Network Analysis for Understanding Dynamics
- Wikipedia, Music & Chaos -

時間発展のネットワーク分析

TAKASHI IBA
井庭 崇
Ph.D. in Media and Governance
Associate Professor
at Faculty of Policy Management, Keio University, Japan
twitter in Japanese: takashiiba
twitter in English: taka_iba

Takashi Iba　井庭 崇

Studying ...

1997 -

Neural Computing (Optimization & Learning)
The Science of Complexity (Social and Economic Systems)
Research Methodology (Multi-Agent Modeling & Simulation)
Software Engineering (Model-Driven Development, UML)

2004 -

Sociology (Autopoietic Systems Theory)
Network Analysis
Pattern Language (A method to describe tacit knowledge)
Creativity

Teaching at SFC, Keio University

Social Systems Theory
Pattern Language
Simulation Design
Science of Complex Systems

Introduction to Complex Systems (1998) in Japanese

Social Systems Theory [Reality+ Series] (2011) in Japanese

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Network Analysis for Understanding Dynamics

- want to capture the process / dynamics of phenomena as a whole.
- introducing network analysis in a new way.
- to build a directed network by connecting between nodes, based on the sequential order of occurrence.
- a new viewpoint "dynamics as network", which is a distinct from "dynamics of network" and "dynamics on network."

Dynamics

Dynamics

When Network Scientists talk about dynamics...

Dynamics of Networks

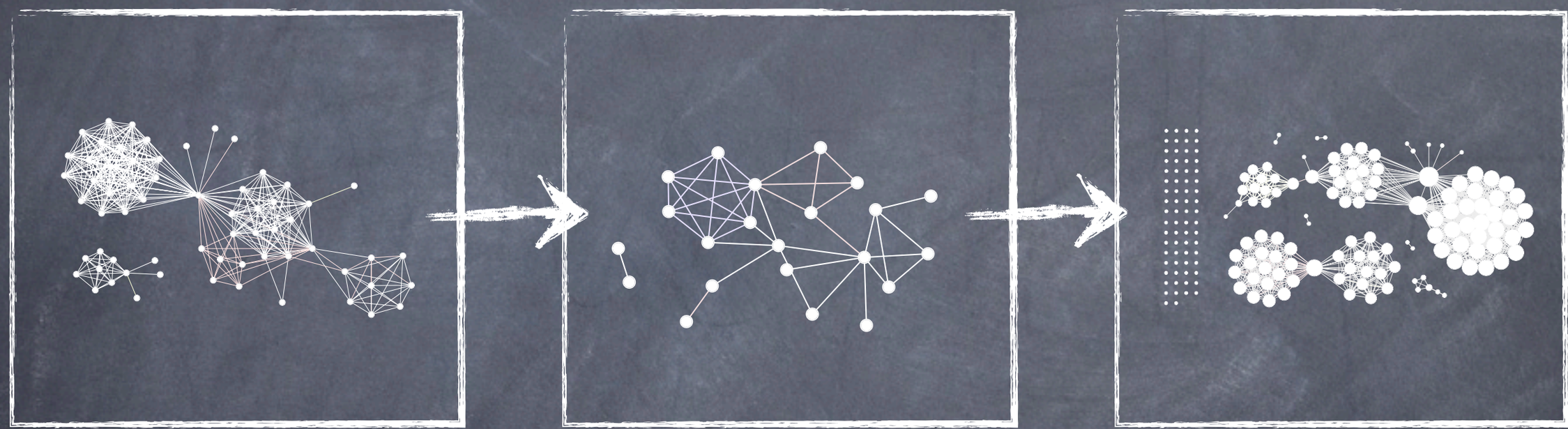
the evolution of network structure in time

Dynamics on Networks

the information spread or interaction on networks

Dynamics of Networks

Structural Change of Alliance Networks of Nations



1850 1946 2000

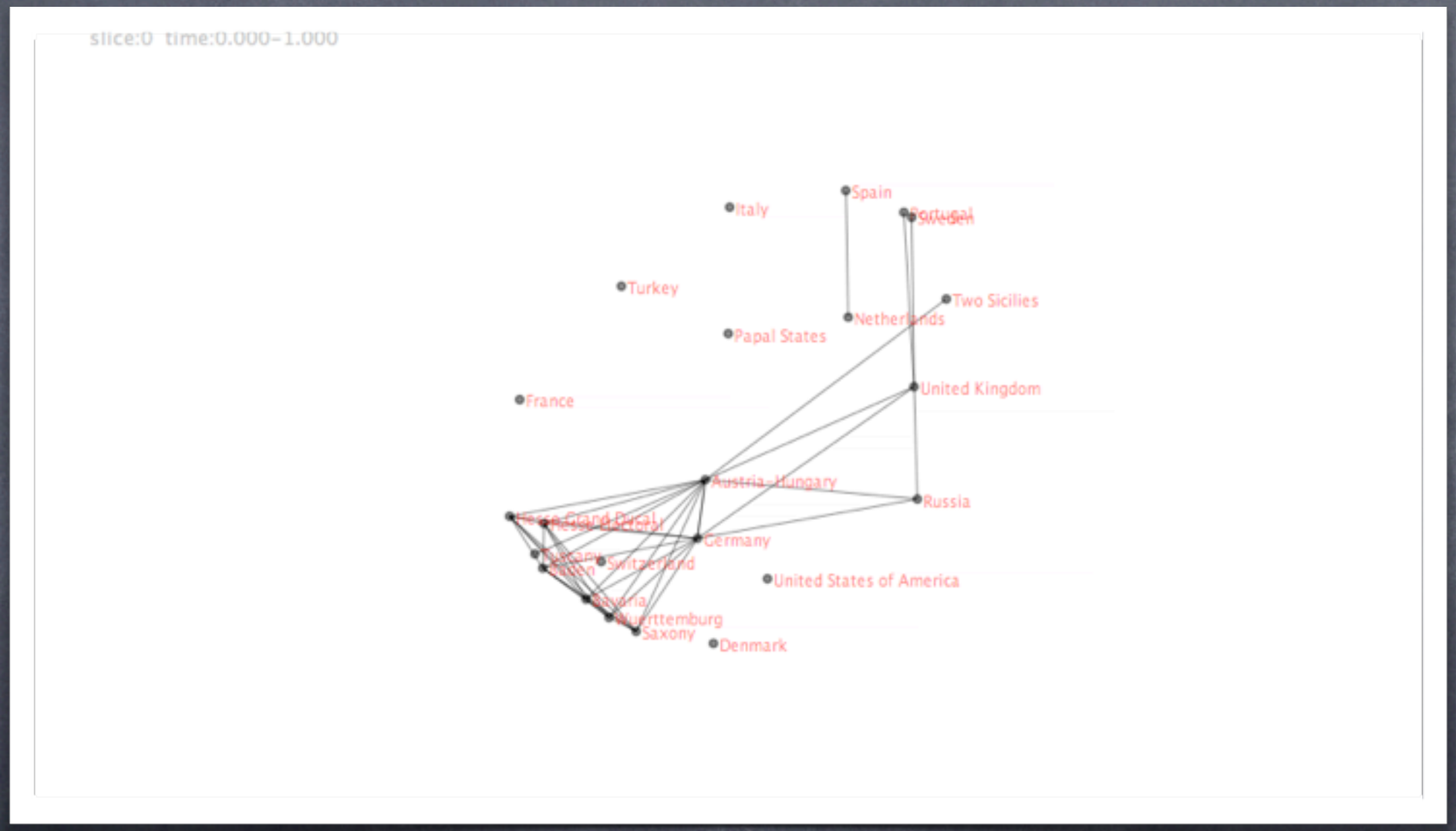
the evolution of network structure in time

T. Furukawazono, Y. Suzuki, and T. Iba, "Historical Changes of Alliance Networks among Nations", NetSci'07, 2007

古川園智樹, 鈴木祐太, 井庭 崇, 「国家間同盟ネットワークの歴史的変化」, 情報処理学会 第58回数理モデル化と問題解決研究会, Vol.2006, No.29, 2006年3月, pp.93-96

Dynamics of Networks

Alliance Networks of Nations



the evolution of network structure in time

When Network Scientists talk about dynamics...

Dynamics of Networks

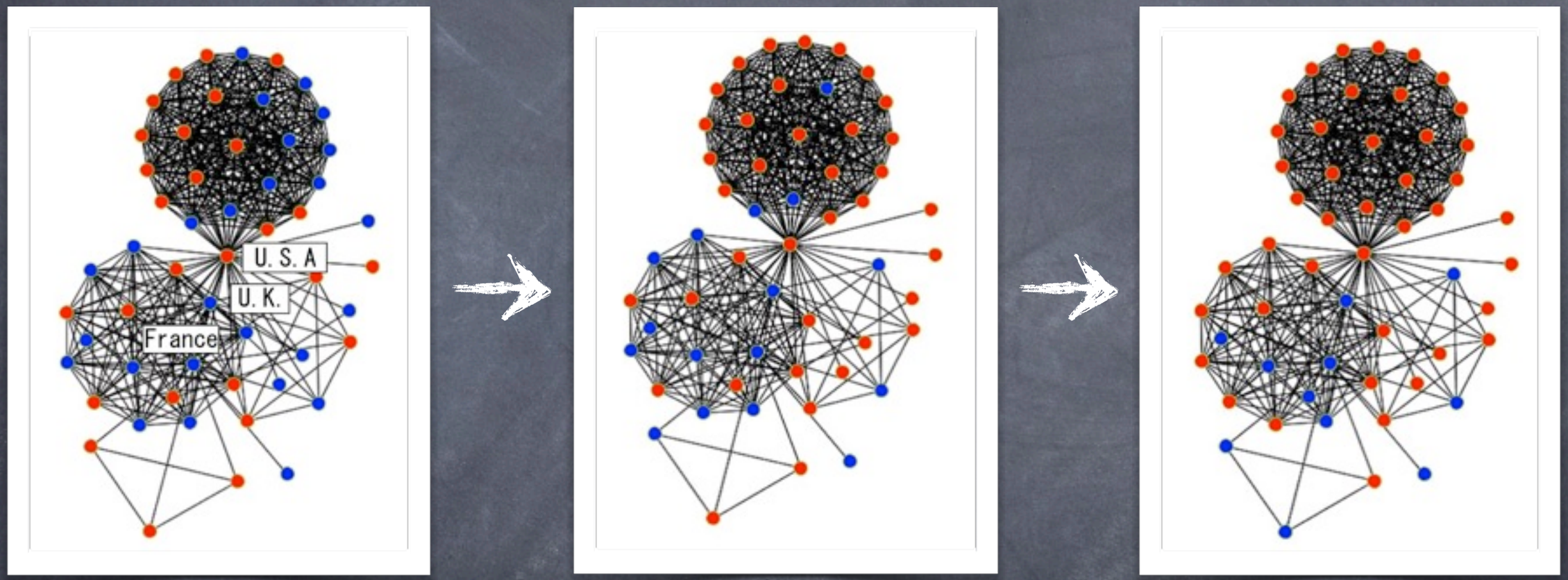
the evolution of network structure in time

Dynamics on Networks

the information spread or interaction on networks

Dynamics on Networks

Iterated Games on Alliance Network of Nations, as an example



the information spread or interaction on networks

T. Furukawazono, Y. Takada, and T. Iba, "Iterated Prisoners' Dilemma on Alliance Networks", NetSci'07, 2007

古川園 智樹, 高田 佑介, 井庭 崇, 「ネットワーク上におけるジレンマゲーム」, 情報処理学会ネットワーク生態学研究会 第2回サマースクール, 2006

When Network Scientists talk about dynamics...

Dynamics of Networks

the evolution of network structure in time

Dynamics on Networks

the information spread or interaction on networks

Dynamics of Networks

Dynamics on Networks

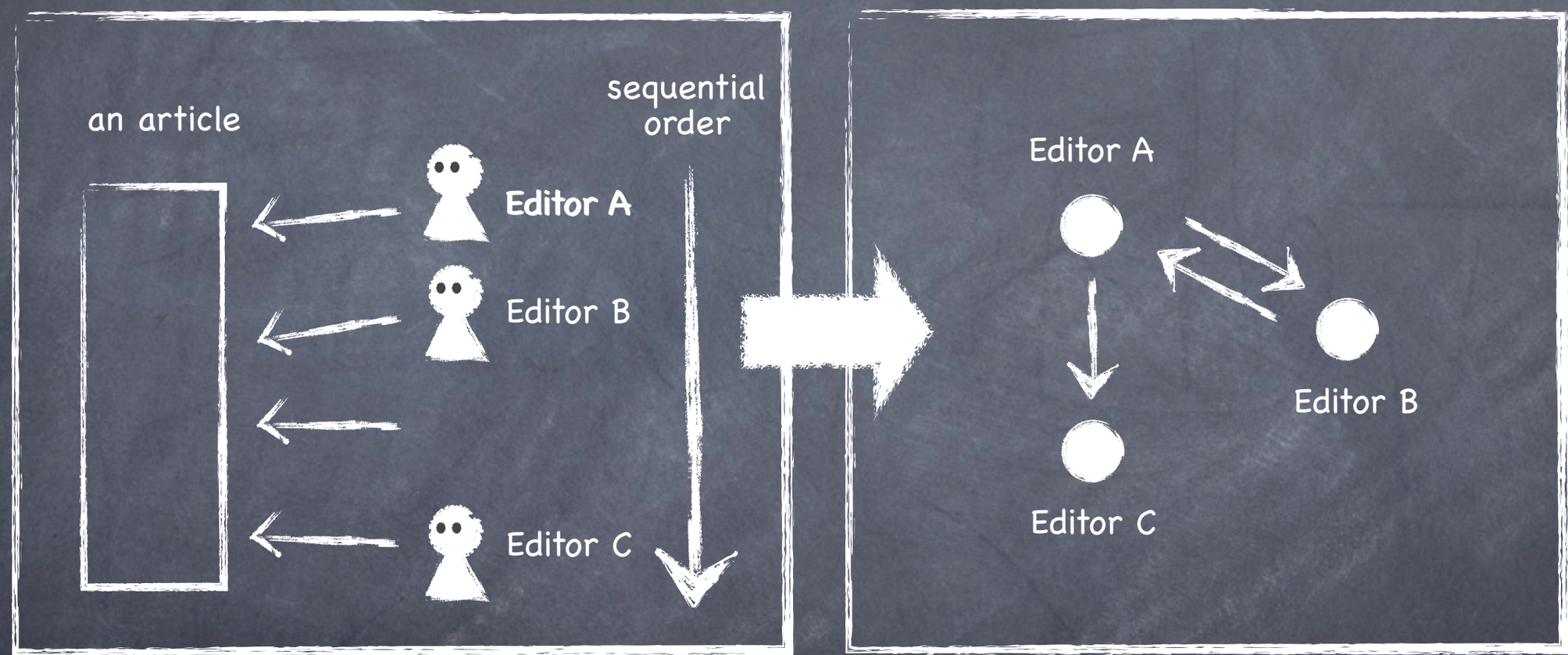
Dynamics as Networks

Dynamics as Networks

the dynamics of system/phenomena are represented as networks

Dynamics as Networks

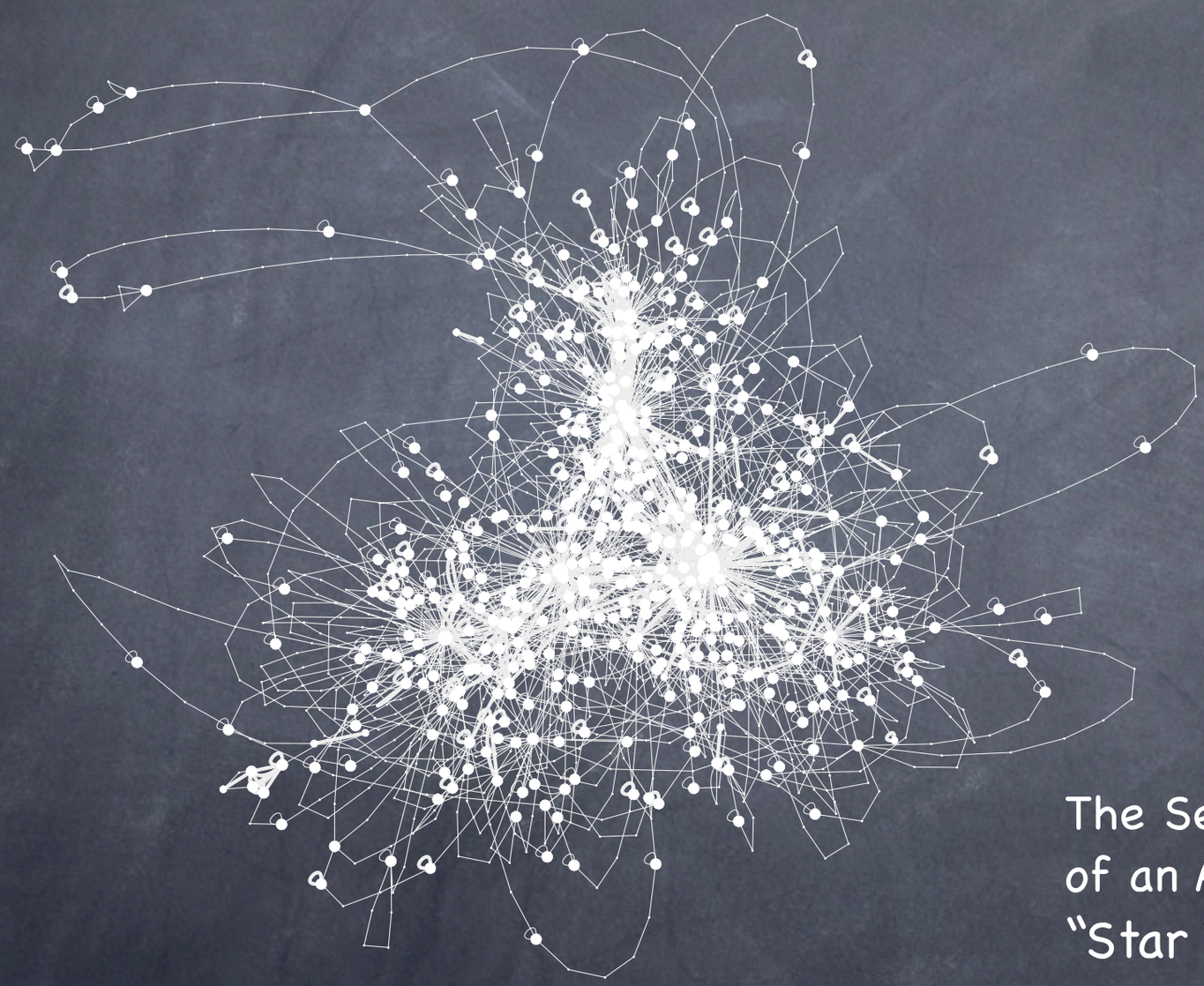
Sequential Collaboration Network of Wikipedians



the dynamics of system/phenomena are represented as networks

Dynamics as Networks

Sequential Collaboration Network of Wikipedians



The Sequential Collaboration Network
of an Article on Wikipedia (English)
"Star Wars Episode IV: A New Hope"

the dynamics of system/phenomena are represented as networks

Dynamics as Networks

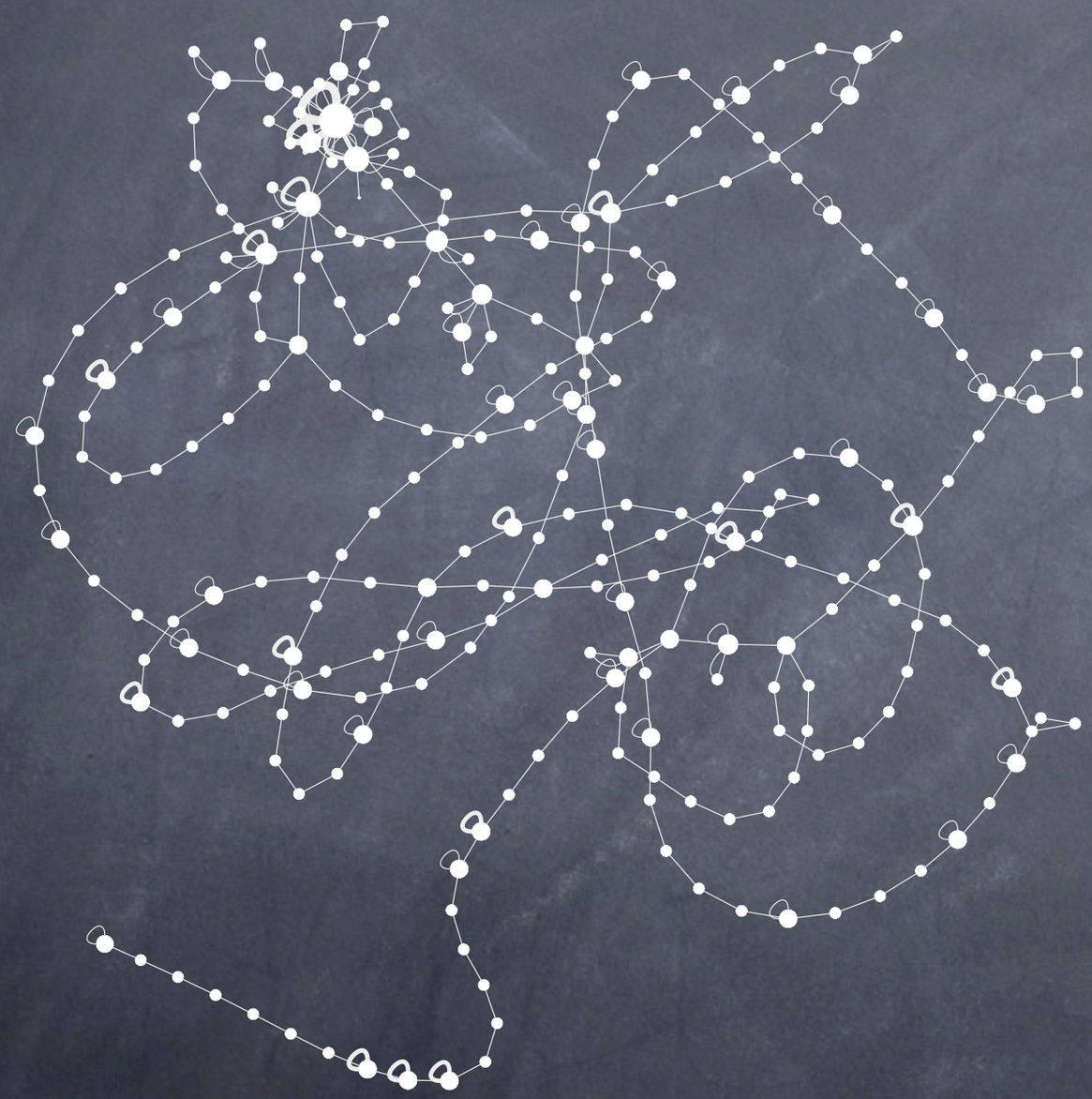
Sequential Collaboration Network of Wikipedians

