Analysis of Induced Current Density inside Grounded and Ungrounded Human Model Exposed to Electric Field

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Abstract: The surface-charge integral equation is applied for the grounded and the ungrounded human model placed at heights of 0.011, 0.128 and 1.23m above the ground under a uniform 60 Hz electric field of 1 kV/m. The calculations lead to the quantification of the characteristics of the internal current density inside the grounded and the ungrounded human model. The calculated results are demonstrated with the experimental results reported by Kaune et al. who used the 3-dimensional human model.

Key word: Human model, Electric field, Induced current density, Surface-charge integral equation.

1. Introduction

The health effect of small currents induced in a human body exposed to the power frequency electric field has been investigated since early 1970s. The introduction of extremely low frequency electric fields with human body has become an increasingly important subject since potential health hazards due to the electric fields emitted by extremely high-voltage power lines. The health effect of the weak current induced in the human body as a result of the interaction between human body and power frequency electric fields has been investigated. Chen et al. reported some results of the induced current densities in a homogeneous body of the realistic shape, but their calculations have not yet been done as to the current densities induced inside the ungrounded human model. Dawson et al. analyzed the induced current densities inside the grounded and ungrounded human model of the 3-dimensional shape, but such internal distributions are not accurately calculated as a function of the model heights above the ground.

In this paper, in order to analyze the effect of the human body placed at heights above the ground for the internal current distributions, the human model of the 3-dimensional shape is used. The surface-charge integral equation is applied for the grounded and the ungrounded human model placed at heights of 0.011, 0.128 and 1.23m above the ground under a uniform 60 Hz electric field of 1 kV/m. The calculations lead to the quantification of the characteristics of the internal current densities inside the grounded and the ungrounded human model. The calculated results are demonstrated with the experimental results reported by Kaune et al. who used the 3-dimensional human model.

2. Surface-charge integral equation

Physically, the body potential \( \phi_s(r) \), which is spatially constant over the body, can be considered as the sum of the potential \( \phi_s(r) \), which is maintained by the induced polarization charge \( \rho(r) \) inside the body, and the potential \( \phi_o(r) \), which is maintained by the impressed electric field. That is,

\[
\phi_s(r) = \phi_s(r) + \phi_o(r)
\]  \hspace{1cm} (1)

Using the quasi-static approximation and considering the ground image effect, \( \phi_s(r) \) can be expressed as

\[
\phi_s(r) = \frac{1}{4\pi\varepsilon_0} \int_{\Delta s'} \frac{\rho(r')}{|r - r'|} ds'
\]  \hspace{1cm} (2)

Where \( \Delta s \) is the body surface and \( \Delta s_i \) is the surface of the body image, \( r' \) is a field point on
the body surface, and r represents a source point on the body surface and the image surface. When equation (2) is point matched at the center of the nth subarea $r_{ni}$, it can be expressed as

$$\phi_{om} = \sum_{n=1}^{N} \frac{1}{4\pi\epsilon_0} \left[ \int_{\Delta S_n} \frac{\rho_n \, ds_n'}{|r_{ni} - r_n|^3} + \int_{\Delta S_{ni}} \frac{-\rho_n \, ds_n'}{|r_{ni} - r_n|^3} \right]$$

(3)

Where $\rho_n$ is the surface-charge density at the nth subarea $\Delta S_n$ and $r_n$ is a source point within $\Delta S_n$. The surface-charge density at the corresponding subarea $\Delta S_{ni}$ of the body’s image is $-\rho_n$ due to the image effect and $r_{ni}$ is a source point within $\Delta S_{ni}$. When $\phi_b$ is the human body potential, $\{\phi_{om}\}$ is equal to $\phi_b$. From equations (1) and (2), the next equation is obtained.

$$\sum_{m=1}^{N} \sum_{n=1}^{N} M_{mn} \rho_n + \sum_{n=1}^{N} \phi_{om} = \phi_b$$

(4)

Here,

$$M_{mn} = \frac{1}{4\pi\epsilon_0} \left[ \int_{\Delta S_n} \frac{ds_{n'}}{|r_{ni} - r_n|^3} + \int_{\Delta S_{ni}} \frac{-ds_{n'}}{|r_{ni} - r_n|^3} \right]$$

Assuming that the human body exist in the uniform electric field E, the potential $\{\phi_{om}\}$ is given as $\{\hat{h}_1 E, \hat{h}_2 E, \cdot \cdot \cdot, \hat{h}_N E\}$. Where, $\hat{h}_1, \hat{h}_2, \cdot \cdot \cdot, \hat{h}_N$ are the heights at the center of the subarea on the body surface. Equation (4) can be used to generate N simultaneous equations when m is varied from 1 to N, that is,

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & \cdots & M_{1N} \\ M_{21} & M_{22} & M_{23} & \cdots & M_{2N} \\ M_{31} & M_{32} & M_{33} & \cdots & M_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{N1} & M_{N2} & M_{N3} & \cdots & M_{NN} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \rho_N \end{bmatrix} + \begin{bmatrix} \hat{h}_1 E \\ \hat{h}_2 E \\ \hat{h}_3 E \\ \vdots \\ \hat{h}_N E \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_N \end{bmatrix}$$

(5)

The total net charge on the body is zero for the isolated case. That is,

$$\Delta S_1 \rho_1 + \Delta S_2 \rho_2 + \cdots + \Delta S_N \rho_N = 0$$

(6)

From equations (5) and (6), N simultaneous equations can be expressed in a matrix form as follow.

$$\begin{bmatrix} M_{11} - M_{1a} & M_{12} - M_{1a} & \cdots & M_{1N} - M_{1a} \\ M_{21} - M_{1a} & M_{22} - M_{1a} & \cdots & M_{2N} - M_{1a} \\ M_{31} - M_{1a} & M_{32} - M_{1a} & \cdots & M_{3N} - M_{1a} \\ \vdots & \vdots & \ddots & \vdots \\ M_{N1} - M_{1a} & M_{N2} - M_{1a} & \cdots & M_{NN} - M_{1a} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \rho_N \end{bmatrix} = \begin{bmatrix} (h_1 - h_a)E \\ (h_2 - h_a)E \\ (h_3 - h_a)E \\ \vdots \\ (h_N - h_a)E \end{bmatrix}$$

$$\begin{bmatrix} \Delta S_1 \\ \Delta S_2 \\ \Delta S_3 \\ \vdots \\ \Delta S_N \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \rho_N \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

(7)

For this case, the unknown surface charge density $\{\rho_n\}$ is determined from equation (7).

