Evaluation of the Coupling between the Electromagnetic Field and a Printed Circuit Board Trace in a Metallic Enclosure

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Abstract: A new analytical method is developed for the terminal responses of a printed circuit board (PCB) trace exposed to electromagnetic field in a rectangular enclosure. The analysis is based on reciprocity theorem in a coupling voltage formulation with the assumptions that the PCB trace is modeled as transmission line and the enclosure is modeled as a length of a rectangular waveguide ended by a short. From the general solutions obtained, several much simpler approximations are derived revealing the principal behavior and indicating the relevant parameters to minimize the coupling. The coupling effect of different mode TE&TM wave is investigated to illustrate the capability of the present technique.

Key words: reciprocity theorem of antenna, transmission line, coupling effect; coupling length

I. INTRODUCTION

Metallic shields are widely used to solve both susceptibility and emission problems of electrical and electronic systems. Due to the great number of applications of shielding devices in the electronic industry, there is a great interest in the coupling of electromagnetic field to a circuit board (PCB) in a metallic enclosure. The external field induces voltages and currents at the trace ends to where sensitive semiconductor components may be connected. The present paper concentrates on the response of a single trace on a PCB.

Some works [1][2][3] focus their attention on the problem of coupling of free-space electromagnetic fields to transmission lines. The analysis, based on a transmission line model with free-space source, is primarily addressed to two-wire transmission line, ignoring the effect of the dielectric around the transmission line.

In [4], for a PCB trace in freespace, the analysis based on transmission line theory in a scattered voltage formulation uses a quasi-TEM propagation model for the trace is developed. But when the frequency of signal is up to GHz, the telegram equation and distributed voltage resource method is not accurate.

Some other works [5] consider higher frequency, but the incident wave is assumed to be uniform planar wave. But in a metallic enclosure, in a much more complex environment, TE and TM wave can be the incident wave.

In this paper, an enhanced model using reciprocity theorem of antenna is proposed. It can calculate the response of a PCB trace excited by nonuniform electromagnetic wave. The paper has deduced an analytic solution of the induced voltage at the terminal of line in the frequency domain.

II. PHYSICAL MODEL AND MATHEMATICAL FORMULATION

The physical model of the PCB trace in a rectangular enclosure is shown in figure 1. We assume the enclosure as a part of a rectangular waveguide ended by a short. The PCB lies at the bottom of the enclosure with a PCB trace, which is parallel to y-axis. The trace’s length is L, and whose width is W, and the thickness of the PCB trace is T, the thickness of the dielectric is h, the relative permittivity is \( \varepsilon_r \). \( Z_c \) is the characteristic impedance of the PCB trace modeled as transmission line. The impedances at each terminal are \( Z_1 \) and \( Z_2 \), respectively. And in this waveguide we assume the incident waves, which are often coupled through aperture in the enclosure, transmit along z-axis and vertical to the PCB. Although this assumption is not rigorous, the above approximation becomes realistic when the PCB is far from the aperture.

![Figure 1: The physical model of PCB trace in a rectangular enclosure](image)
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1. The Calculation of Voltage Coupled on the terminal of PCB trace

When incident electromagnetic waves couple to transmission line, the current on the wires is induced, as well as the coupling voltage is generated on the terminals of the line. But only the terminal voltage is considered since it is the practical interest and can be easily evaluated.

According to the reciprocity theorem, we assume the transmission line as a receiving antenna and transform this scattering problem into a radiation problem of the antenna. The open voltage \( V_0 \) at the terminal of the PCB trace induced by external electric field \( E \), is

\[
V_o = \frac{1}{I(0)} \int [E_x I_x e^{i\beta x} - G_x e^{i\beta x}] dy
\]

where \( I(0) \) is the inspiriting current in the input port of the antenna when the antenna acts as an emission antenna, \( I(0) \) is the current distribution in the emission antenna.

