Macromodeling of Lossy and Dispersive Multiconductor Transmission Lines

F. Canavero, S. Grivet-Talocia

Dept. of Electronics, Politecnico di Torino, C. Duca degli Abruzzi 24, 10129 Torino, Italy
Email: flavio.canavero@polito.it, stefano.grivet@polito.it

Abstract: This paper presents an automatic procedure for the generation of efficient macromodels of lossy and dispersive multiconductor transmission lines. The lines are assumed to be specified via frequency tables of per-unit-length matrices. The proposed technique translates the telegraphers equations into an equivalent circuit made of ideal (lossless) scalar transmission lines, resistors, capacitors, and dependent sources. The fundamental steps are the extraction of the line modal delays, the rational approximation of admittance and delayless propagation operators, including passivity constraints, and the synthesis of an equivalent circuit. The latter can be directly used for transient analysis using SPICE-like circuit solvers, thus allowing accurate signal integrity analyses of complex interconnected systems with realistic (nonlossy) terminations. Several examples of package-level and board-level transmission lines are presented.

1 Introduction

Electrical interconnects at chip, multichip, package, and board level constitute one of the most critical parts for the signal integrity of all electronic systems. Nonetheless, an accurate and efficient transient simulation of electrical interconnects is still a challenging task even in the most advanced circuit solvers. This is due to the intrinsic difficulties in the design of stable algorithms for the time-domain analysis of structures with frequency-dependent parameters. Indeed, it is well known that accurate interconnect models must take into account metal (skin effect) and dielectric losses, which lead to possibly large attenuation at increasing frequency. The underlying physics is best captured using a frequency-domain approach, leading to constitutive parameters with a complex dependence on frequency. A robust approximation is therefore required for the conversion to time domain of the constitutive line equations and the subsequent generation of a line macromodel to be employed in a transient simulation.

In this paper we present a particular line macromodeling strategy in the general framework of the well-known Method of Characteristics (MoC). This technique was first introduced for lossless transmission lines by Brainn [1]. Here we employ a generalized version that allows to deal with multiconductor lines whose per-unit-length parameter matrices have a possibly complex dependence on frequency. In such case, both the characteristic admittance and the propagation operators of the line are frequency-dependent. A line macromodel is obtained via the following steps. First, the line equations are projected onto their high-frequency asymptotic modes. This leads to a particular form of the propagation operator that allows an easy extraction of the line modal delays. A second step is the generation of a rational approximation of characteristic admittance and delayless propagation operators in the Laplace domain. In this step, particular care is taken in the enforcement of passivity so that the final macromodel is guaranteed to be stable under any termination condition. The last step is the translation of the rational expressions into equivalent circuits that can be implemented into a standard SPICE-like circuit solver. In this step, ideal lossless scalar transmission lines are used to synthesize the modal delays, since all commonly used circuit solvers have efficient models for ideal lines (usually based on the standard MoC in the original formulation).

The proposed strategy is here applied for the transient analysis of complex transmission line structures for high-speed digital signaling at package (Multichip Module, MCM) and board (Printed Circuit Board, PCB level). We considered structures with up to nine conductors (plus reference) with excellent accuracy and efficiency. Some results are presented to support this conclusion.

2 Macromodel generation

We consider a lossy multiconductor transmission line governed by the telegraphers equations, here stated in the Laplace ($s$) domain

$$-\frac{d}{dz} V(z, s) = Z(s) I(z, s), \quad -\frac{d}{dz} I(z, s) = Y(s) V(z, s),$$

(1)
where \( z \) represents the longitudinal coordinate along which signals propagate according to the quasi-TEM mode. The length of the line will be denoted as \( L \). The transmission line per-unit-length matrices \( Y(s) \) and \( Z(s) \) are defined as

\[
Y(s) = G(s) + sC(s) \quad \text{and} \quad Z(s) = R(s) + sL(s)
\]

(2)

with \( G(s), C(s), R(s) \), and \( L(s) \) denoting the per-unit-length conductance, capacitance, resistance, and inductance matrices, respectively. These four matrices are collectively indicated as frequency-dependent, per-unit-length (f-PUL) parameters. These parameters are specified at fixed frequency points \( \{ \omega_k = j\omega_k = j2\pi f_k \} \) by means of transverse 2D electromagnetic simulation.

We remark that one important aspect that is often neglected is the self-consistency of the frequency tables of the f-PUL matrices to be used as raw data. More precisely, the real and imaginary parts of both transverse admittance \( Y(s) \) and impedance \( Z(s) \) in (2) must be related by Hilbert transform according to the well-known Kramers-Kröning conditions [7]. Therefore, some causality check must be performed before attempting any macromodeling procedure. A possible strategy is described in [5].

The transmission line segment is treated as a multiport, where we denote by \( V_1(s), I_1(s) \) the input (near end) and by \( V_2(s), I_2(s) \) the output (far end) terminal voltage-current port quantities. A straightforward derivation allows to cast the telegrapher's equations (1) in the following form,

\[
I_1(s) = Y_c(s) V_1(s) - J_1(s), \quad I_2(s) = Y_c(s) V_2(s) - J_2(s)
\]

(3)

where \( J_1(s), J_2(s) \) are controlled current sources defined as

\[
J_1(s) = H(s)[Y_c(s)V_2(s) + I_2(s)], \quad J_2(s) = H(s)[Y_c(s)V_1(s) + I_1(s)]
\]

(4)

with

\[
\Gamma^z(s) = Y(s)Z(s), \quad Y_c(s) = \Gamma^{-1}(s)Y(s), \quad H(s) = e^{-\alpha L(s)}
\]

(5)

being the squared propagation matrix, the characteristic admittance matrix, and the propagation operator, respectively. The main difficulty in the generation of line macromodels is the conversion of the above expressions into time-domain. This is a difficult task due to the complex frequency dependence of all involved transfer functions. In order to overcome these difficulties, we extract the line delays and we perform rational approximations, as detailed below.