The Sequential Collaboration Network
of an Article on Wikipedia (English)
"Australia"

the dynamics of system/phenomena are represented as networks

Dynamics as Networks

Sequential Collaboration Network of Wikipedians



The Sequential Collaboration Network
of an Article on Wikipedia (English)
"Damien (South Park)"

the dynamics of system/phenomena are represented as networks

Dynamics as Networks

the dynamics of system/phenomena are represented as networks

Dynamics as Networks

the dynamics of system/phenomena are represented as networks

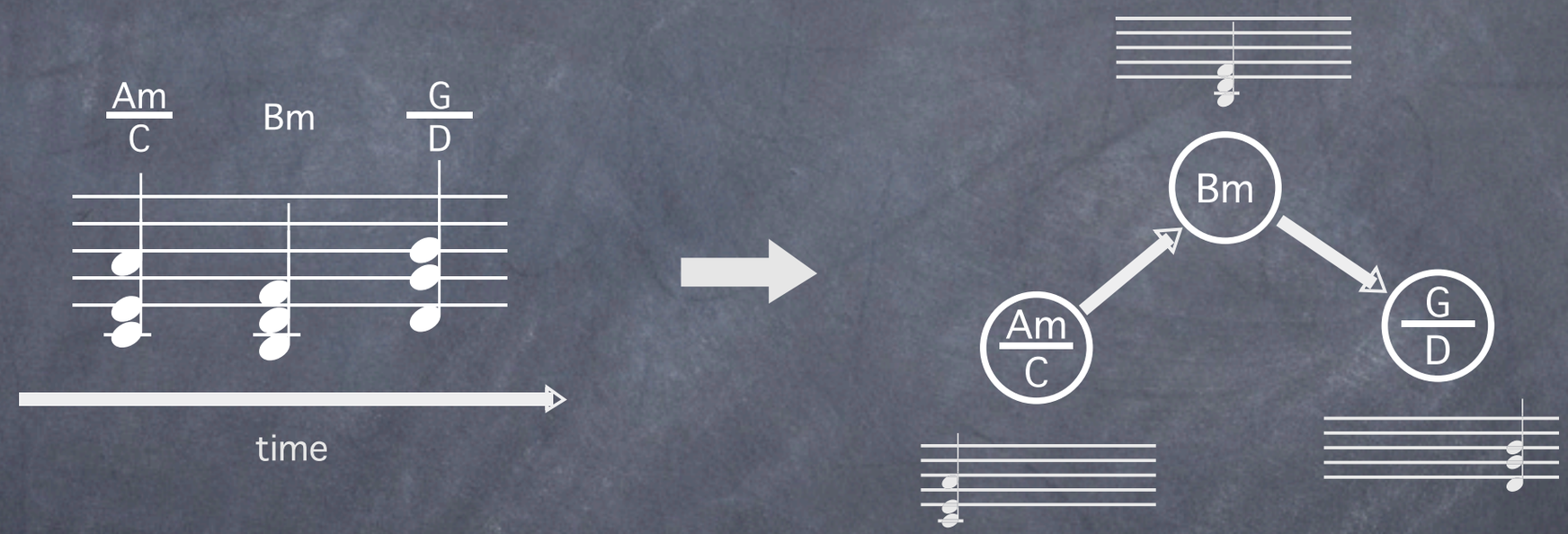
- Chord Networks of Music
- Chord-Transition Networks of Music
- Collaboration Networks of Wikipedia
- Collaboration Networks of Linux
- Co-Purchase Networks of Books, CDs, DVDs
- State Networks of Chaotic Dynamical Systems

Chord Networks of Music

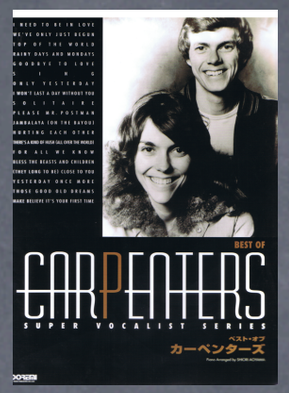
Dynamics as Networks

the dynamics of system/phenomena are represented as networks

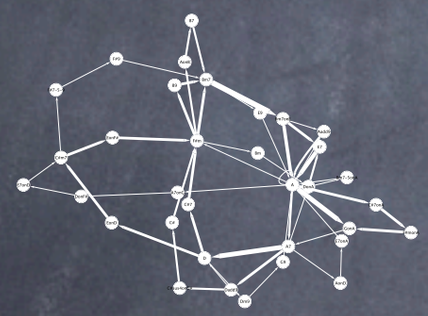
Sequential Chord Network of Music



Sequential Chord Network of Music (Carpenters)

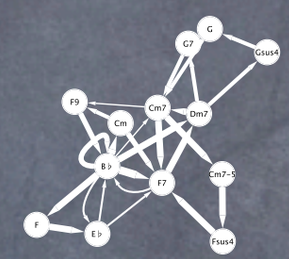


I NEED TO BE IN LOVE



WE'VE ONLY JUST BEGUN

TOP OF THE WORLD



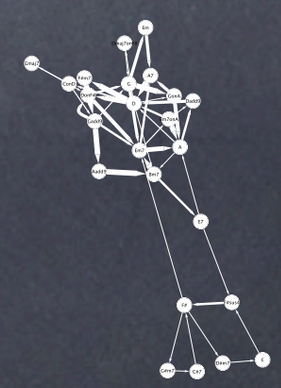
RAINY DAYS AND MONDAY

GOODBY TO LOVE

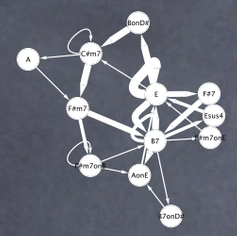
SING

ONLY YESTERDAY

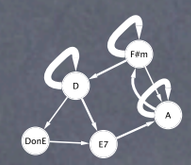
I WON'T LAST A DAY WITHOUT YOU



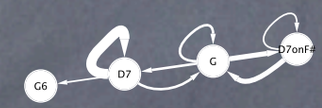
SOLITAIRE



PLEASE MR.POSTMAN



JAMBALAYA
(ON THE BAYOU)

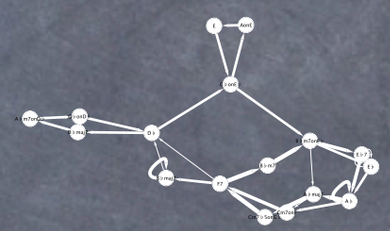


HURTING EACH OTHER

THERE'S A KIND OF HUSH
(ALL OVER THE WORLD)

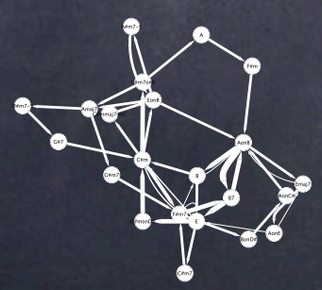
FOR ALL WE KNOW

BLESS THE BEASTS
AND CHILDREN

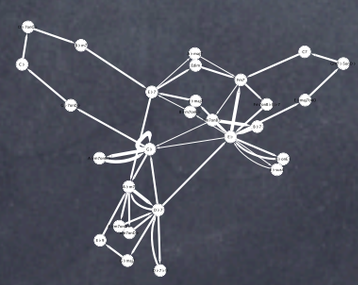


(THEY LONG TO BE)
CLOSE TO YOU

YESTERDAY ONCE MORE



THOSE GOOD OLD DREAMS

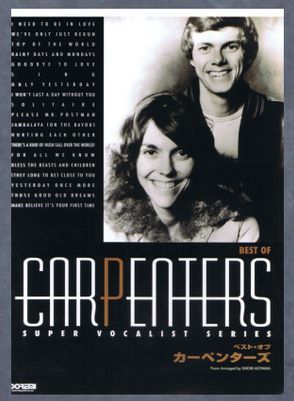


MAKE BELIEVE IT'S
YOUR FIRST TIME

Sequential Chord Network of Music [Integrated]

Carpenters

major 19 songs



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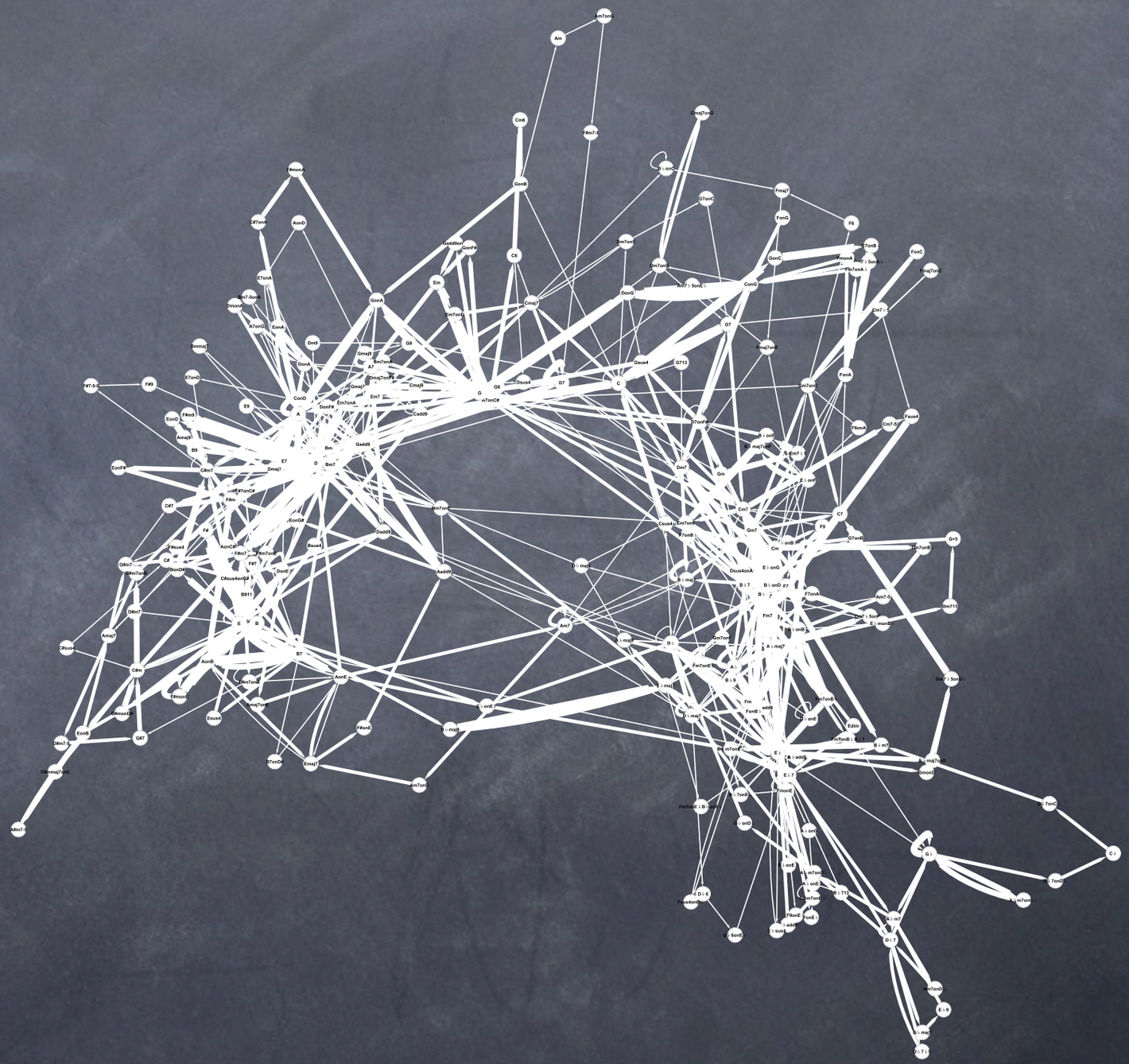
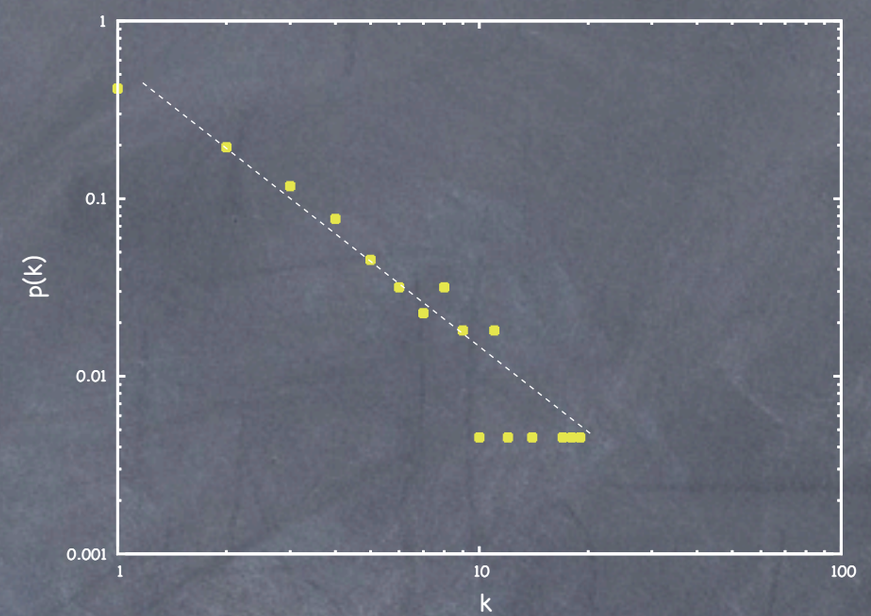


Sequential Chord Network of Music [Integrated]

