MODELLING TECHNIQUES FOR THE ACCURATE SIMULATION OF INTERCONNECTS

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Abstract: The paper describes the hierarchy of models used in the modeling of interconnects in high-speed designs and focuses on the development of techniques needed for the high-frequency simulation of complete modules.

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1. Introduction

The design of high-speed circuits in modern IT and communication systems relies on strict signal specifications, noise and interference limits well up to microwave frequencies. What hitherto had been simple wiring where the only consideration had been the correct circuit topology, has now become an important part of design encompassing disciplines such as RF and microwave design. It is not an exaggeration to say that designs are now interconnect rather than device dominated. The requirements to make numerous connections in a limited space with the minimum of cross-talk and minimum signal distortion are extremely challenging for designers.

At relatively low frequencies, models of interconnects may be constructed using a few lumped elements. Simple RC models may suffice at low-frequencies with the addition of inductances for models in the 10s of MHz frequency band. At higher frequencies in the 100s of MHz region, propagation effects and associated time delays become important and full-field models are necessary for accurate modeling. As a rule, full-field modeling requires a spatial resolution at least as good as a tenth of the lowest wavelength of interest. For circuits clocked at 2 GHz interconnect characterization to encompass several harmonics is required. This is in itself a formidable computational task. However, an additional challenge is presented by the need to model accurately the geometrical and material details of the interconnects and associated source, load and adjacent circuits. These features are normally much smaller than the wavelength but must nevertheless be modeled accurately. This is a classic multi-scale problem and a major challenge for the modeler. Efficient solutions to this problem must be found before simulation of complete practical systems can take place. The aim of this paper is to discuss approaches to this problem and to present the Modal Expansion Technique (MET) as an efficient multi-scale modeling method.

In the next section we present the conventional approaches to wire modeling, followed by a section describing the MET approach. Results are also shown together with the outlook for whole system modeling at high-frequencies.

2. Conventional Approaches to Multi-scale Modelling

2.1 Multi-grid and Hybrid Meshes

The straightforward response to the requirement of modelling fine features in a general numerical mesh is to have a fine mesh only where it is needed (e.g. around a thin wire) and a coarse mesh elsewhere. This gives a multi grid-mesh as shown in Figure 1a.

Fig. 1 Schematic of a multi-grid (a) and of a hybrid mesh

Several techniques of interfacing at the fine-coarse junction have been developed [1]. In the time-domain, which is most suited to broad-band characterization, it is clear that this interface needs to implement some form of time and space averaging. This can be problematic leading to instabilities in many such schemes. Similarly as the fine and coarse meshes have a different capacity to propagate high-frequency signals the fine-coarse interface is effectively a
discontinuity causing numerical artifacts. Nevertheless such schemes have been developed and used in research codes.

An alternative approach is illustrated in Figure 1b. In this, the so called hybrid mesh, the mesh is made finer in the desired location, but distorted shape mesh is established well beyond the area where a fine resolution is required. The advantage of this mesh is that across all interfaces there is one-to-one correspondence in space and in time and hence no averaging is required. These schemes are stable and effective but result in a smaller time-step determined by the aspect ratio of the computational elements. More details and original references may be found in [2].

Both these schemes distort the mesh in order to accommodate fine features. It would appear profitable to maintain a background mesh determined by global consideration and interface the fine features (e.g. a thin wire) to the mesh using knowledge of local field behaviour in the vicinity of the fine feature. This option is explored in the next subsection.

2.2 Interfacing Static Wire Solutions to a Coarse Mesh

The ground rules for interfacing thin wires to a coarse numerical mesh were described in [3]. The essence of this approach is that near a wire the electric and magnetic field may be approximately related to the current and electric charge on the wire using,

\[ E(r) = \frac{q}{2\pi r}, \quad H(r) = \frac{i}{2\pi r} \]  

(1)

It can be shown that at some reference return conductor radius \( r_0 \) the electric field for a z-directed wire is related to charge and current by,

\[ E_z(r_0) = L_d \frac{\partial i}{\partial t} + \frac{1}{C_d} \frac{\partial q}{\partial t} \]  

(2)

where, \( L_d, C_d \) are the inductance and capacitance per unit length between the wire of radius ‘a’ and a return at radius \( r_0 \) to be determined. This radius is approximately the half the mesh size but for maximum accuracy must be empirically determined.