Since in this paper we assume the transmission line is far from the wall of enclosure, the sine current distribution is assumed which can reach the requirement of engineering. Consider that the line don't match to the terminal load \( Z_2 \) and match to \( Z_1 \), the expression of open voltage is

\[
V_o = \frac{1}{(1-\Gamma_y)} \int E_x I_x e^{i\beta x} - G_x e^{i\beta x} dy
\]

where \( \Gamma_y \) is the reflection coefficient on \( Z_2 \). \( \beta \) is the propagation constant. \( E_x \) is electric field at the boundary of dielectric, \( E_{z1} \) and \( E_{z2} \) are the electric field on the terminal \( Z_1 \) and \( Z_2 \), respectively.

When electromagnetic waves couple in the transmission line, the voltage on the terminal is induced by \( E_x \) and \( E_y \). According to (1). Because the thickness of the PCB \( h \) is much smaller than the length \( L \), the effectiveness of \( E_y \) compared to the voltage induced by \( E_y \) can be ignored, (we also calculate that the voltage coupled by \( E_y \) is less than 1 percent of the voltage coupled by \( E_x \) when \( L=10 \text{cm}, h=0.4 \text{mm} \)). So that

\[
V_o = \frac{1}{(1-\Gamma_y)} \int E_x e^{i\beta x} dy
\]

Now we deduce \( E_y \) on PCB trace. According to the superposition principle of the field, the field in the enclosure space \( (E_0, H_0) \) can be expressed as following:

\[
\begin{align*}
E_{z1} &= E_{z0} + E_{zr} = E_{z0} (1 + \Gamma_1(z)) \\
E_{z2} &= E_{z0} + E_{zt} = E_{z0} (1 - \Gamma_1(z))
\end{align*}
\]

And the field in the substrate \( (E_r, H_r) \) can be expressed as following:

\[
\begin{align*}
E_{z1r} &= E_{z0} + E_{zrr} = E_{z0} (1 + \Gamma_1(z)) \\
E_{z2r} &= E_{z0} + E_{ztr} = E_{z0} (1 - \Gamma_1(z))
\end{align*}
\]

In the expression (4) and (5), the subscripts i, r and t represent the incident wave, reflected wave and the refraction wave respectively. (Figure 2) \( \Gamma_1(z) \) and \( \Gamma_2(z) \) is the reflection coefficient in section I and section II, respectively.

According to former assumption, the incident wave is propagating along \( z \)-axis shown in figure1, then \( \theta = 0 \). With the boundary condition at \( z = -h \text{ (m)} \) and \( z = 0 \), field distribution can be solved. And the

\[
E_y(z) = E_{z0} (1 + \Gamma_2(-z))
\]

where \( \Gamma_2(-z) = \frac{A-1}{A+1} \)

and \( A = \frac{k_z e^{-i2\beta h}}{1 + e^{-i2\beta h}} \)

\( k_z \) is the propagation constant of \( z \)-direction in the dielectric medium.

So by using relation (3) and (6), we have the open voltage coupled on the terminal of transmission \( V_o \).

Then we calculate the voltage on the terminal load. Figure 3 shows the model of the transmission line.

![Figure 2: The distribution of field](image)

![Figure 3: Model of the terminal of transmission line](image)
So that
\[ Z_{in} = Z_t \frac{Z_{te} + jZ_{tg} \beta l}{Z_{te} + jZ_{tg} \beta l} \]  \hspace{1cm} (9)
And the voltage on the terminal AB of transmission line can be expressed as,
\[ V_1 = \frac{Z_{te}}{Z_{te} + Z_2} V_0 \]  \hspace{1cm} (10)
Now with \( E_y \) of incident wave and relations before, we can get the coupling voltage on the PCB.
Next we focus on \( E_y \) of the incident wave. In this modeled waveguide, the wave mode, which can propagate in the enclosure, is TE_{mn} and TM_{mn}. Here we can calculate the coupled voltage by TE and TM wave.
For TE_{mn} mode, \( E_y \) can be expressed as
\[ E_y = \frac{j \mu_0 \mu_m n \pi}{k_e^2} \sin \left( \frac{m \pi}{a} x \right) \cos \left( \frac{n \pi}{b} y \right) e^{-j \beta z} \]  \hspace{1cm} (11)
where \( m, n \) is the mode of TE wave. \( H_{mn} \) is constant.
And \( E_y \) of the TM_{mn} mode can be expressed as
\[ E_y = \frac{j k_z}{k_e^2} \sin \left( \frac{m \pi}{a} x \right) \cos \left( \frac{n \pi}{b} y \right) e^{-j \beta z} \]  \hspace{1cm} (12)
where \( E_{mn} \) is constant, \( m, n \) is the mode of TE wave.
2. The Coupling length of The Transmission Line
The terminal voltage can be written as
\[ V = |E| h_e \]  \hspace{1cm} (13)
where \( |E| \) is the magnitude of the incident electric field. Then define \( h_e \) as the coupling length of the transmission line.
It shows that the terminal voltage can be calculated as long as the magnitude of the electromagnetic field is known. By this definition, the coupling capability of different transmission line and different incident electromagnetic wave can be compared.