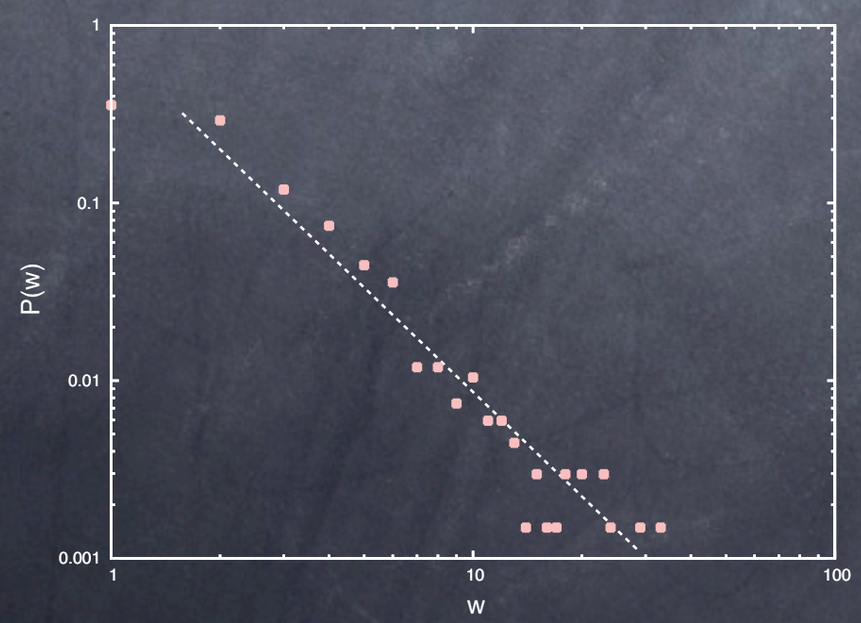
Carpenters

major 19 songs

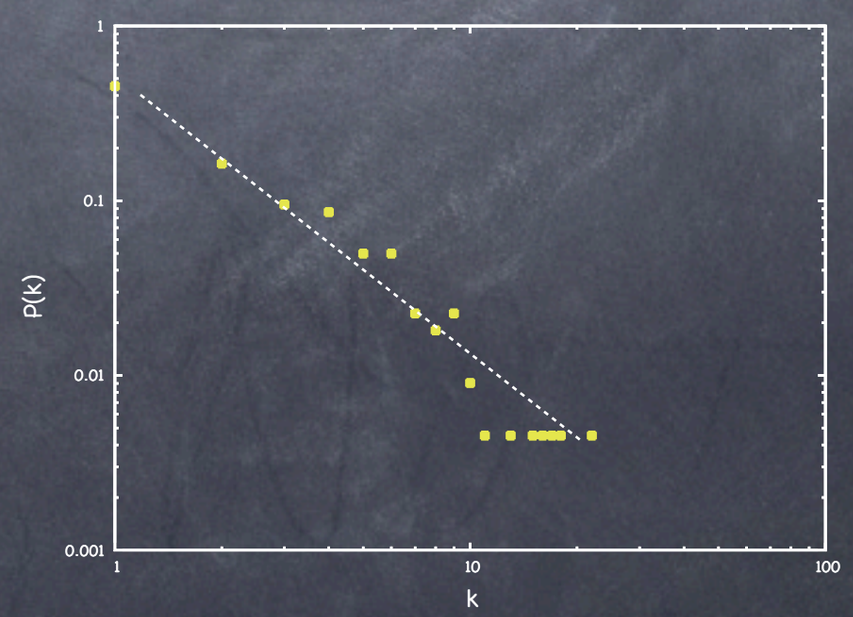
in-degree distribution



weight distribution



out-degree distribution

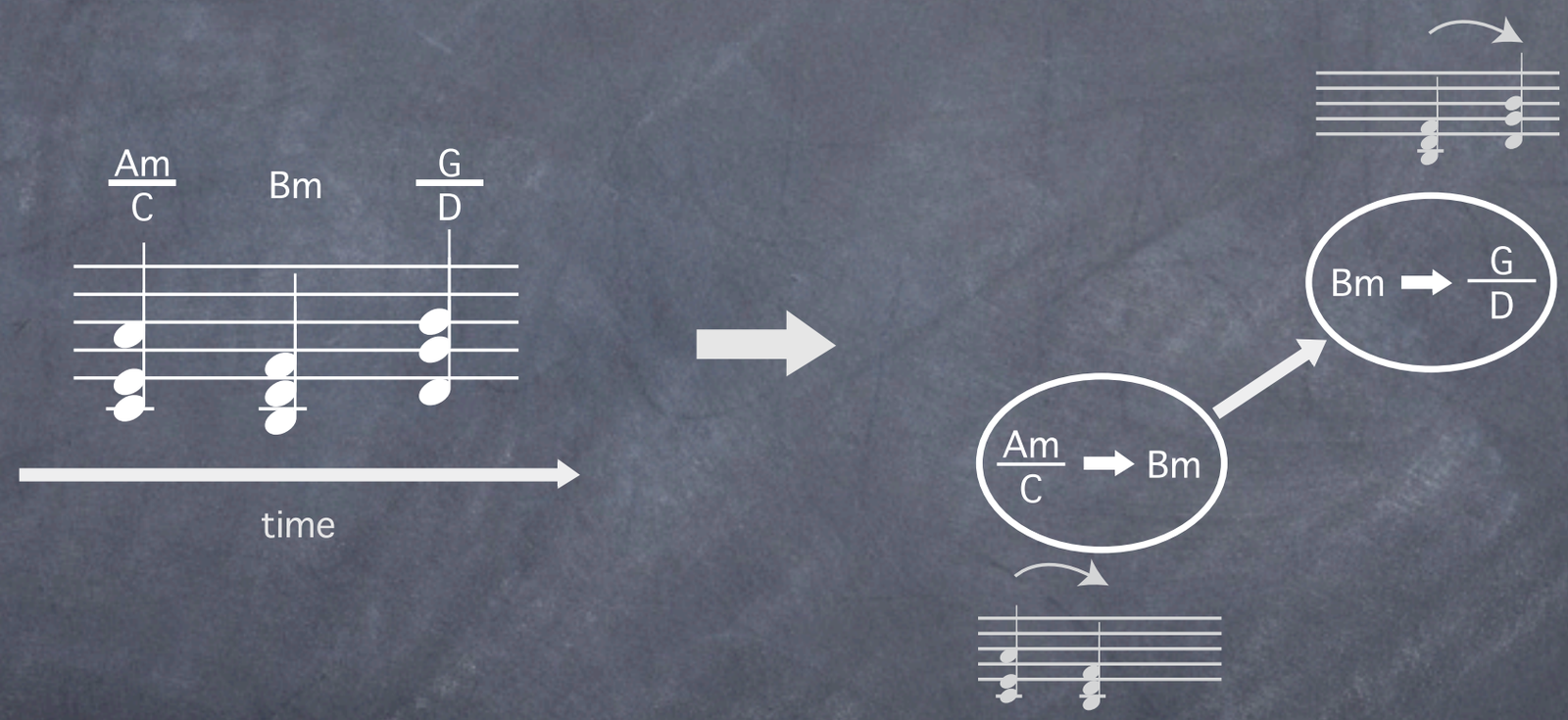


Chord-Transition Networks of Music

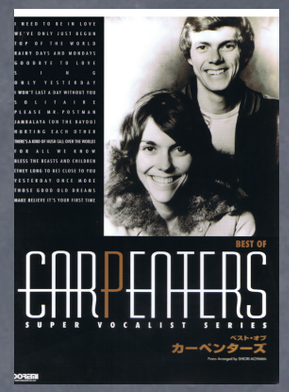
Dynamics as Networks

the dynamics of system/phenomena are represented as networks

Sequential Chord-Transition Network of Music



Sequential Chord-Transition Network of Music



I NEED TO BE IN LOVE

WE'VE ONLY JUST BEGUN

TOP OF THE WORLD

RAINY DAYS AND MONDAY

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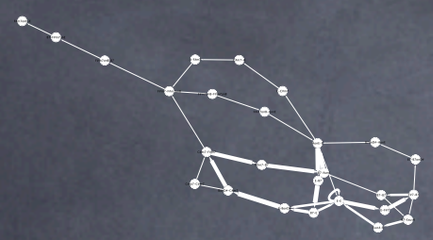
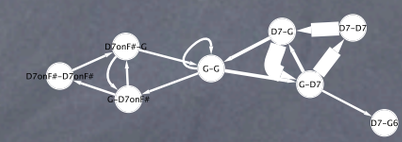
SOLITAIRE



PLEASE MR.POSTMAN

JAMBALAYA
(ON THE BAYOU)



HURTING EACH OTHER

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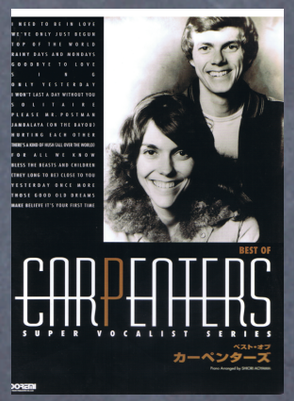
Carpenters

major 19 songs



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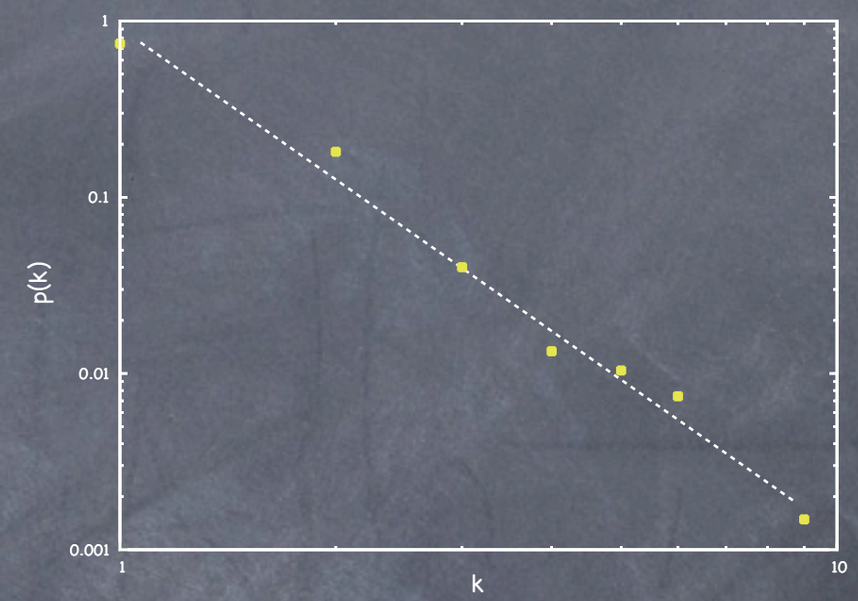
MAKE BELIEVE IT'S YOUR FIRST TIME

Sequential Chord-Transition Network of Music [Integrated]

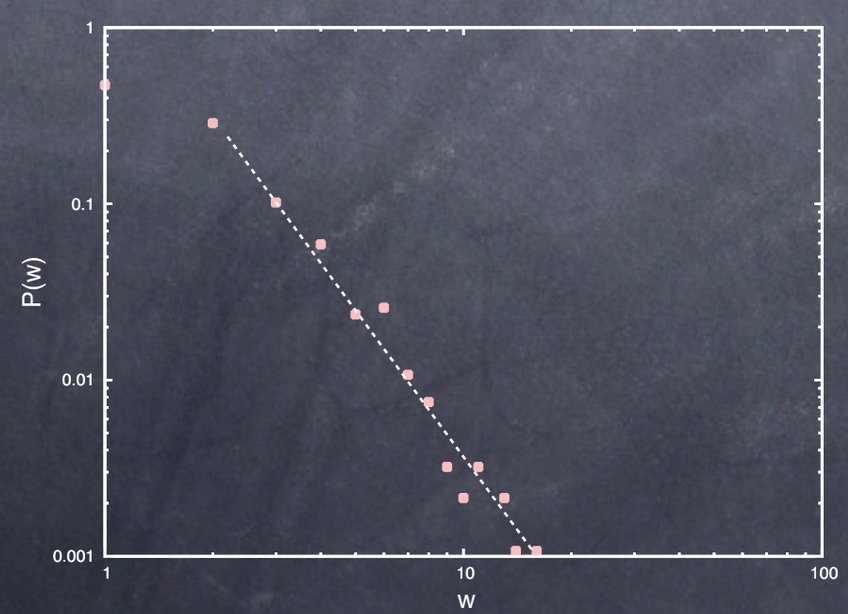
Carpenters

major 19 songs

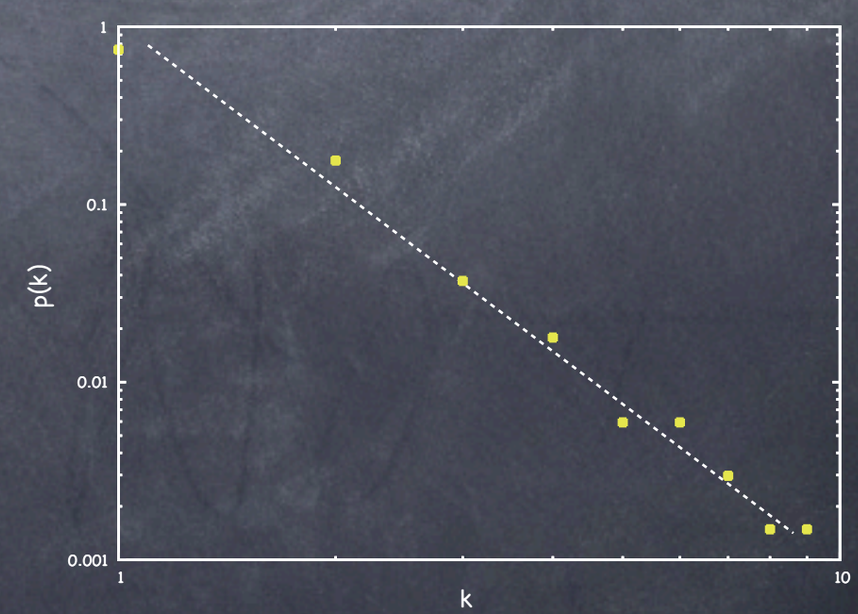
in-degree distribution



weight distribution



out-degree distribution



Papers & Presentations

- 広瀬 隼也, 井庭 崇, 「コードを基にした楽曲ネットワークの可視化と分析」, 情報処理学会 第5回ネットワーク生態学シンポジウム, 沖縄, 2009年3月

- T. Iba, "Network Analysis for Understanding Dynamics", International School and Conference on Network Science 2010 (NetSci2010), 2010

Collaborators

Junya Hirose

Former Graduate Student

Collaboration Networks of Wikipedia

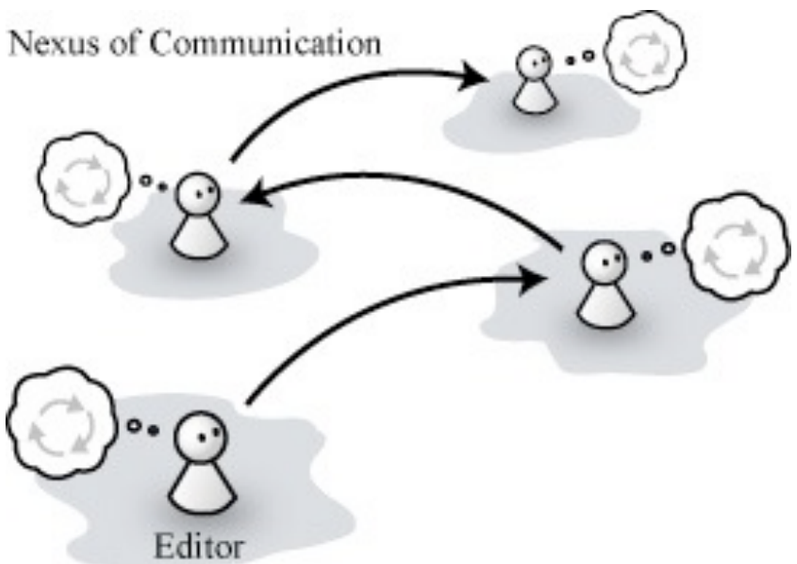
Dynamics as Networks

the dynamics of system/phenomena are represented as networks

Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- The characteristics of collaboration patterns of *all* articles in a certain language.

- The commonality and differences of collaboration patterns among Wikipedias written in various languages.



Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles

Editorial Collaboration Networks of Wikipedia Articles in Various Languages

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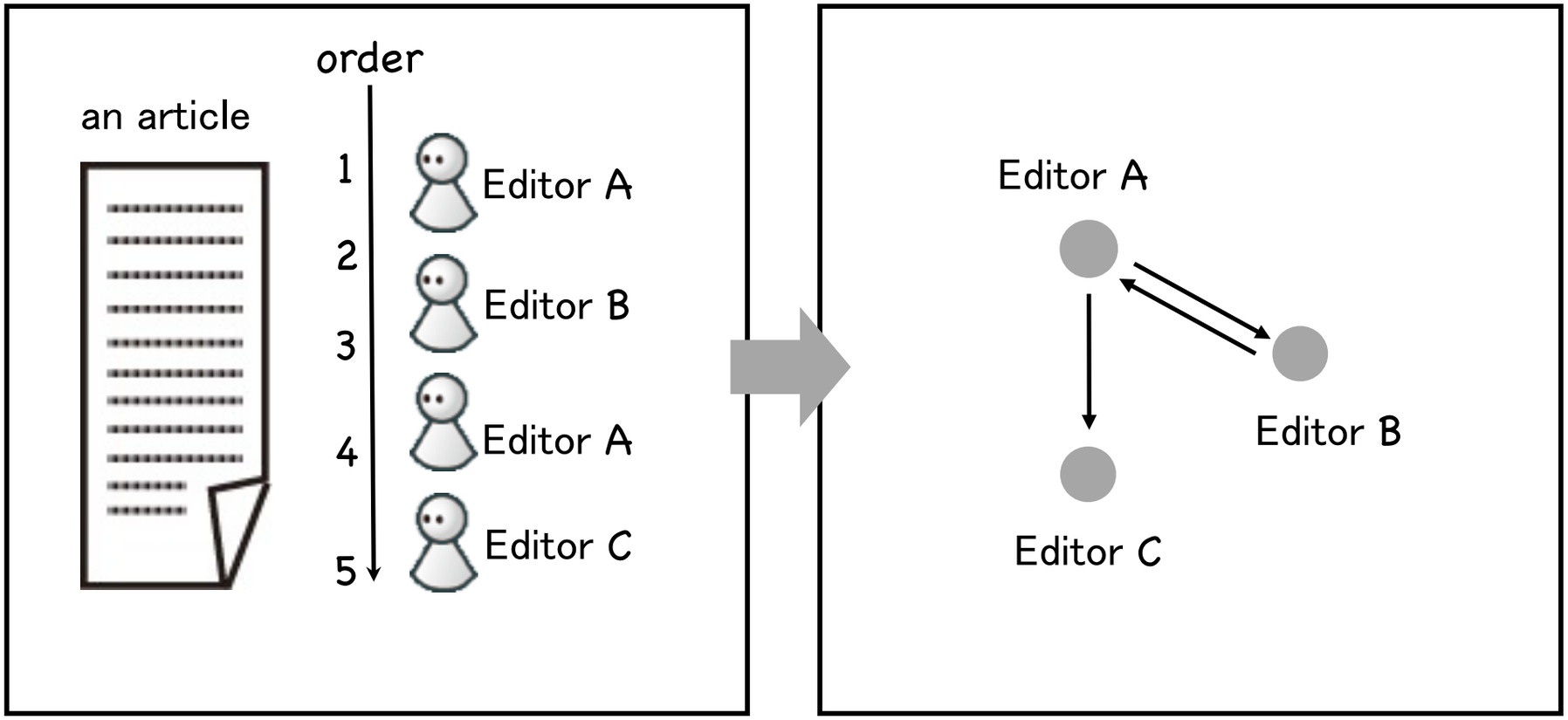
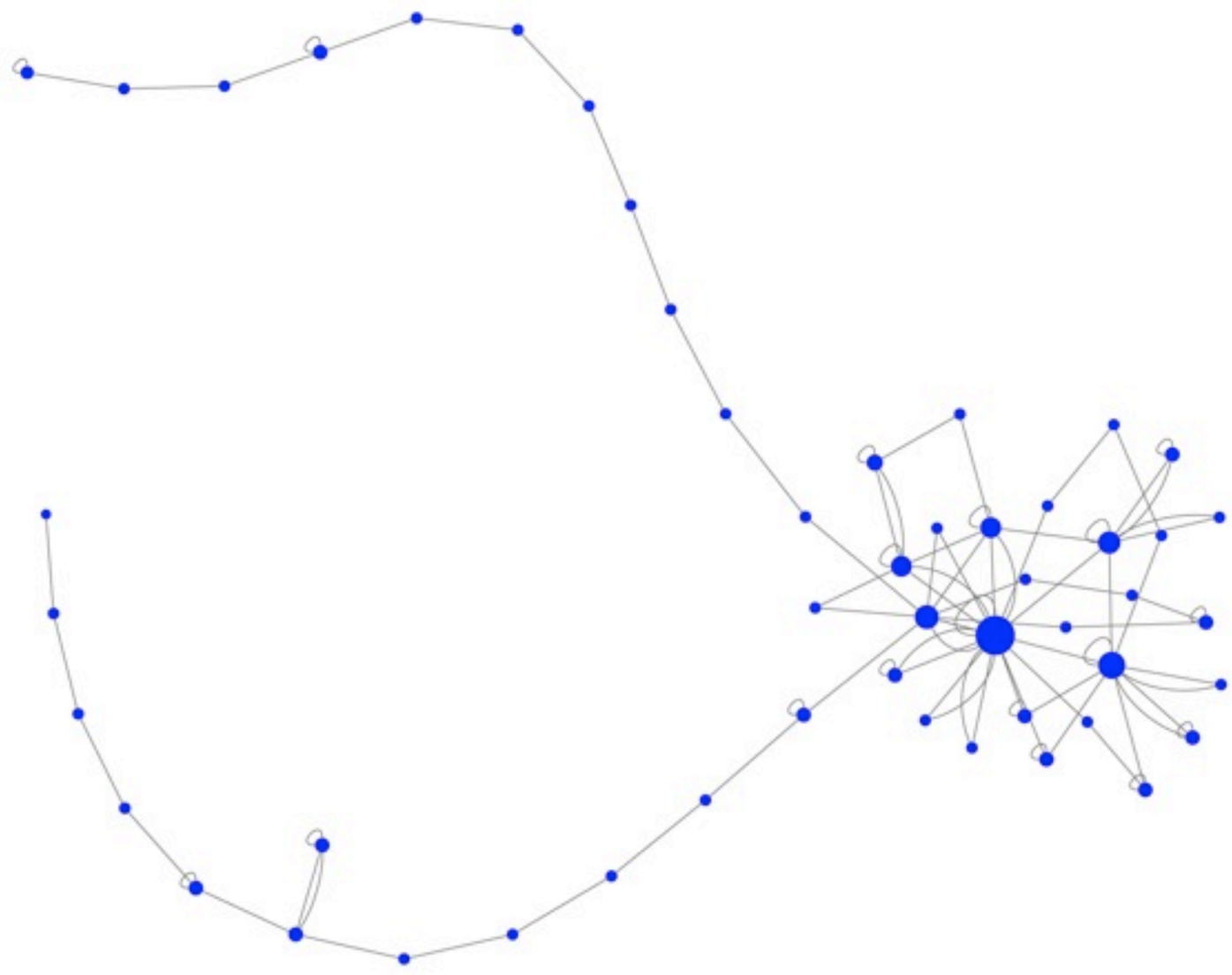
■ Method: Sequential collaboration network

Building a *sequential* collaboration network, connecting a relation from editor A to editor B, if editor B follows on work done by editor A.