The extraction of the line delays is performed first. The set of modal delays \( \{ \tau_k \} \) are computed as

\[
\tau_k = L \sqrt{\lambda_k},
\]

where \( \lambda_k \) are the eigenvalues of matrix \( C_\infty L_\infty \). A very high-frequency value for the per-unit-length matrices must therefore be present in the data. Also, we denote the matrix collecting the eigenvectors corresponding to \( \{ \lambda_k \} \) as \( M_\infty \).

It should be noted that these eigenvectors correspond to the line modes at very high frequency. We use these asymptotic modal decomposition matrices in order to extract the modal delays by introducing the following delayless propagation operator

\[
P(s) = \text{diag} \{ e^{\tau_k} \} M_\infty^{-1} H(s) M_\infty.
\]

(6)

Since constant (high-frequency) models are used, the decoupling of the line equations is only partial. In fact, if we attempted to perform full decoupling, we would be forced to use frequency-dependent modal decomposition matrices, characterized by a complex frequency behavior. These would have to be processed and approximated by rational expressions. Unfortunately, our tests proved that this approximation is particularly challenging. Instead, we use the high-frequency asymptotic modes in order to reduce the complexity of the rational approximation algorithm and to obtain more robust macromodels. The conversion into time-domain of (3)-(4) is an easy task if all transfer matrices are rational. Therefore, we approximate both characteristic admittance and delayless propagation operator as

\[
Y_c(s) \approx \sum_n \frac{R_n}{s - p_n} + Y_\infty \quad \text{and} \quad P(s) \approx \sum_n \frac{R_n^2}{s - q_n} + P_\infty.
\]

(7)

The results that will be presented in this paper are based on the algorithm sketched in [2], although excellent results can also be obtained by the well-known Vector-Fitting (VF) algorithm [6]. The employed approximation procedure is based on the determination of the location of real poles only within the band of available frequency points by means of an iterative bisection placement. Due to the smoothness of the functions being approximated, very good accuracies can be achieved with relatively few poles (on the order of 6-10) for all the lines that were tested. As an example, we report in Figure 1 the rational approximation for relevant entries of \( Y_c(s) \) and \( P(s) \) for a two-conductor PCB line [8].
Figure 1: Rational approximation of characteristic admittance and propagation operator elements for a PCB line.

The passivity of the rational expressions (7) is tested a posteriori via a recent methodology based on the spectral properties of associated Hamiltonian matrices (see, e.g., [4, 3]). This technique allows also a compensation of passivity violations that may possibly arise during the rational approximation stage. The derivation in [4] shows that this compensation is achieved with the minimal perturbation on the accuracy of the approximation.

The final step of the macromodeling procedure is the generation of a circuit equivalent that can be used in a standard circuit simulation environment. The synthesis of lumped equivalent corresponding to rational expansions of characteristic admittance and delayless propagation operators in the form (7) is a standard task. These are also easily combined, using dependent sources and ideal lossless lines, in order to recover the fully-delayed propagation operator

\[ H(s) = M_\infty \text{diag} \{ e^{-sT_L} \} P(s) M_\infty^{-1}. \]  

(8)

We omit for conciseness the detailed derivation of the equivalent circuit, which is nonetheless quite straightforward. As a result, we obtain a direct circuit synthesis of Eqs. (3)-(4) using standard circuit elements.

3 Numerical Results

The first example that we present is a two-conductor PCB line whose parameters and terminations are specified in [8]. This paper presents three interesting benchmark lossy and dispersive lines that can be used for the assessment of macromodeling algorithms. We note that this structure is the same that was used in Fig. 1 for the illustration of the rational approximation algorithm. One of the line conductors is excited with a step voltage source, and no excitation is applied to the other conductor (victim). For this particular structure we have linear termination networks. Therefore, a full validation is possible by using a standard frequency-domain solution and inverse FFT for the computation of the transient responses. The results for some of these responses on a 20cm line segment are depicted in Fig. 2. The transient solution employing the proposed macromodel was computed in a fraction of a second on a Pentium IV-based PC (1.8 GHz) using IBM PowerSpice circuit solver. The results are not distinguishable on this scale from the reference solution. Therefore, we can conclude that the accuracy of the proposed macromodeling algorithm is very good.

The second example is a more challenging nine-conductors (plus reference) MCM line. A fully-coupled model was used for the per-unit-length parameters. All the line conductors are terminated by a parametric macromodel based on sigmoidal expansions of a high-speed IBM CMOS driver (\( V_{dd} = 1.8 \) V) at the near end (for details on this device see [9]). This model takes into account the full nonlinear and dynamic behavior of the actual device at a reduced computational cost. All the drivers are switching a pulse except one, which is kept quiet. The other end of the conductors are terminated by simple capacitive receivers. Figure 3 reports the transient results obtained using the line macromodel. This example shows that the proposed technique is applicable for the signal integrity analysis of complex structures including both transmission lines links with many conductors and complex dynamic/nonlinear termination networks.

4 Acknowledgments

We would like to thank Dr. Ruchli, H. M. Huang, and E. Klink from IBM for their continuous support and for providing the line examples that were used in this work.
Figure 2: PCB line (20 cm). Transient voltages at near and far terminations of active line (left) and victim line (right). The proposed macromodel is labelled as "TOPLIne".

References


Figure 3: MCM 9-conductors line. Transient voltages at near and far terminations of victim line (left) and one of the driven lines (right).