After the induced surface-charge density $\rho_i$ is determined, the induced electric field at the body surface $E_i$ is simply obtained from

$$E_i = \rho_i / \epsilon_0$$

(8)

One of the most important quantities concerning the body current density is $J_p$, which is defined as the current densities from the head top to the one part cross section area of the model as shown in Fig.1.

Fig.1. Explanatory diagram for calculating the current density in the model

$$J_p = j \omega \sum \rho \Delta S_i / s_p$$

(9)

Here, $\Sigma \Delta S_i$ and $\Sigma \rho$, are the model surface area and the induced charge density from the head top to one part cross section area of the model.

3. Electric field distributions

The quarter model of the human body is divided into 429 triangular elements as shown in Fig.2. The number of nodes in the division pattern is 273. In the case of the grounded and the ungrounded human model whose distance d (d: the distance between the bottom of the human foot and the ground: see Fig.1.) are 0.011, 0.128, and 1.23m, the current densities induced inside the human model are analyzed. As an example of the calculated results, Fig.3 and Fig.4 show the electric field distributions on the surface of the grounded and the ungrounded model (d= 0.128m). From Fig.3 and Fig.4, it is observed that the electric field strengths are occurred strongly at the top on the head part for the grounded model and for the ungrounded model, the electric field strengths become largest at the bottom on the model foot. Fig.5 shows the effect on the distance d for the electric fields at the top on the head part, the pelvis and the bottom on the foot of the human model. From this figure, the maximum electric field strength at the top of
the head part is approximately 22.5 times as strong as the external field.

Fig.2. Triangular division pattern on the human model.

Fig.3. Electric field strengths on the surface of the grounded human model.

Fig.4. Electric field strengths on the surface of the ungrounded human model (d=0.128m).

Fig.5. Relation between distance d and the electric field strengths at head top, pelvis and bottom on the model foot.

In the case that the distance d is 0.011m, the electric field strength at the bottom on the model foot becomes largest under all calculations. That value is approximately 75.7 times as strong as the external field. Similarly, it is seen from this figure that the electric field strengths on the pelvis are small in comparison with ones of the head top and the bottom on the foot.

4. Current density distributions

Fig.6 shows the characteristics of the current density distributions induced inside the human model of the 3-dimensional shape in the case of the grounded and ungrounded model whose distance d are 0.011, 0.128 and 1.23m

Fig.6. Induced current densities in the human model.

It is seen from this figure that the current densities induced inside the human model increase when the distance d decreases. The current densities induced inside the human model become large at the neck or the ankle part which close section areas are small.

5. Comparisons with the experimental results

The analytical results are compared with the experimental results induced inside the 3-dimensional human model reported by Kaune et al. as shown in Table 1. As an example, the analytical results are compared with the experimental ones for the grounded and ungrounded model whose distance d is 0.128m as shown in Fig.7 and Fig.8. Since the cross section areas of the experimental model differ from ones of the analytical model a little, the induced current densities induced inside the experimental model of are equivalently converted into ones of the analytical model as shown in Table 1. It is obvious from Table 1, Fig.7 and Fig.8 that the analytical results of the induced current densities at neck, chest pelvis and ankle agree well with the experimental ones. That is, the analytical results are demonstrated by the experimental results of Kaune et al. who used the
3-dimensional human model.

Fig.7. Comparison between the experimental results of Kaune et al. and the analytical results for the grounded human model.

Fig.8. Comparison between the experimental results of Kaune et al. and the analytical results for the ungrounded human model (d=0.128m).

Table 1. Comparison between the experimental and the analytical results at neck, chest, pelvis and ankle part.

<table>
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<tr>
<th></th>
<th>Height (m)</th>
<th>neck</th>
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<th>pelvis</th>
<th>ankle</th>
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<td>0.005</td>
<td>0.010</td>
<td>0.050</td>
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<td>Kaune et al.</td>
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<td>0.190</td>
<td>0.250</td>
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<tr>
<td></td>
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</tr>
</tbody>
</table>

6. Conclusions
In the case of the grounded and the ungrounded human model placed at the heights of 0.011, 0.128 and 1.23m, the electric fields on the surface of the human model and the current densities induced inside the model are discussed. The conclusions are as follows.

(1) The basic characteristics of the electric fields on the surface of the grounded and the ungrounded human model are made clear.

(2) The basic characteristics of the current densities induced inside the grounded and the ungrounded human model are made clear.

(3) Under our calculations, the maximum electric field strength at the top on the head part of the model is obtained with the grounded model. That value is approximately 22.5 times as strong as the external field. In the case that the distance d is 0.011m, the electric field strength at the bottom on the model foot becomes largest within all calculations. That value is approximately 75.7 times as strong as the external field.

(4) The analytical results are demonstrated by the experimental results of Kaune et al. who used the 3-dimensional human model. In future publications on this subject, we will report the analysis of the current densities induced inside the grounded and the ungrounded human model in detail.

References

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