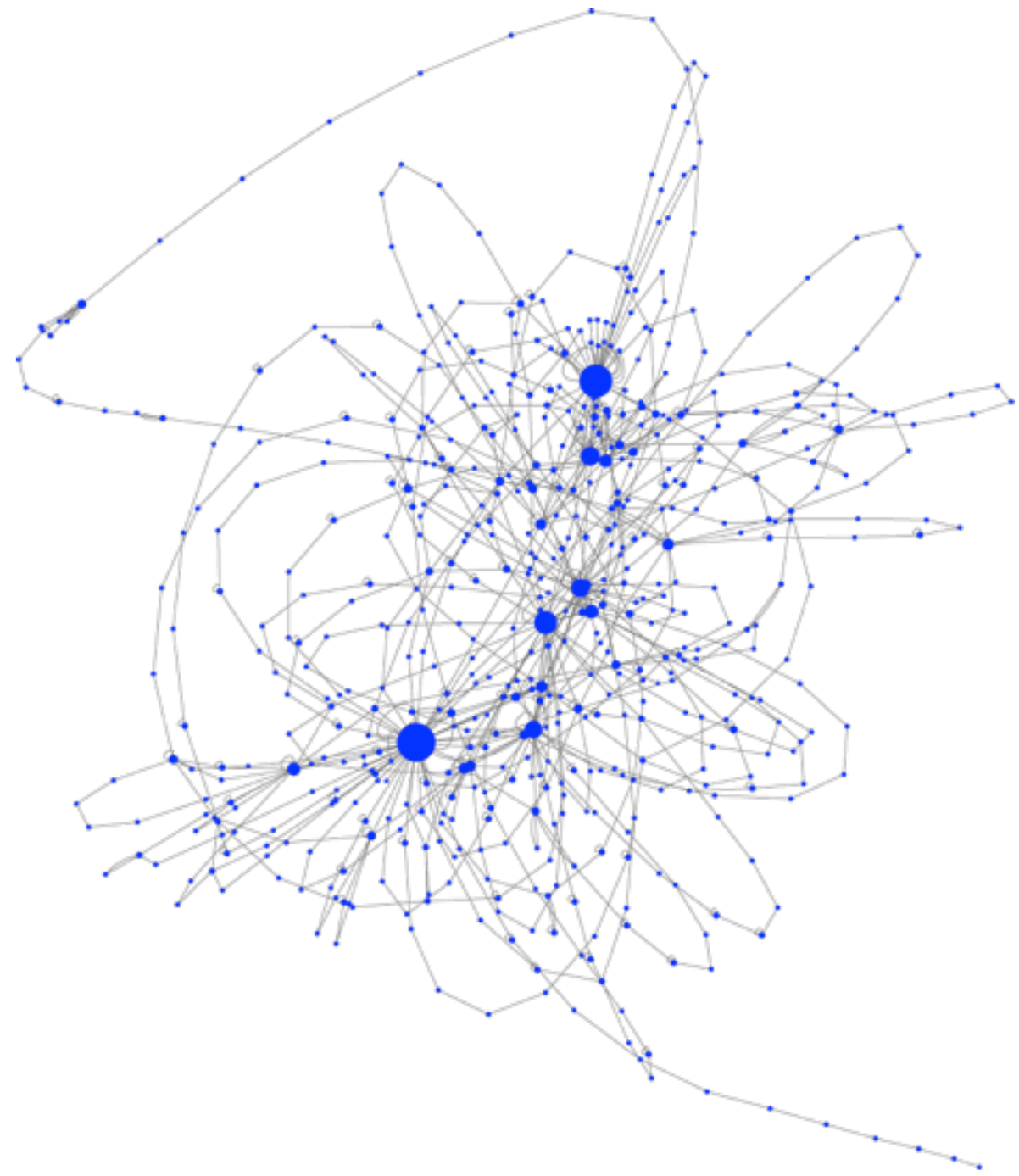


Sequential Collaboration Network of Article “Collaborative Innovation Networks” in English Wikipedia



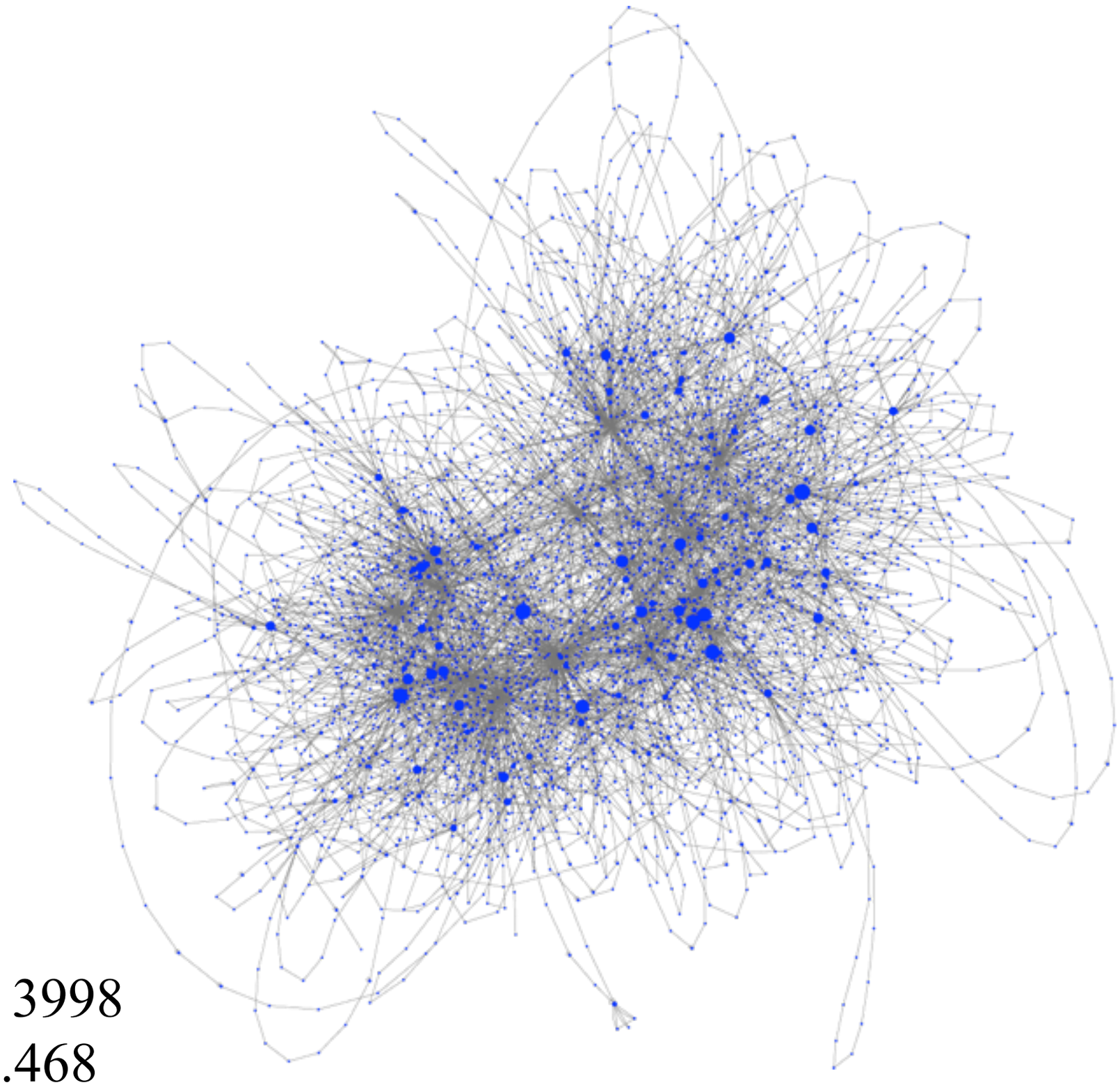
The number of Nodes = 51
Average path length = 6.399

Sequential Collaboration Network of Article “Basel” in English Wikipedia



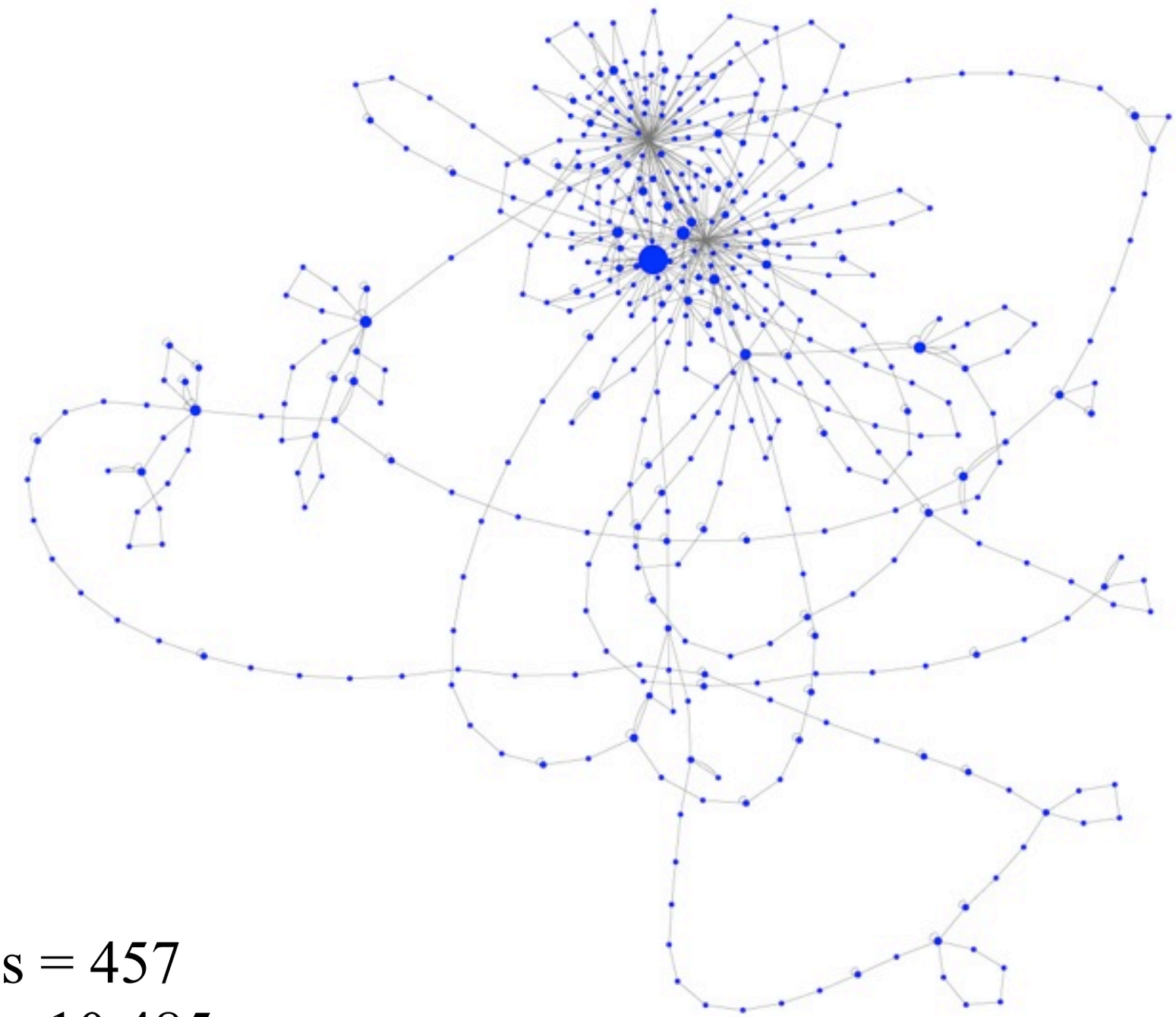
The number of Nodes = 594
Average path length = 6.577

Sequential Collaboration Network of Article “Switzerland” in English Wikipedia



The number of Nodes = 3998
Average path length = 5.468

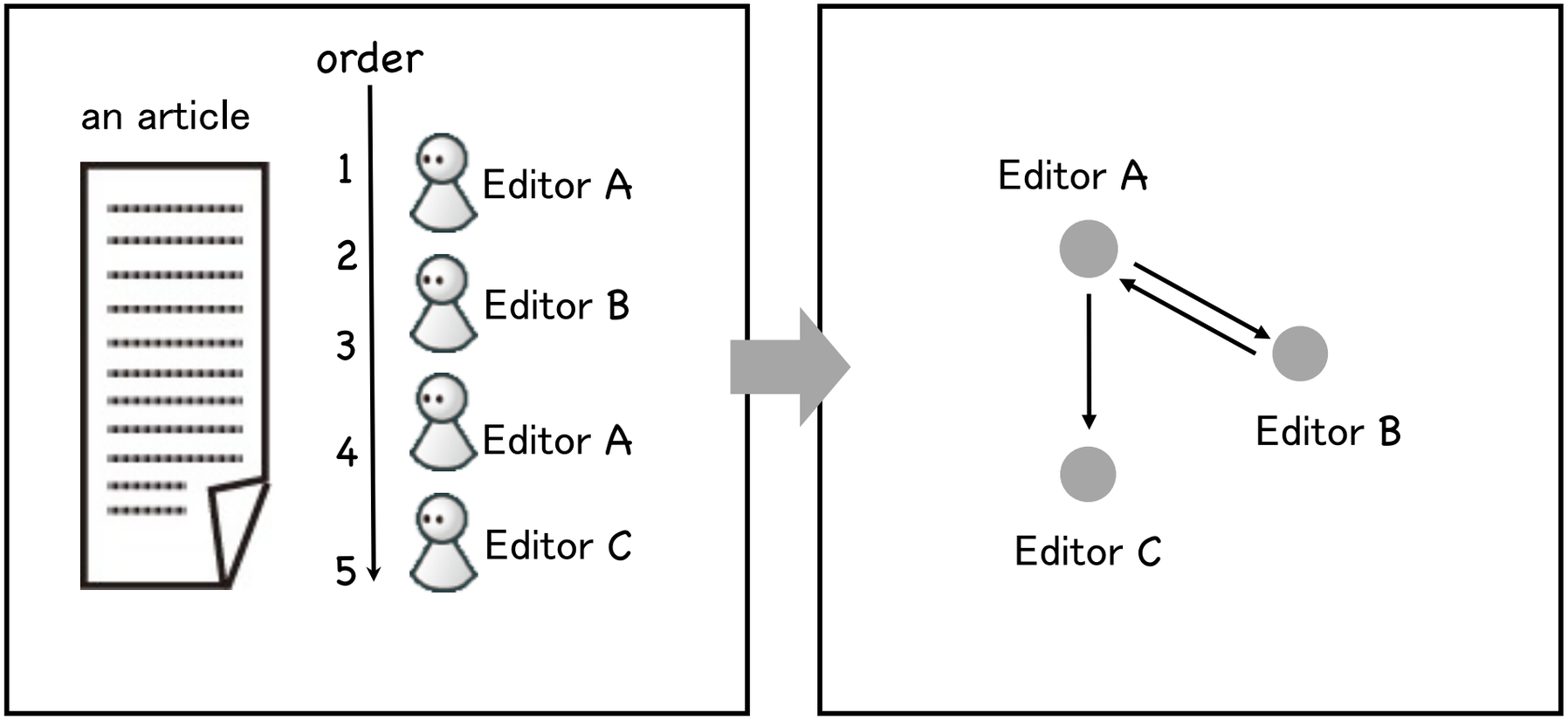
Sequential Collaboration Network of Article “Fondue” in English Wikipedia



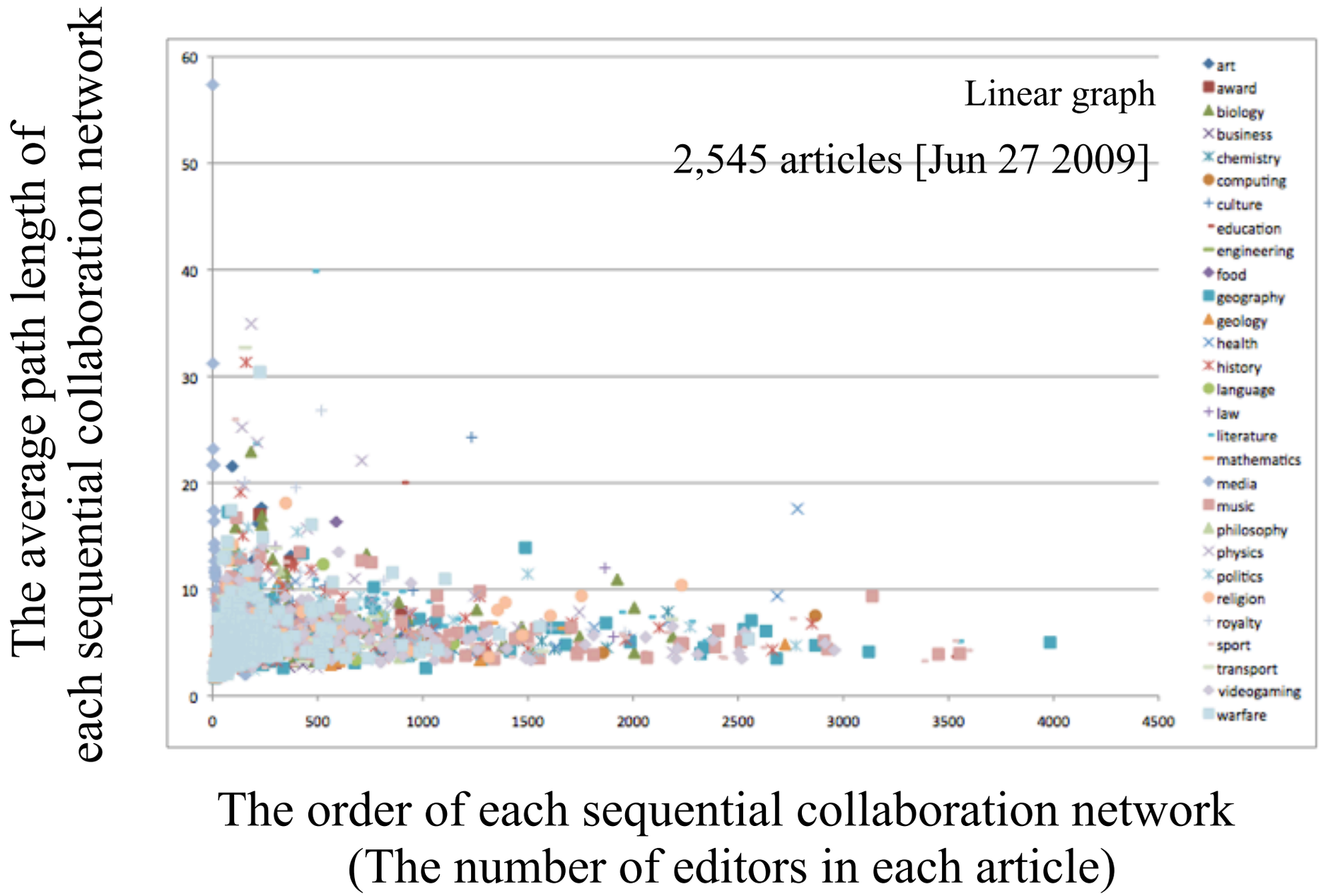
The number of Nodes = 457
Average path length = 10.485

■ Method: Sequential collaboration network

Building a *sequential* collaboration network, connecting a relation from editor A to editor B, if editor B follows on work done by editor A.



Our Previous Study: Featured Articles in English Wikipedia



T. Iba, K. Nemoto, B. Peters & P. Gloor, "Analyzing the Creative Editing Behavior of Wikipedia Editors Through Dynamical Social Network Analysis", COINs2011, 2009
T. Iba and S. Itoh, "Sequential Collaboration Network of Open Collaboration", NetSci'09, 2009

Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
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- Analysis 3: Distribution of Featured Articles

■Analysis 1: Comparison of 12 different languages

Analyzing ALL articles as of January 1st, 2011 in each language.

The ranking based on the data as of January 6th, 2011.

