes in the first or second derivatives, and in general phenomena in which the change is localized in time.

By applying wavelet transforms to the radiated electric field signal, the excitation of the victim system, we transform the radiated signal in the time-scale plane, thereby providing time-localisation. Through suitable choice of the wavelet basis and compression techniques we obtain a dramatic reduction in the information content of the signal, thereby extracting the high-frequency content of the electromagnetic interferences. Respect to their former investigations on this subject with the wavelet approach [5], the authors extended their analysis beyond the piecewise-linear Haar basis, thereby improving the signal time-frequency localization.

By dealing with the electromagnetic field radiated by a tortuous and branched lightning return stroke [6], we were able to fit the simulation results to the experimental ones, and through wavelet analysis, to extract significant information, correlating this information content to the physics and geometry of the problem. By splitting the contribution of the straight channel from the contribution of the tortuous and branched channel, we extract the high frequency content of the signal, due to both straight and tortuous channel geometry, and localize this disturbance in time.

The analysis in the time-scale domain yields insight in the characterisation of the lighting phenomena.

2. Modelling the lightning discharge

The first step in our study is to simulate, through a suitable numerical code, a typical lighting discharge path, characterised by a tortuous channel geometry, with kinks and branches – Fig.1.

Figure 1: lightning discharge path of the parametric-stochastic simulator

For this purpose the authors [7] have developed a 3D parametric-stochastic simulator, which generates the discharge path and computes the currents in the tortuous channel and in each branch.
From the spatial current distribution we were able to calculate the vertical component of the radiated electric field, computed at ground level in the Fraunhofer region. The simulations yield results that fit with the experimental results, evidencing the typical jagged shape due to branches and kinks.

3. Wavelet analysis
Our investigations deal with complete orthogonal wavelet bases for the $L^2(\mathbb{R})$ space, representing the typical signal generated by a tortuous and branched lighting discharge path. Wavelet vectors are formed by dyadic scaling and shifting the mother wavelet $\psi(t)$:

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k) \quad j, k \in \mathbb{Z}$$

Discrete wavelet transform can be therefore considered a decomposition at increasingly finer resolution levels of subspaces $W_j$ that span the $L^2(\mathbb{R})$ space. The scale in the wavelet domain is an extension of the concept of frequency in the Fourier domain, due to the time-resolution of the wavelet coefficients. The scale-level $L$ is made up of the set of wavelets of same frequency. Through sampling at rate $S$, we obtain $L = \log_2(S)$ frequency-levels.

4. Optimal wavelet selection
Through wavelet analysis, we aim at extracting the time-located high-frequency content of the discharge radiated field. In our study we selected the Daubechies wavelets (Fig.2), as the most suitable family of wavelet bases for our purpose, for several reasons:

![Figure 2: Mother wavelets of the Daubechies family, from $2^{nd}$ to $10^{th}$](image)

First, these wavelets have optimal resolution in the time-axis, due to their compact support, with support length equal to 2N-1, where N is the wavelet order.

Second, these wavelet family satisfies certain moment-vanishing conditions: the $dbN$ wavelet has N vanishing moments in the time domain. As a result, the regularity increases with the order, thereby providing better frequency-resolution with higher order bases, in accordance with the uncertainty principle. Finally, these wavelets have a characteristic jagged shape, similar to that of the analyzed interference signals, or in other word, the vector of these wavelet bases have a significant correlation with the interference. In fact, wavelet coefficients may be interpreted as correlation coefficients, where the correlation is maximum as the signal energy is locally equal to the corresponding wavelet energy.

The drawback of these wavelets is that they have no explicit expression except for the $db1$, or Haar wavelet. Another drawback, which is not significant for our application, is that these wavelet have a very pronounced asymmetry. However, among the wavelets proposed by Daubechies, there are also nearly symmetrical wavelets, referred to as $symN$.

In figure 3 we represent the coefficients in the time-scale plane, in which the horizontal and vertical axes represent respectively the sampled time axis and the scale level. Each coefficient, due to its time-scale resolution, is represented by a rectangle in the plane, whose colour represents the coefficient magnitude. By noting the higher amplitude coefficients with darker colours, we obtain a bright matrix structure, crowded with near to zero wavelet coefficients. By simply removing near-zero values, a 48% compression is achieved. By zeroing out the coefficients below a target global threshold, we obtain a compressed spectrum which still holds most of the energy of the original signal.

![Figure 3: time-scale representation of the wavelet coefficients of the electric field, using a $db2$ basis, near zero, 80%, 88% and 92% compression.](image)

Figure 3 shows that the wavelet spectrum consists of few significant coefficients, whereas near to zero coefficients can be neglected. The resulting sparse spectrum, is suitable both for noise time-frequency localisation as well as for optimal signal reconstruction.

For non-periodic signal, such as the discharge signals, zero-padding setting is required. In fact, by convolving the signal through the discrete wavelet
transform, we introduce boundary coefficients which are not represented in the graphics above. As a result, the visualized compression ratio results to be higher than the effective and measured one. By reverse-transforming in the time-domain, we reconstruct the radiated field (Fig. 4).