An example of the empirical factors involved for TLM models is shown in equations (3) where \( 2\Delta \) is the mesh resolution,

\[ C_d = \frac{2\pi e}{\ln\left(\frac{0.4\cdot2\Delta}{a}\right)}, \quad L_d = \frac{\mu}{2\pi} \ln\left(0.15\frac{2\Delta}{a}\right) \]  

(3)

There are moreover restrictions imposed on the maximum wire radius which may be thus modeled as a fraction of the mesh resolution. The most serious limitation of this formulation is the requirement, inherent in the use of static symmetrical solutions for the field near the wire, that the wire is located centrally inside the basic computational element. True route wire placement is not possible. A consequence of this is that multiple conductors (wire looms, closely spaced tracks) cannot be accurately modeled as they all appear centrally located irrespective of any adjustments made to their inductance and capacitance matrices to account for the actual relative placement. Critical timing information is thus lost which at high frequencies makes the calculation of common-mode currents, glitches etc difficult to predict accurately. Yet, this is exactly the situation (closely packed interconnects) found in practical high-density high-speed circuits.

Notwithstanding these limitations, the quasi-static approximation described above is widely used in TLM and FDTD models both for single and multi-conductor systems [1].

In the next section we describe the MET approach which removes most of the limitations identified above and thus offers the prospect of accurate and efficient computation in practical systems.

3. The Modal Expansion Technique (MET)

The formulation described above uses the quasi-static approximation to provide a link between the wire and the surrounding mesh. This is an approximation which however can be improved if a more complete wire solution is employed. Such a solution is available in the literature based on an infinite number of modes. For a z-directed wire,

\[ E_z(r, \varphi) = \sum_{n=-\infty}^{\infty} B_n e^{jnp} \left[ J_n(k_0r) - \frac{J_n(k_0a)}{N_n(k_0a)} N_n(k_0r) \right] \]  

(4)

\[ H_\varphi(r, \varphi) = \frac{1}{j\omega\mu_0} \frac{\partial E_z}{\partial r} \]  

(5)

where, \( J \) and \( N \) are Bessel and Neumann functions and \( a \) is the wire radius. It is clear that the quasi-static solution is only one term in a full modal expansion of the field around the wire. If one were to include more than one mode in the interface between the wire and the numerical mesh, then a greater accuracy and flexibility should result. The theoretical development of this interface (MET) is described in more detail in [4]. We describe here the two-dimensional case as it illustrates the essence of the MET.
The first question to address is the number of modes that may be included in the model. Considering that there are four sampling points (ports) in a 2D mesh, it is clear that four modes can be included in contrast to the conventional approaches which account for only one (quasi-static) mode. It seems physically reasonable to select the four lowest modes, $n=0, 1, -1, 2$. The general approach to seeking a solution is to calculate the impedance seen by each mode of the field at the boundary of the computational element (node). The electric and magnetic field are related by an admittance operator,

$$\hat{Y} = \hat{H}$$

(6)

The structure of the admittance operator and its eigenvalues $\gamma_n$ may be found more simply by physical reasoning. Sampling modes $n=0, 1, -1, 2$ at the ports located at angles $\varphi=0, \pi/2, \pi, 3\pi/2$ gives the following matrix relating voltage at its sampling points to its modal components,

$$[U] = \begin{bmatrix}
1/2 & 1/\sqrt{2} & 0 & 1/2 \\
1/2 & 0 & 1/\sqrt{2} & -1/2 \\
1/2 & -1/\sqrt{2} & 0 & 1/2 \\
1/2 & 0 & 1/\sqrt{2} & -1/2
\end{bmatrix}$$

(7)

where the mapping between voltage samples $V$ their modal components $X$ is,

$$V = [U]X$$

(8)