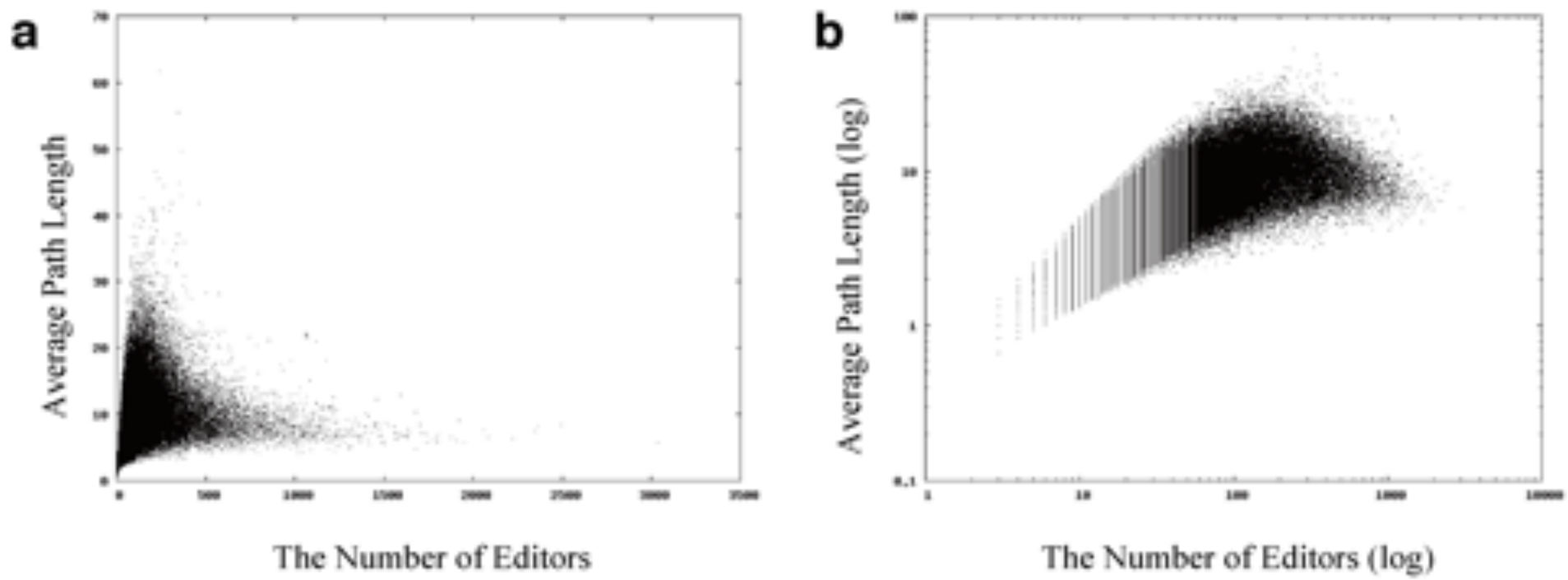
Target Languages

Rank 1: English
Rank 2: German
Rank 3: French
Rank 4: Polish
Rank 5: Italian
Rank 6: Japanese
Rank 7: Spanish
Rank 8: Dutch
Rank 9: Portuguese
Rank 10: Russian
…
Rank 15: Finnish
…
Rank 20: Turkish

English

Rank 1
3,490,325 articles

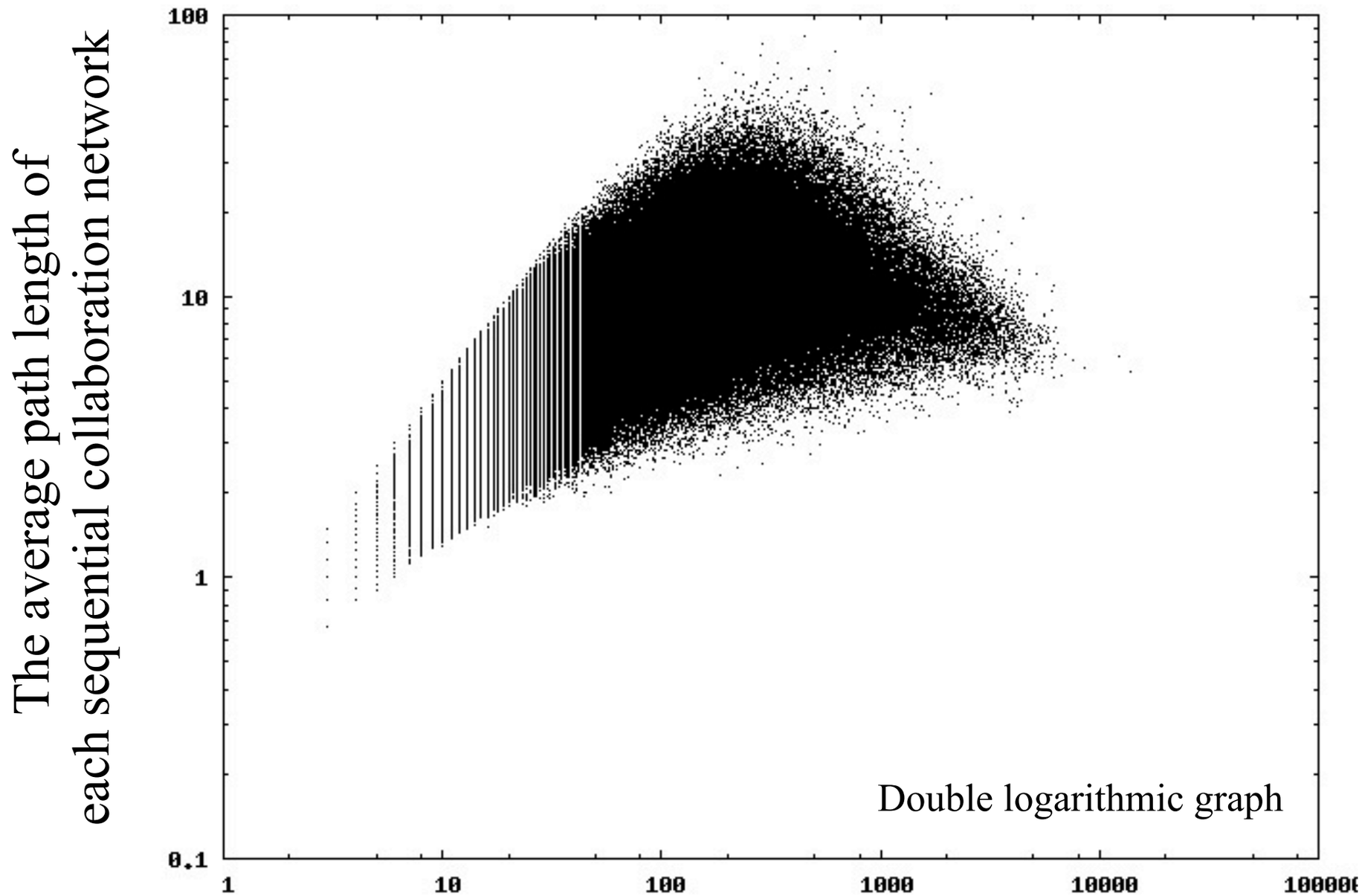
The average path length of each sequential collaboration network



The order of each sequential collaboration network
(The number of editors in each article)

English

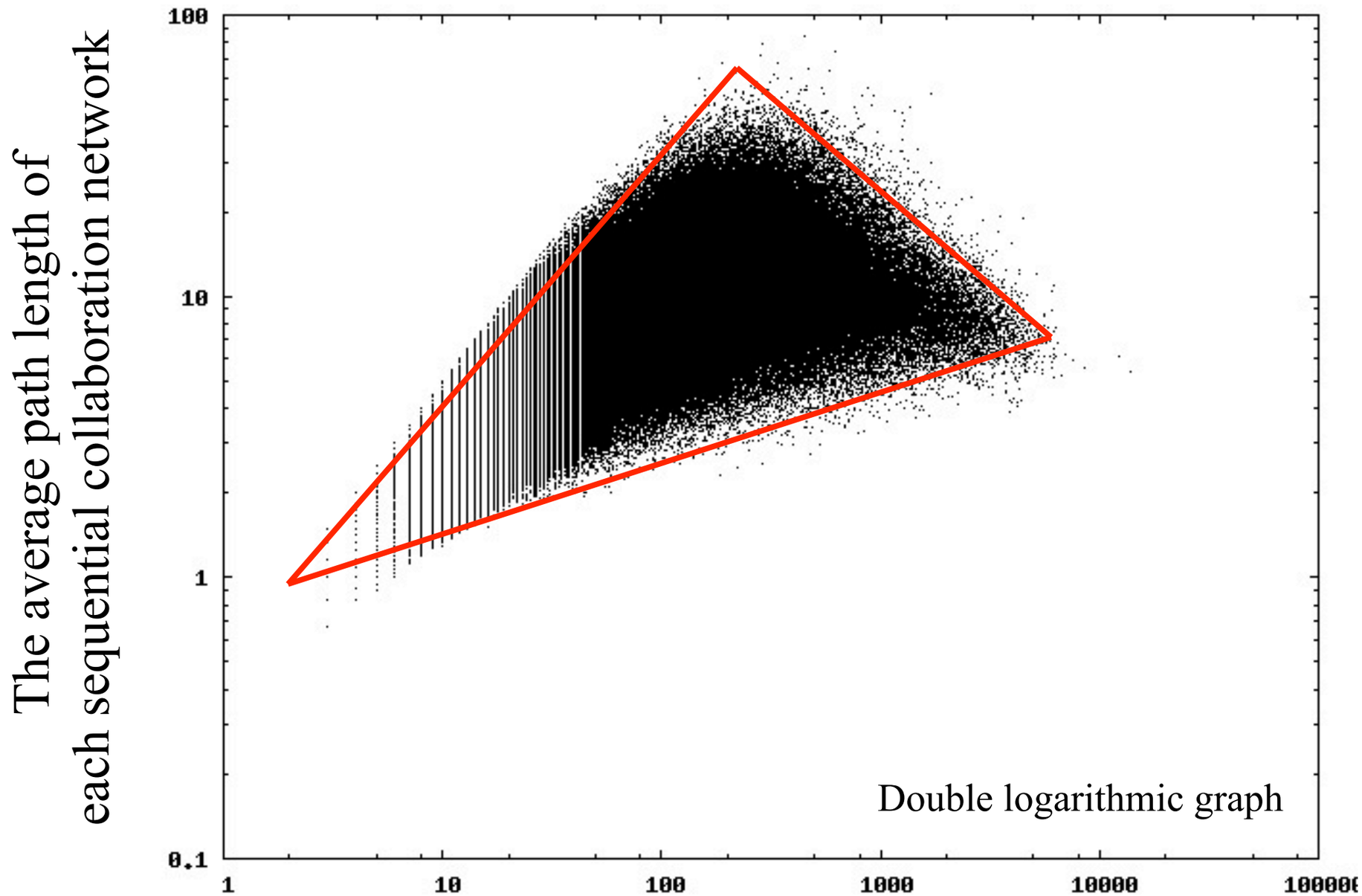
Rank 1
3,490,325 articles



The order of each sequential collaboration network
(The number of editors in each article)

English

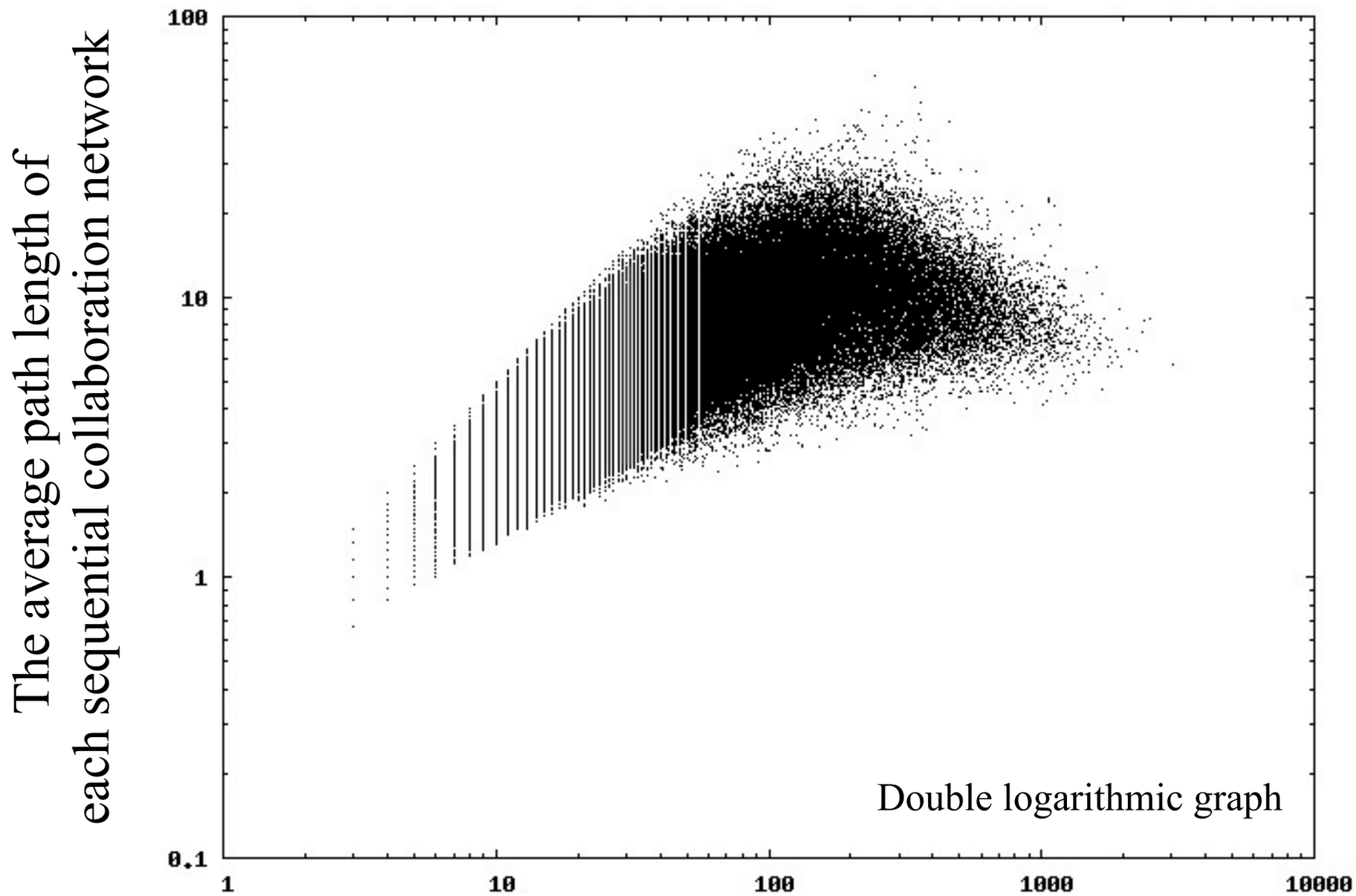
Rank 1
3,490,325 articles



The order of each sequential collaboration network
(The number of editors in each article)

German

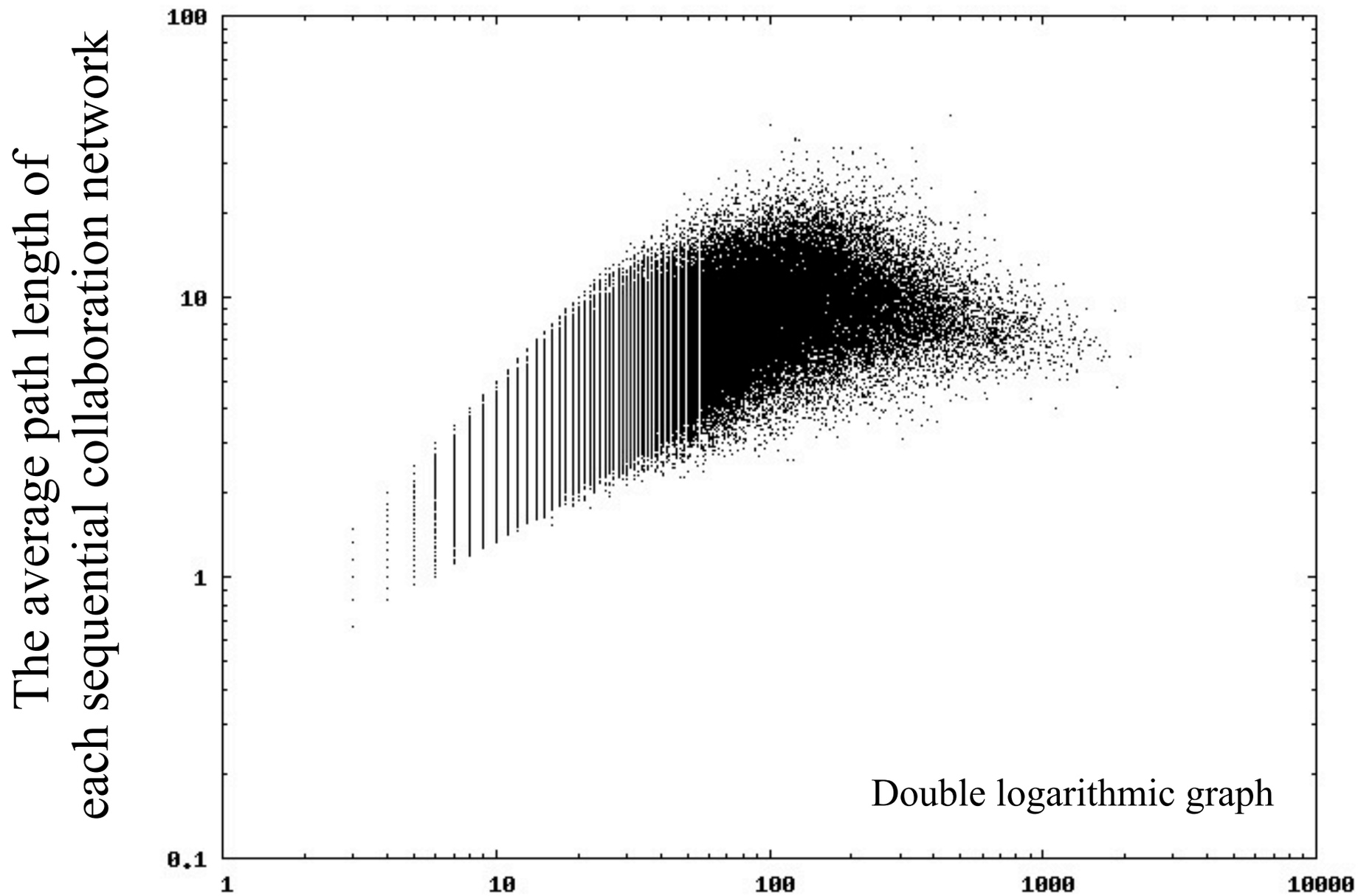
Rank 2
1,155,210 articles



The order of each sequential collaboration network
(The number of editors in each article)

French

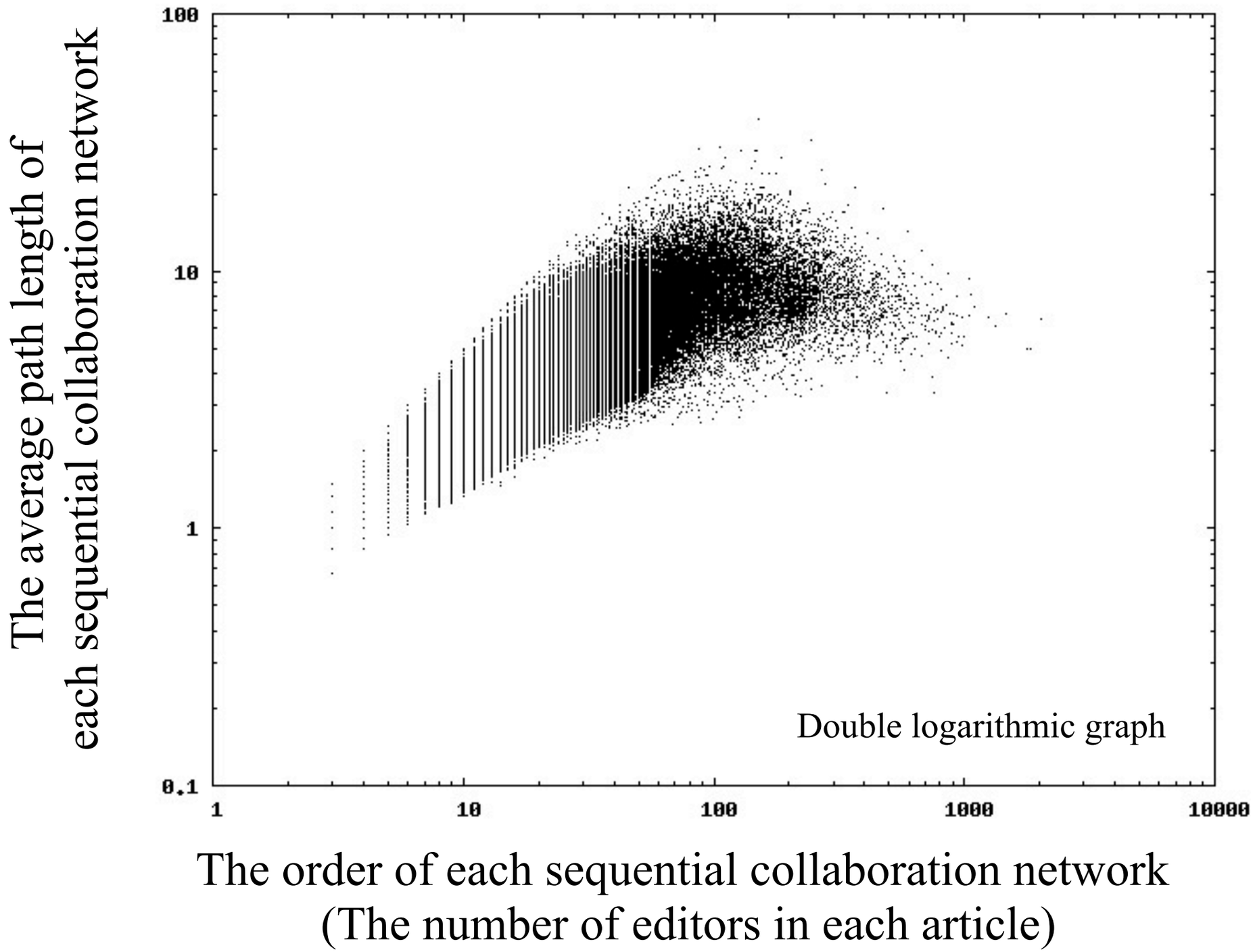
Rank 3
1,039,251 articles



The order of each sequential collaboration network
(The number of editors in each article)