![Figure 4: 80% compression of the radiated field – db2 in the time domain vs. original signal](image)

### 5. Optimal wavelet compression

Among several orthogonal bases, we have to define a criteria for selecting the most suitable wavelet basis for each analysed signal or for each type of signals. The main criteria is to choose those bases which, for a given compression ratio, retains most of the energy of the original signal, both on a global and a local scale. Most wavelet transforms yield good results in terms of the total energy retained. On the other hand we observe different results in terms of local correlations. We therefore compute the energy in the residuals between the original signal and the signal reconstructed after the compression. Alternatively, by leveraging on the conservation properties of the Fourier transform, we operate directly in the Fourier domain (Fig.5).

![Figure 5: residual analysis via Fourier transform of the db2 and db3 compression (88%)](image)

The *a priori* approach, on the other hand, leverages on the wavelet auto-similarity properties, since all the wavelet of a certain basis are derived by the mother wavelet through scaling and shifting, we can somehow correlate *a priori* the most suitable wavelet basis to the signal at hand. In particular, for the Daubechies wavelet, we consider the property of regularity increasing with the wavelet order. As a result, we expect a better correlation with the signal discontinuities by adopting the db1 and db2 bases, and higher correlations with sinusoidal and smooth waveforms by selecting the db10-20 bases, as the zoom-in of the reconstructed signal shows (Fig. 6).

![Figure 6: zoom-in of the wavelet compressed signal (90%) via db2 (continuous) and db10 (dotted) bases vs. original signal (dashed)](image)

We note that the approximation via higher order wavelet bases results in smoother reconstructed waveforms. The lowest order basis (db2) better correlates with the jagged-shaped signal, and better describes the radiated-field discontinuities of branches and kinks of the tortuous channel. On the other hand, the higher order wavelets results in better correlation in case of smooth signals.

### 5. Selection Criteria

In the *a-posteriori* approach, by operating in the Fourier domain we do not properly leverage on the multi-resolution properties of the wavelets. Therefore we introduce a compression-quality indicator that allows us to directly operate in the wavelet domain. The ‘Weighted Average Compression Index’, defined as follows:

$$ WACI = \frac{1}{\sum_{l=1}^{L-1} w_{(L-l)} \sum_{k=1}^{L-1} \zeta_{(L-l),k} \cdot \frac{2^{l-1}}{2^{l-1}} + c_{(L-l)}} $$

where $z_j$ is the number of zeros for each level $j$ and $\{w_j = 2^{\alpha (L-1-j)}; j = 1, ..., L-1; \alpha \geq 0\}$ is a sequence of weights, that opportunely weights by-level compression ratios. If the sequence $w_j$ is set equal to the number of level coefficients ($\alpha=1$), it yields a level-independent compression ratio, while if the sequence is set to one ($\alpha=0$), it yields a time-independent compression ratio. For $\alpha > 1$ we select compressing criteria with focus on high-frequency coefficients.

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The additive $c_j$ sequence counts for the border effects imposed by zero-padding or in general non-periodic settings. For example, by comparing in the time-scale plane the results of two different compressions, through $db2$ and $sym3$ wavelets, with equal energy retained, we observe that the $sym3$ wavelet has less lower-level components, that means that $sym3$ converges faster to a significant representation of the original signal (Fig.7).

![Figure 7: 97% compression through $db2$ / $sym3$](image)

The same results could be difficult to infer via Fourier spectra (Fig.5). On the other hand, by applying the weighted compression index method, we find that the $sym3$ basis has a better weighted compression ratio for $\alpha>1$, due to the higher frequency-resolution, which yields a sparser time-scale spectrum at lower levels than the one of the $db2$ wavelet. As a result, a significant compression is achieved easily at the lowest level.

6. **Straight and tortuous discharge paths**

By transforming the interpolation of the electric field signal in the wavelet domain, we obtain sparse spectra (Fig.8), which are quite similar to the compressed ones in the left part of the time-axis.

![Figure 8: $db2$ and $db3$ spectra of the interpolated electric field signal](image)

These spectra can be thought as the ones radiated by the straight channel of equivalent energy. These spectra are more sparse than the one generated by the tortuous channel discharge, and has localised high-frequency components in correspondence of the signal peak. Since, high frequency components are the forcing functions of a line illuminated by an indirect lightning, in the field-to-transmission line modeling, it is important to extract this information in both cases of straight and tortuous channel. By comparing the corresponding spectra in the wavelet domain, we were able to extract the high frequency components due to branches and kinks. A non-negligible noise in 3rd and 4th wavelet levels is extracted (Fig.9).

![Figure 9: Original and 88% compressed spectra due to kinks and branches – $db3$ wavelet](image)

According to our analyses, the wavelet analysis results in a suitable tool for isolating mid and high-frequency components of the electric field radiated by a lightning discharge, correlating the line-induced noise to current peaks, kinks and branches.

7. **Conclusions**

In this paper we show how the wavelet transform is particular suitable for representing jagged-shaped disturbances with non-negligible high-frequency content, provided that the best wavelet bases is chosen. In particular, through Daubechies wavelets, we were able to represent the electric field radiated by a lightning return stroke of a tortuous channel discharge, as well as to correlate the field high-frequency spectrum to the tortuous and branched channel geometry of the lightning discharge.

**References**