The eigenvalues are essentially the admittance seen by each mode at the node boundary and thus can be obtained from equations (4) and (5),

$$\gamma_n = \frac{1}{j\omega \mu_0} \frac{\partial}{\partial r} \left\{ J_n(k_0r) - \frac{J_n(k_0a)}{N_n(k_0a)} N_n(k_0r) \right\}_{r=\Delta}$$

\[ J_n(k_0\Delta) - \frac{J_n(k_0a)}{N_n(k_0a)} N_n(k_0\Delta) \]

(9)

Applying the usual low-frequency approximations to obtain small argument expansions for the Bessel function one obtains the following expressions for the impedance seen by each field mode,

$$\frac{E_{z0}}{H_{\varphi0}} = j\omega \mu_0 \Delta \ln \left( \frac{\Delta}{a} \right), \quad n = 0$$

$$\frac{E_{zn}}{H_{\varphi0}} = \frac{j\omega \mu_0}{n} \Delta \frac{\Delta^2|l| - a^2|l|}{\Delta^2|l| + a^2|l|}, \quad n \neq 0$$

(10)

Based on these expressions a 2D shunt TLM mesh may be devised to study TM propagation as shown in Figure 2.

**Fig. 2 Schematic of a 2D node with a centrally placed fine wire (Z is for link lines, $Z_s$ for the stub)**

Based on the approximations in (10), the parameters of this node are,

$$Z = Z_L \Delta^2|l| - a^2|l|$$

$$Z_s = Z_L \Delta \ln \left( \frac{\Delta}{a} \right) - Z$$

(11)

In these expressions $Z_L$ is the impedance of the standard TLM nodes surrounding the node containing a fine wire. Certain advantageous features of this formulation are already apparent:

- The formulation is simple and incurs a minimum of computational cost.
- Since four modes are included and some of these are asymmetrical (see (7)) the wire can be accurately positioned away from the center of the node. This generalization has been implemented using the properties of the Bessel addition theorems.
- There are no empirical factors involved or restrictions on the diameter of the wire. The algorithm has been tested successfully for wire radii up to $\Delta$.
- The MET should be more accurate than solutions based on quasi-static solutions alone. This is to be expected as four modes are taken into account and can be seen intuitively from Figure 2-conventional approaches only have a correction based on a stub which can only produce delay. The MET also changes the link-line impedance (to $Z$ rather than $Z_L$) thus offering an earlier
reflection than possible from a centrally placed stub. These comments are confirmed by tests of the algorithm.

Multi-conductor problems may also be dealt with using the MET. A typical problem is of several closely packed wires in an interconnect which all lie within a single node. We show the case of two thin wires inside a single computational element centered at the coordinate axis in Figure 3.

The electric field on the surface of wire 1 is the sum of the incident field, the field scattered from wire 1, and the field scattered from wire 2, i.e. it is in the form,

$$E_z = \sum_n \left\{ J_n(kr_0) + S_{w1}H_n^{(2)}(kr_0) \right\} X_n$$  \hspace{1cm} (12)

Demanding that at the surface of each wire the tangential component for each mode is equal to zero permits the calculation of the unknown coefficients and the impedances seen by each mode in a manner similar to that already described. In this way multi-conductor systems can be accurately modeled.

A MET formulation for a centrally placed thin wire in a three-dimensional model is described in [4].

4. Illustrative results

We show some results in Figures 4 and 5 to illustrate the effectiveness of this approach.

In both figures the electric field is shown near (2 nodes) and far (8 nodes) from the fine feature. In Figure 4 the thin wire ($a=0.1\lambda$) is offset from the center of the node along a diagonal by a distance $0.5\lambda$. In Figure 5, a node centered dielectric coated wire ($a=0.25\lambda$, dielectric thickness=$0.25\lambda$, $\varepsilon_r=50$) is modeled. The results from MET and analytical results are compared and show excellent agreement even for frequency above $f_{max}$ corresponding to ten nodes per wavelength.

7. Conclusions

The basic modeling procedures for accurately describing complex wiring structures and interconnects in full-field codes without incurring excessive computational penalties was presented. Results show excellent agreement with analytical formulations. Work is in progress to extend the technique to encompass a general arbitrary placement of wires and wire clusters in three dimensions.

References