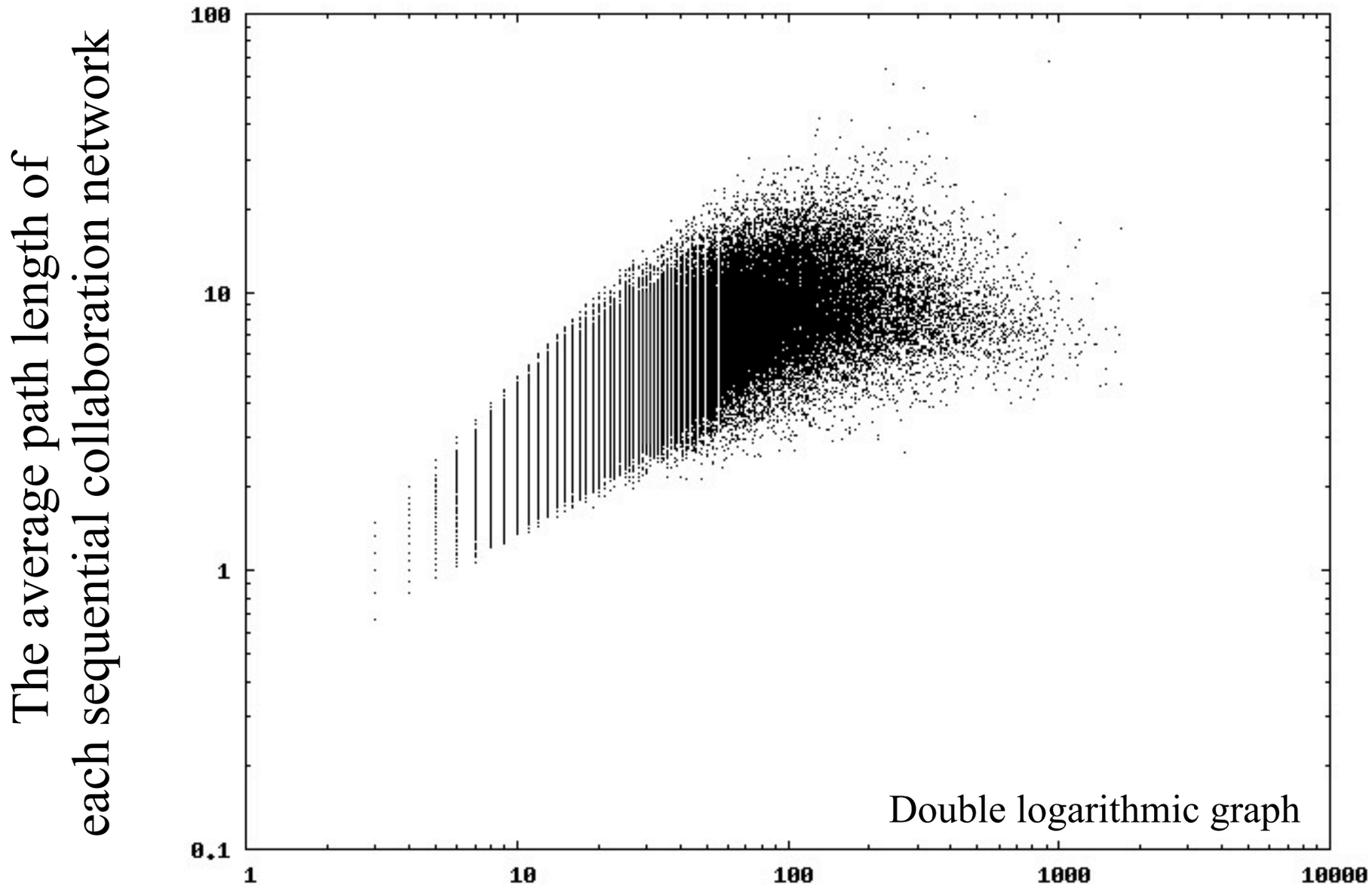
Polish

Rank 4
752,734 articles



Italian

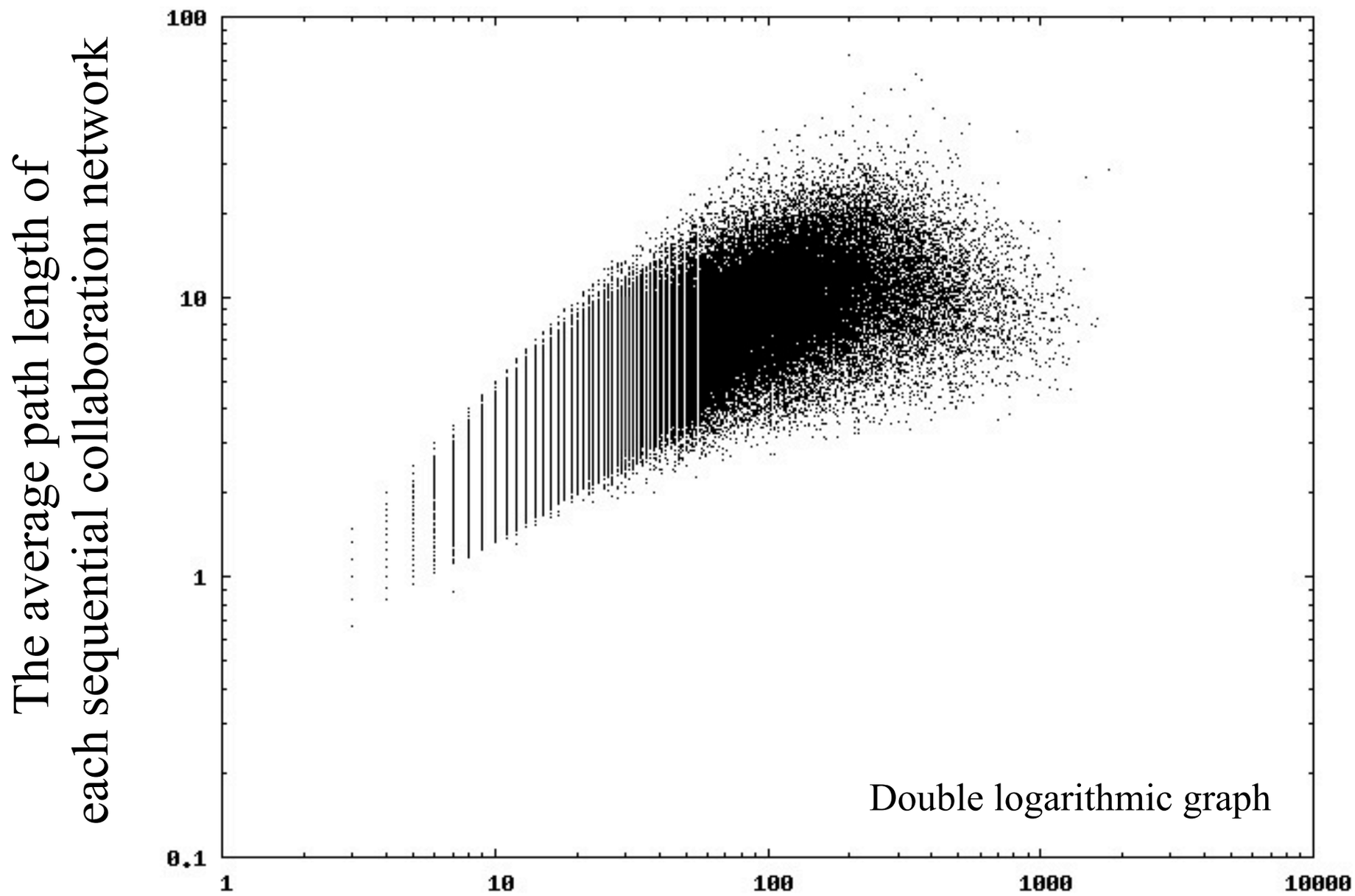
Rank 5
750,634 articles



The order of each sequential collaboration network
(The number of editors in each article)

Japanese

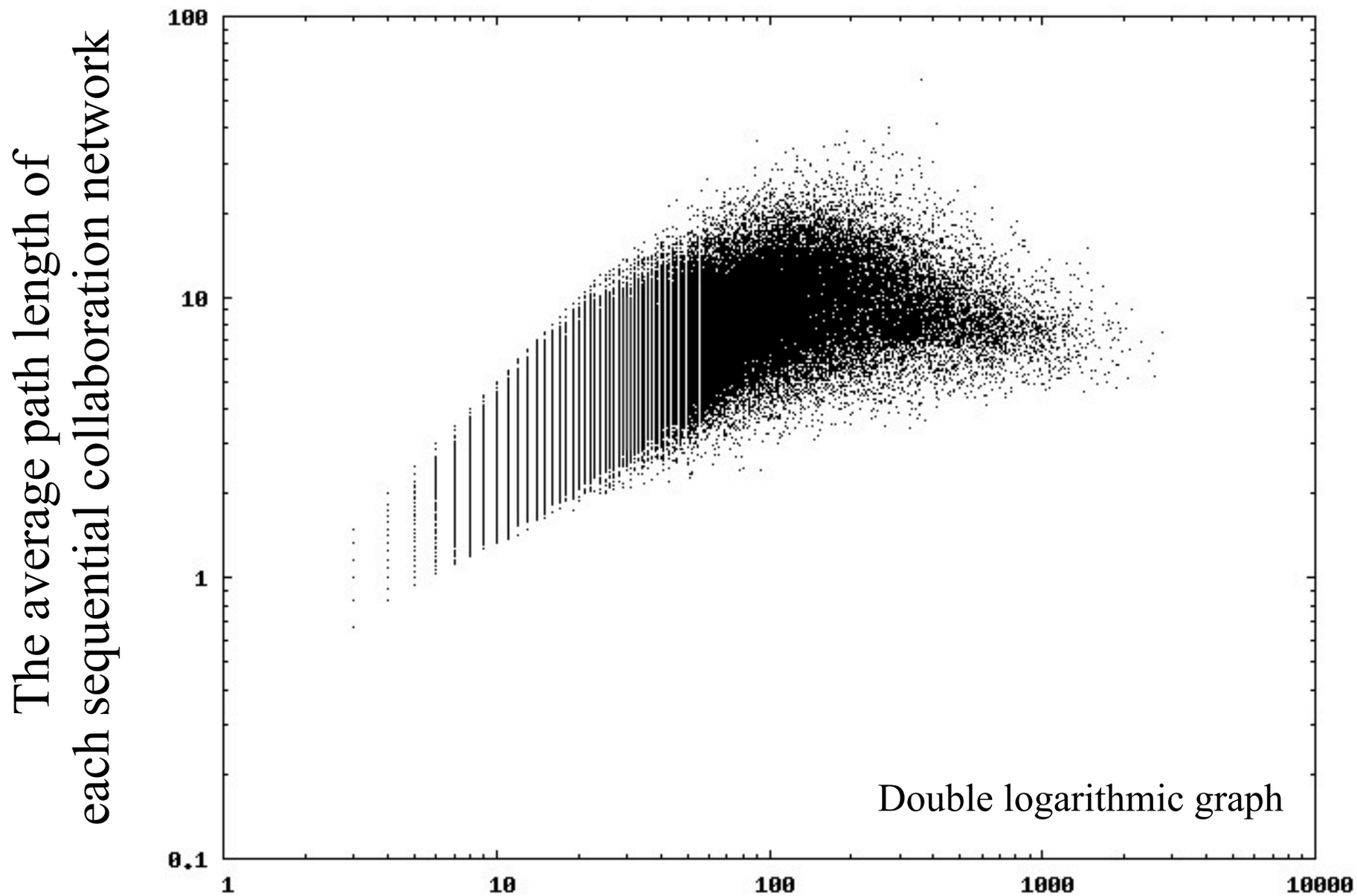
Rank 6
718,974 articles



The order of each sequential collaboration network
(The number of editors in each article)

Spanish

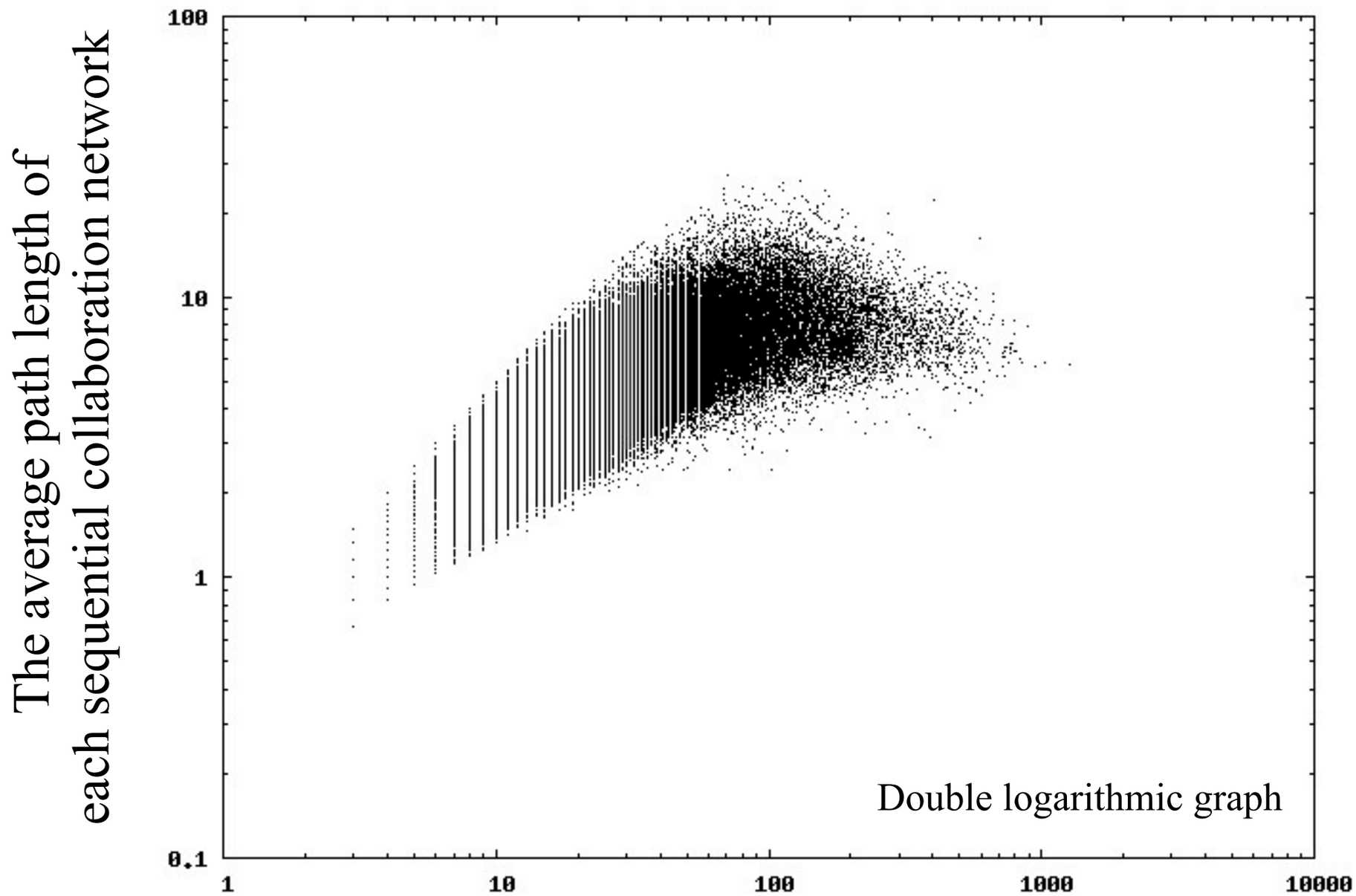
Rank 7
676,866 articles



The order of each sequential collaboration network
(The number of editors in each article)

Dutch

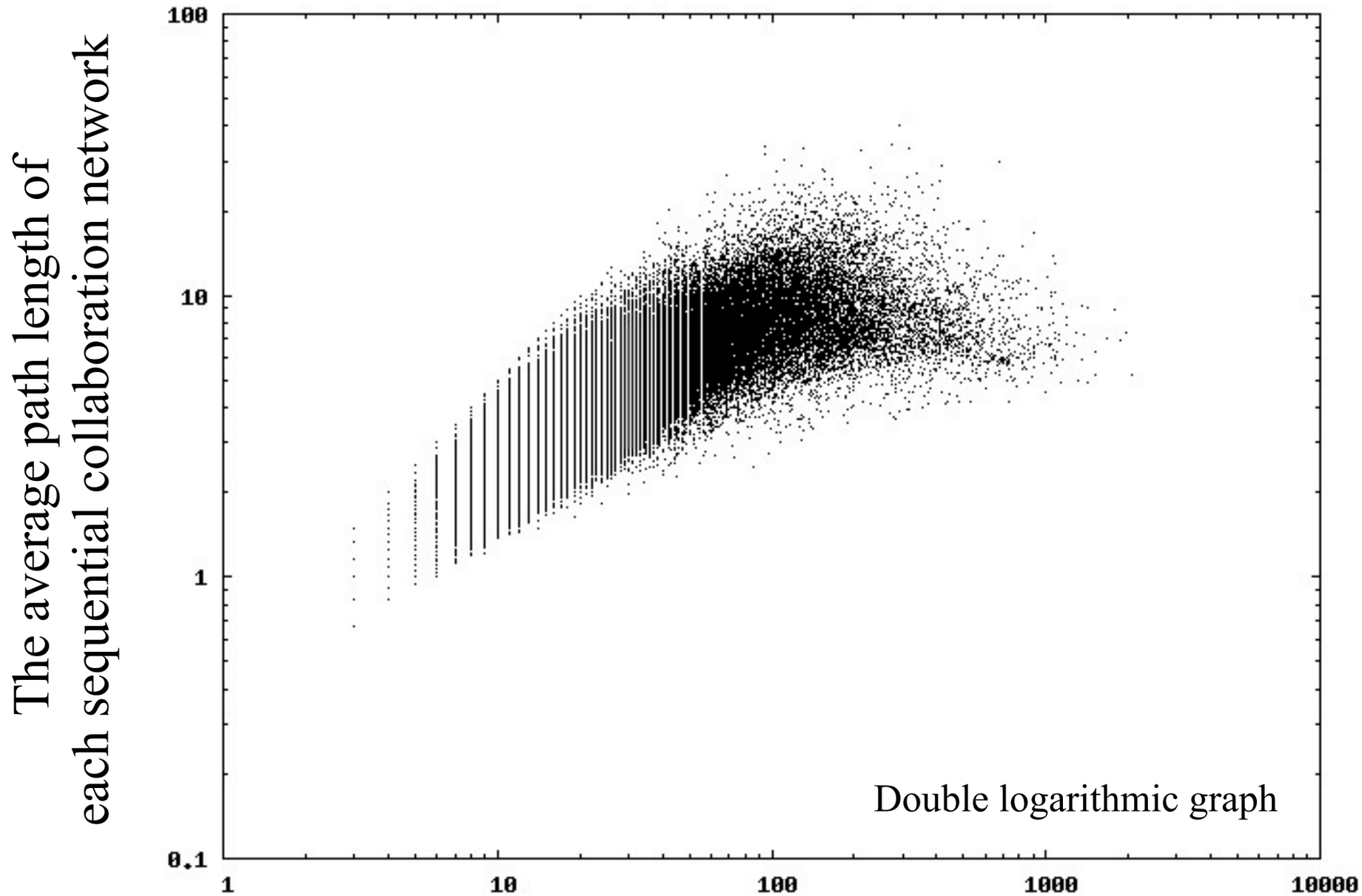
Rank 8
656,079 articles



The order of each sequential collaboration network
(The number of editors in each article)