III. RESULT AND ANALYSIS
In this section, the coupling effectiveness of different mode of TE and TM wave are calculated by using the method developed in the previous sections.
A PCB structure in a metallic rectangular is shown in figure 1 and 2, wherein the size of the enclosure as \( a=40cm, b=20cm \), and TE_{10}, TE_{11}, TE_{30}, TM_{11}, TM_{21} wave can propagate in waveguide when the frequency of the wave \( f \geq 1GHz \). And width of the conductor strip \( w=0.2mm \), lost \( \tan \theta=0.01 \), \( \varepsilon_r = 4.2 \), and the thickness of dielectric \( h=0.4mm \), and the characteristic impedance of the transmission line \( Z_c = 96.41\Omega \). The incident wave irradiates upon the PCB vertically.

First we study the coupling voltage of different mode incident wave. Figure 4 and 5 shows \( V/H_{mn} \) of TE_{10}, TE_{11}, TE_{30} wave. The range of the frequency is varied from 1GHz to 10GHz. It is calculated with matched load \( Z_2 = Z_c \) in figure 4 and \( Z_2 = 2\Omega \) in figure 5. The length of the transmission line \( L=15cm \). It can observe that the voltage coupled by TE wave is varied periodically by frequency and length, and the internal between adjacent peaks is \( f \). For the \( V/H_{mn} \) curves show that by different mode, and on most frequency points, the coupling effects of TE_{10} wave is greater than that of TE_{11} and TE_{30}. And for the same mode at different frequency point, the peaks of coupling voltage are about the same.

Figure 6 shows the voltage coupled by TM wave. The range of the frequency is varied from 1GHz to 10GHz. The impedance of the terminal is \( Z_2 = Z_c \) and \( Z_2 = 2\Omega \). It shows that the coupling capability of TM_{11} and TM_{21} is about the same when the transmission line has the same load.
Then we focus on the coupling length of transmission line induced by different mode of TE waves and uniform planar wave. According to (13), we assume \( |\mathbf{E}| \) is the magnitude of the incident electric field at \((x=a/2; y=b/2; z=-h)\). Figure 7 shows that, for different incident waves, the peaks of coupling length of uniform planar wave is larger than that of TE_{10} and TE_{30}.

IV. CONCLUSION

This work has developed a fast method to evaluate the coupling effectiveness on PCB trace in a metallic enclosure. It regards the transmission line as a receiving antenna. The analytic solution of the voltage at the terminal of the transmission line induced by the different mode of electromagnetic field in a rectangular enclosure is got by the use of the reciprocity theorem. The coupling length is defined.

With some calculated instances, we can know some characters of the coupling voltage and coupling length: The voltage coupled on the terminal of the transmission line is seasonal with the incident wave's frequency. The more the transmission line unmatched, the greater coupling length gets. When the mode of the incident wave gets higher the effectiveness of it gets weaker. With the selected parameters and the matched load, the coupling length of the transmission line is about the magnitude of \(5 \times 10^{-4}m\).

Still it is useful for the estimation of coupling effect of nonuniform incident wave with different incident direction (Figure 2). And the measurement is needed to get the electromagnetic field in a metallic enclosure and analyze the coupling in time domain with previous technique.

REFERENCES