Portuguese

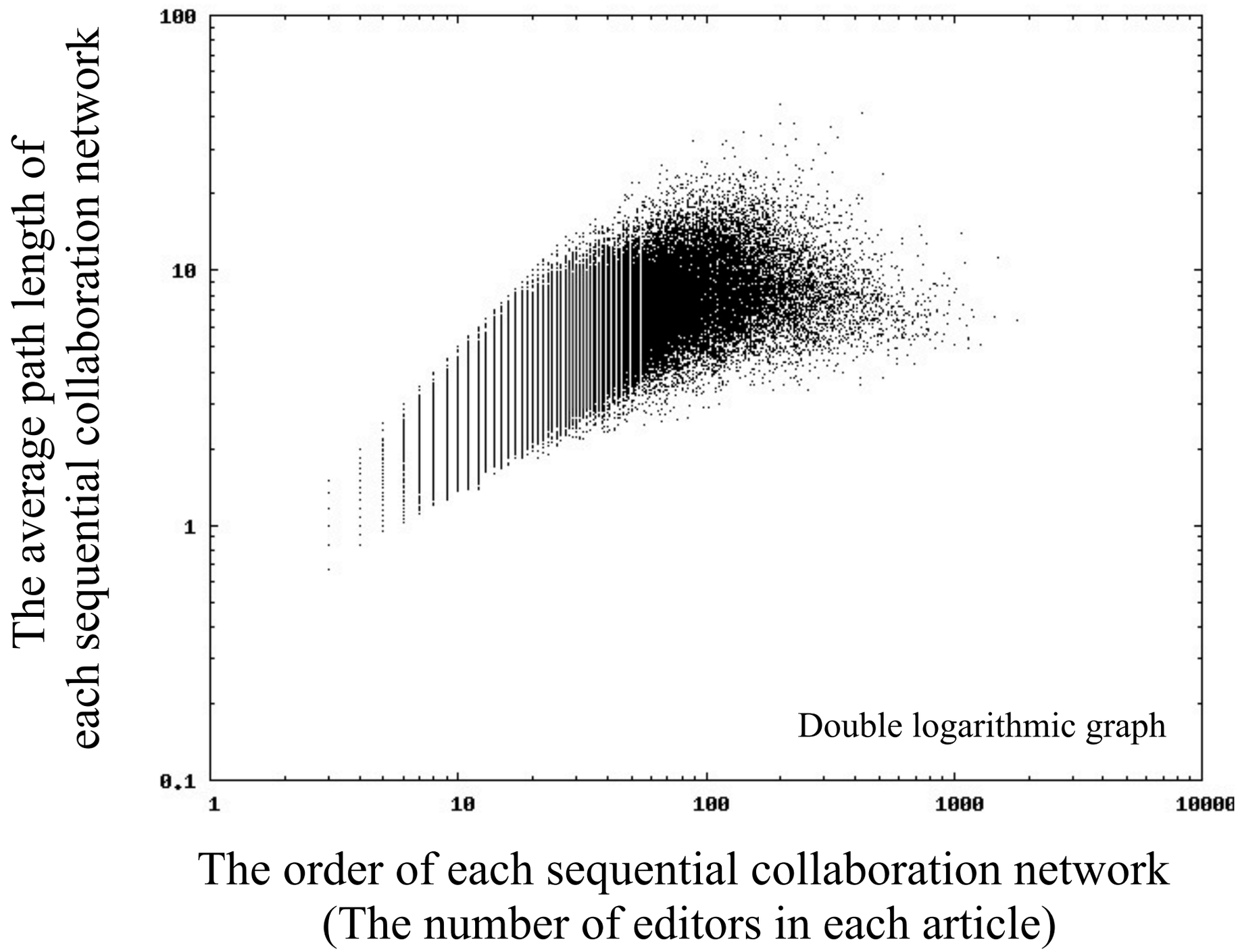
Rank 9
638,747 articles



The order of each sequential collaboration network
(The number of editors in each article)

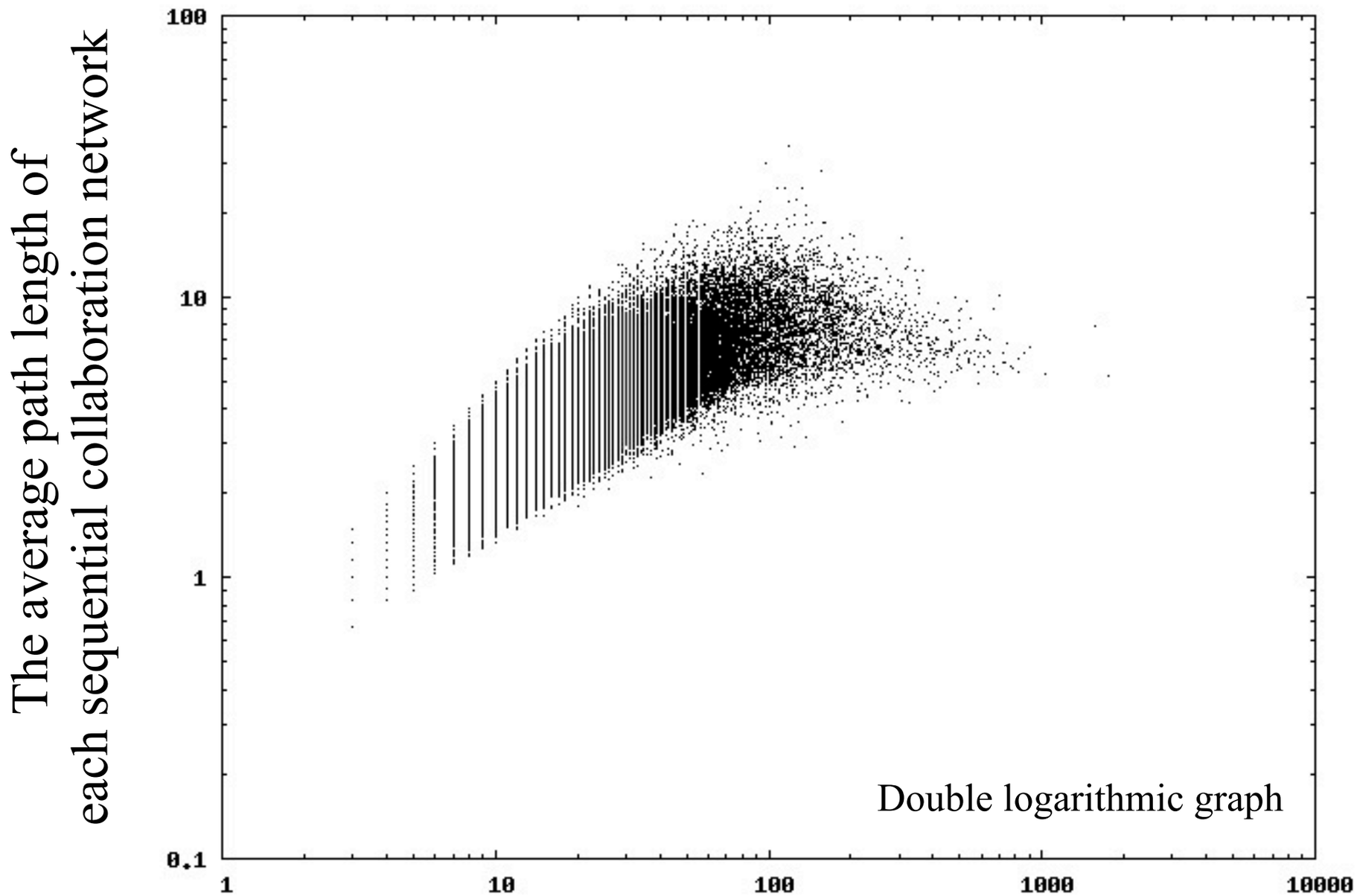
Russian

Rank 10
627,139 articles



Finnish

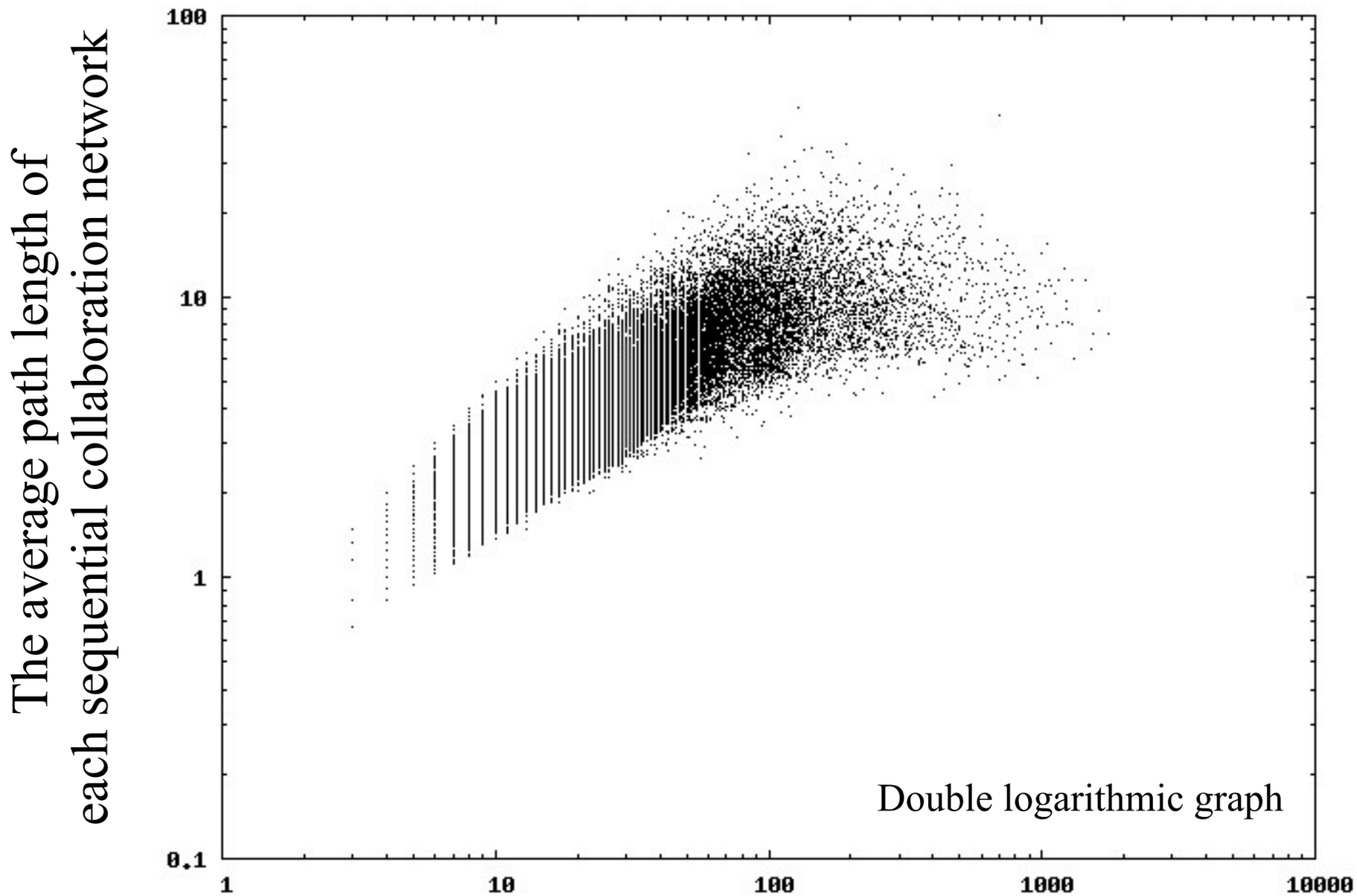
Rank 15
255,712 articles



The order of each sequential collaboration network
(The number of editors in each article)

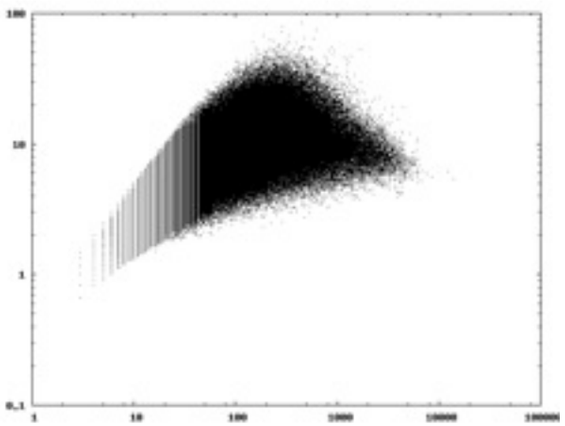
Turkish

Rank 20
152,262 articles

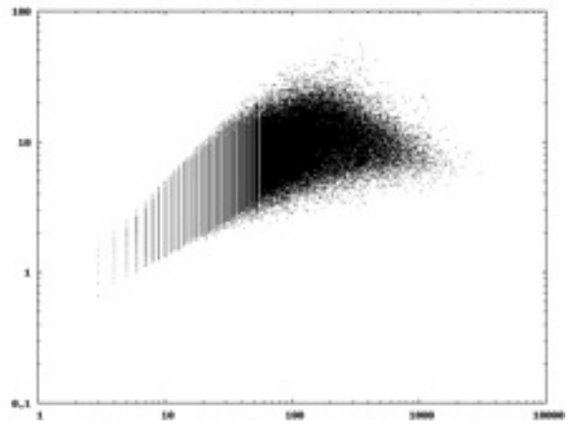


The order of each sequential collaboration network
(The number of editors in each article)

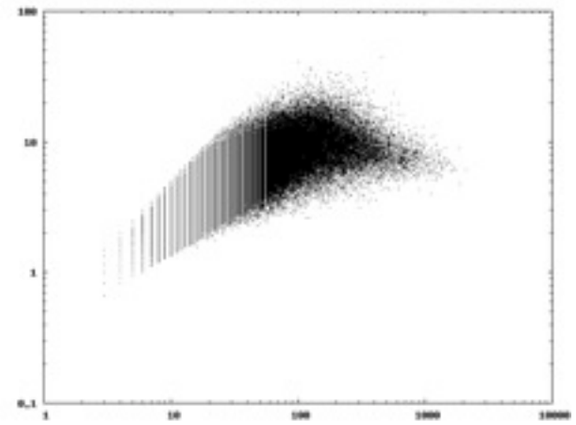
English



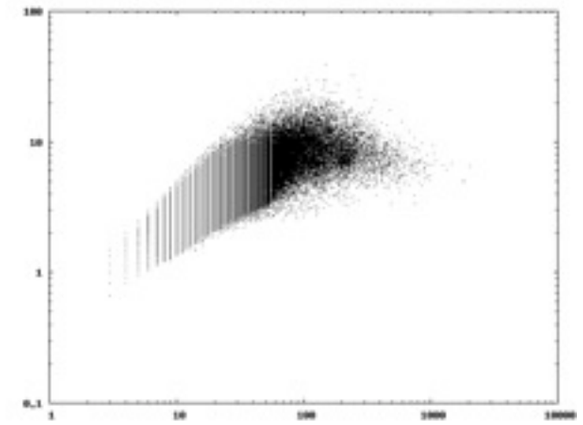
German



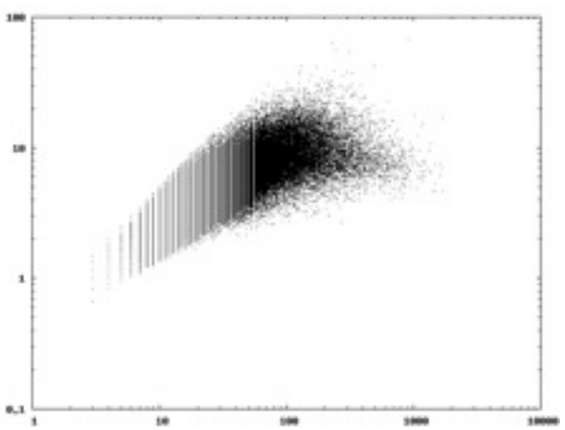
French



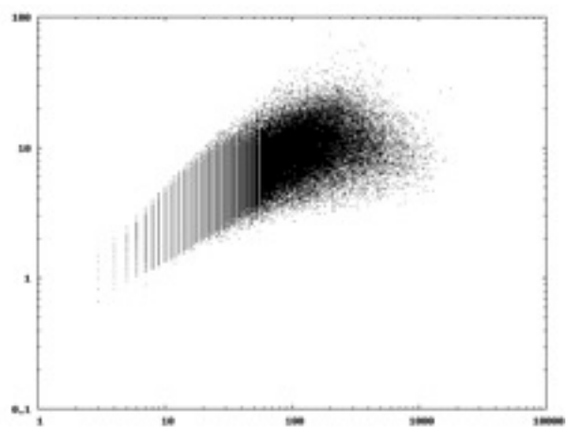
Polish



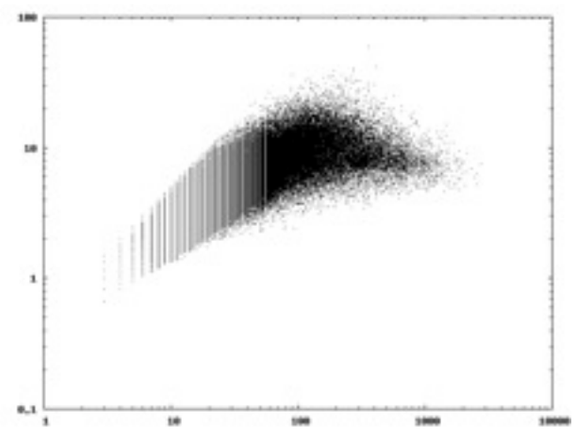
Italian



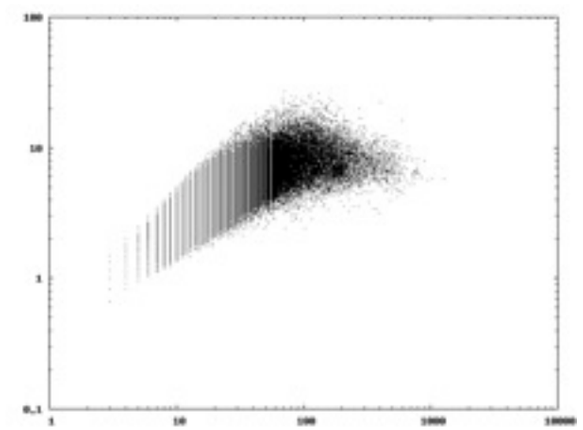
Japanese



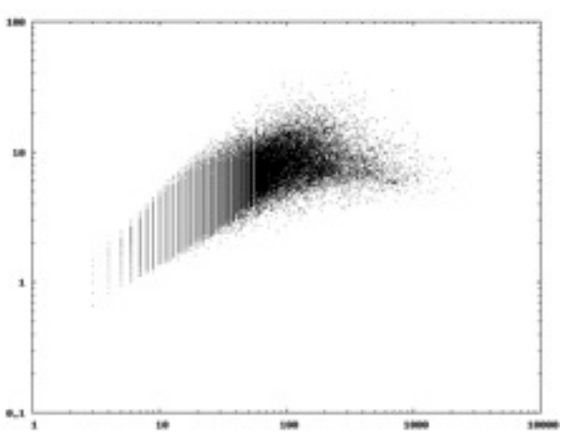
Spanish



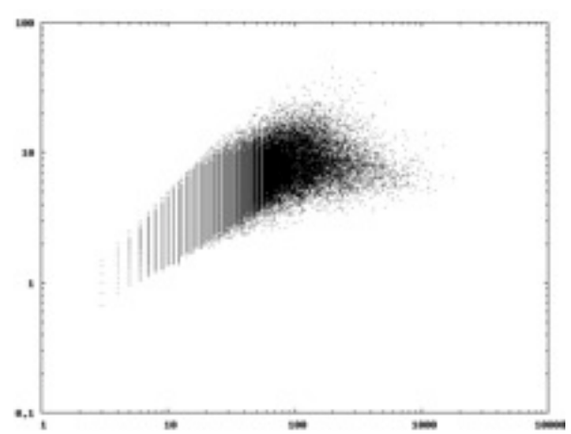
Dutch



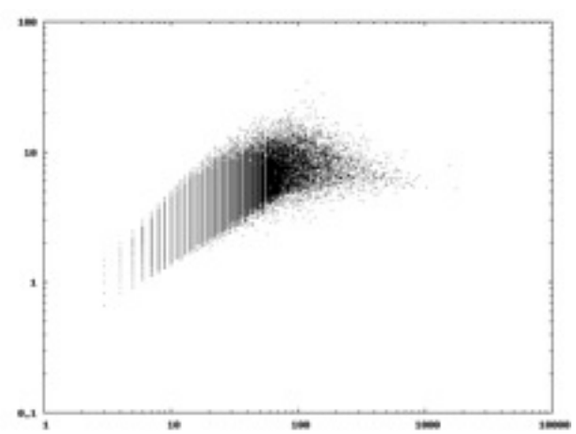
Portuguese



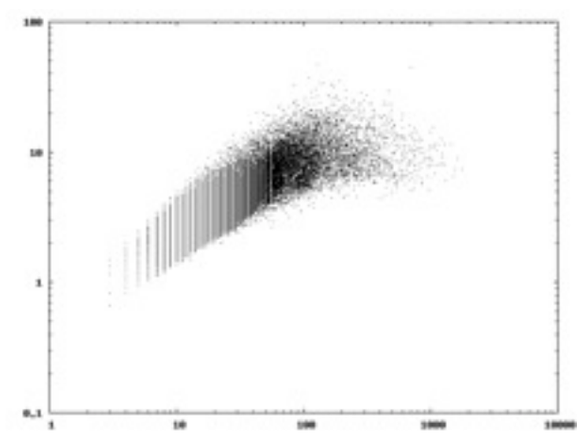
Russian



Finnish



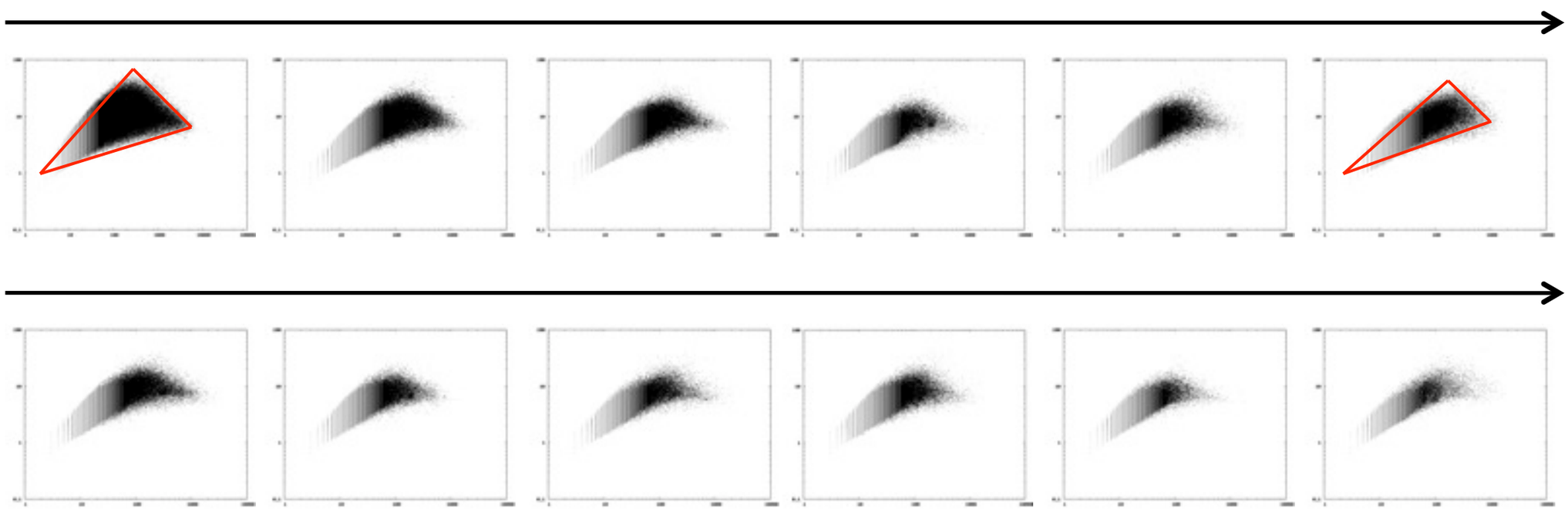
Turkish



Result of Analysis 1: Comparison of 12 different languages

- Scatter plot of all articles exhibits a tilted triangle in all languages.

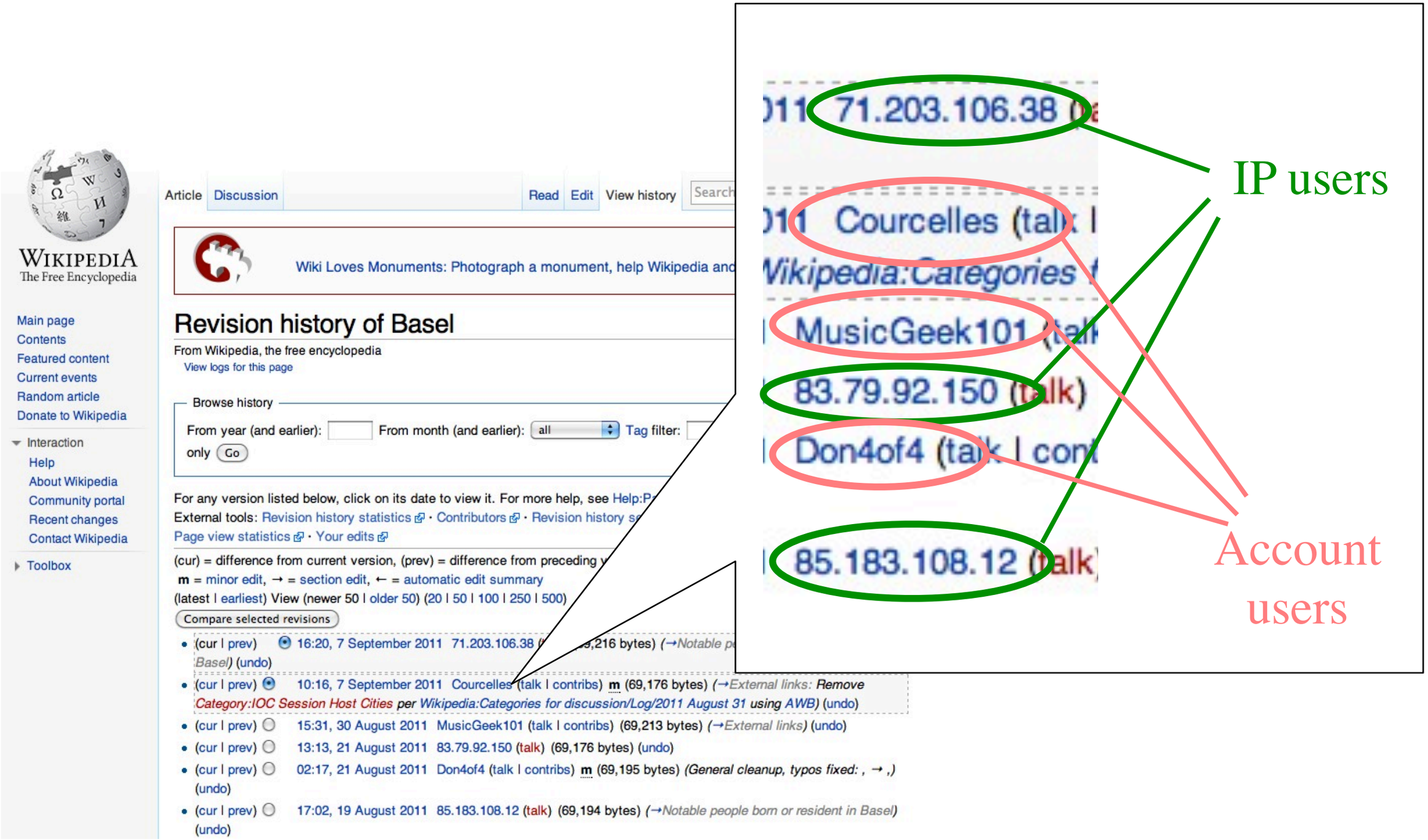
- The height of triangle gets shorter as the number of articles decreases.



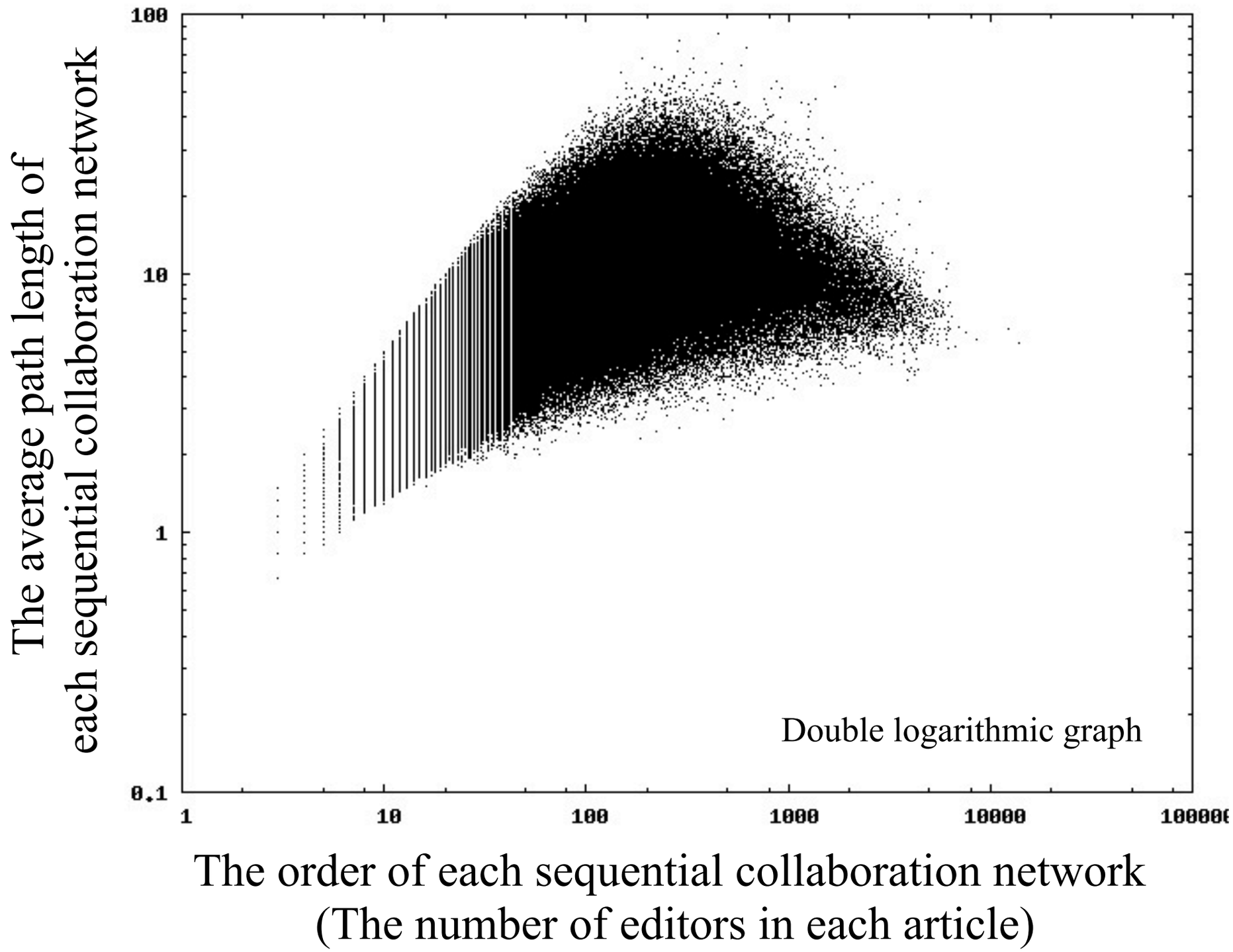
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles

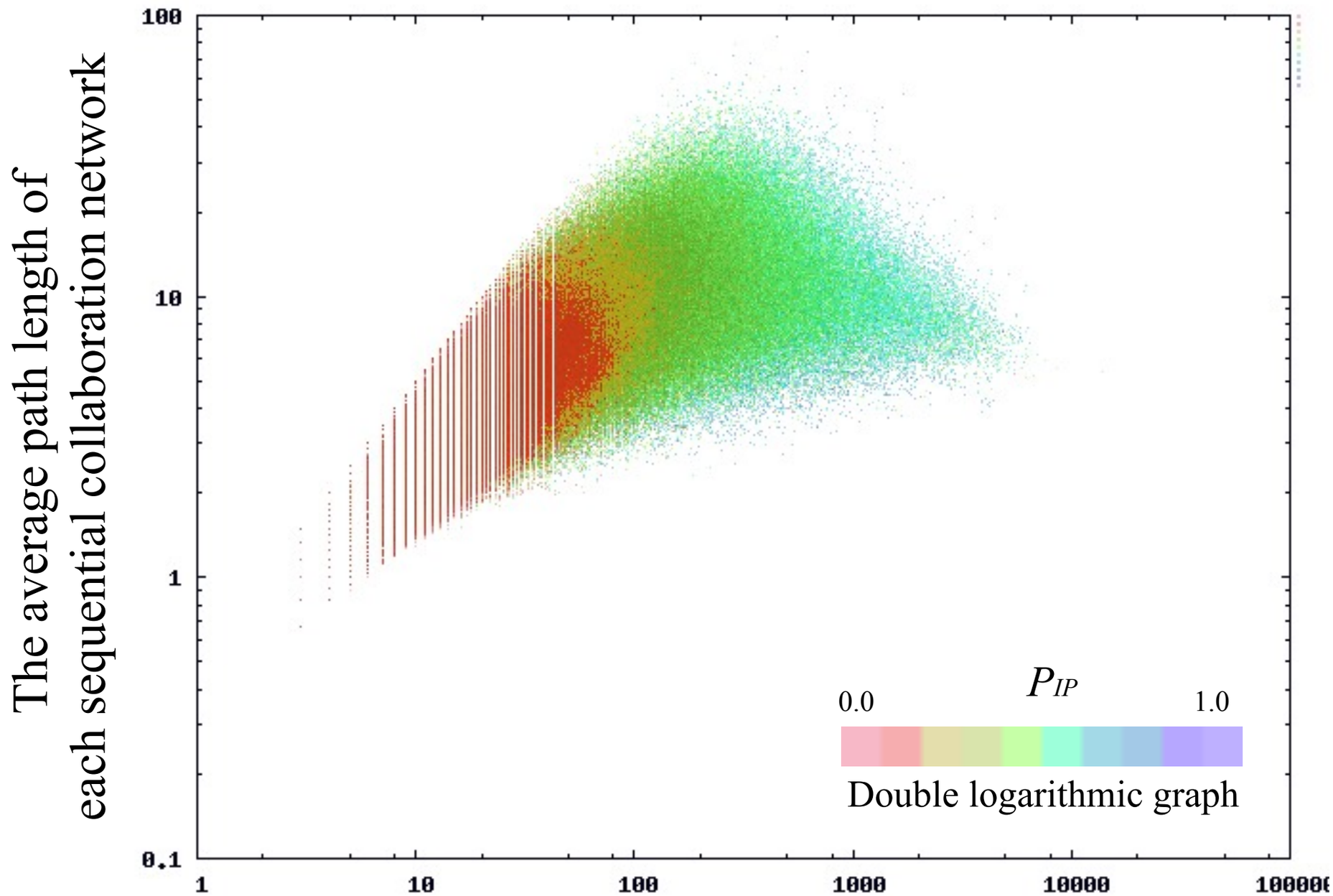
■Analysis 2: Distribution of account and IP users



Scatter plot of articles in English Wikipedia

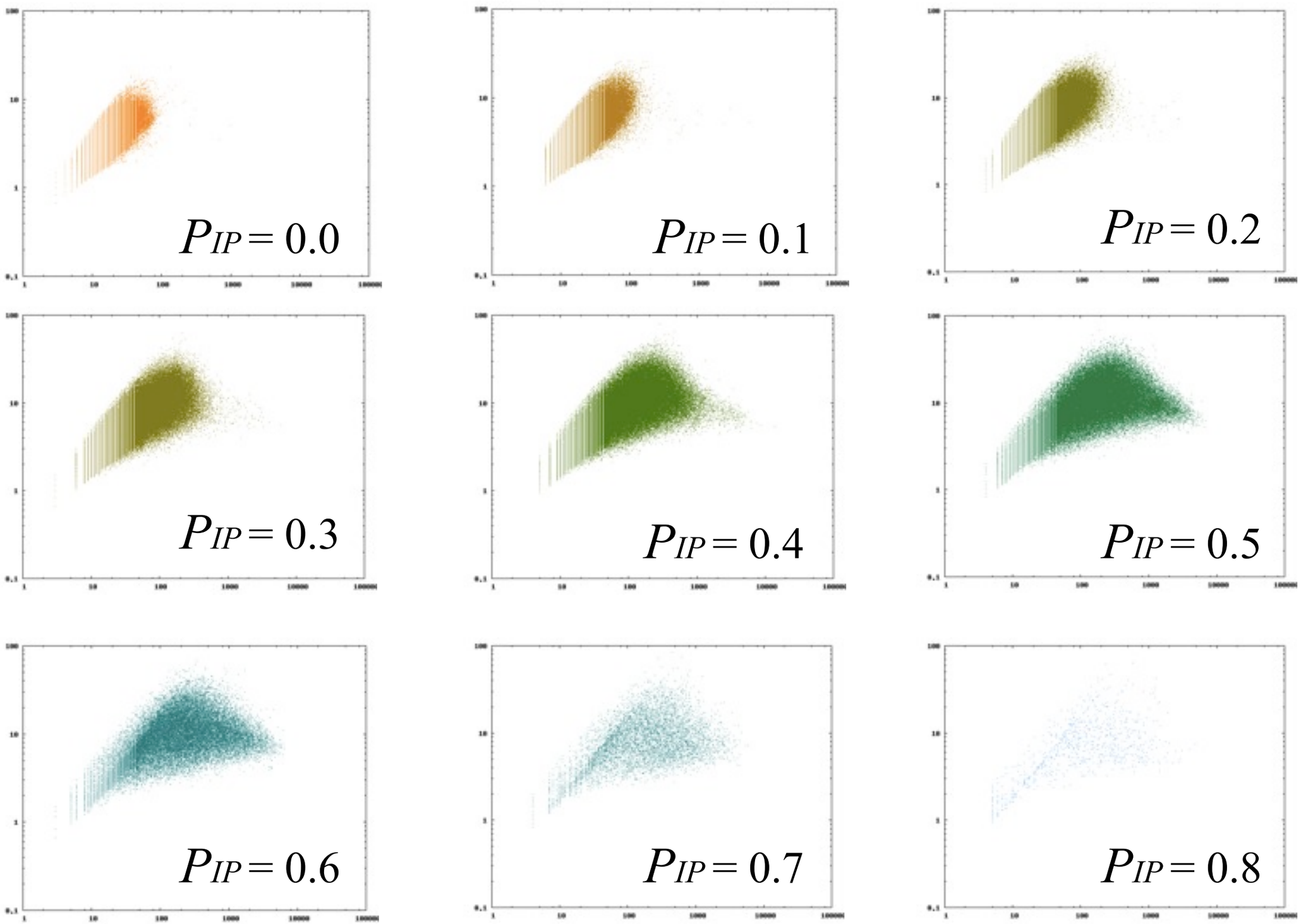


Scatter plot of articles with number of IP users / number of total editors

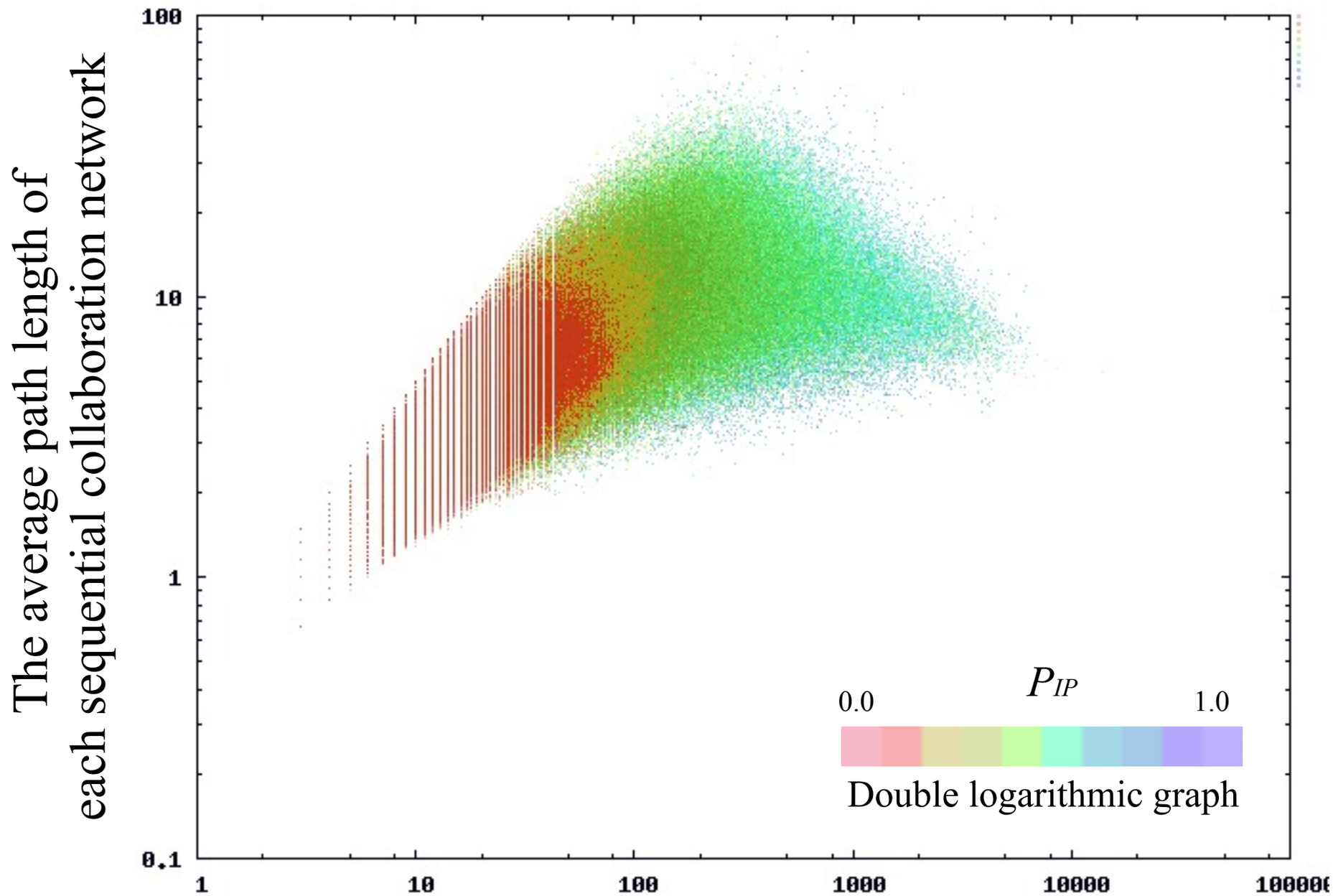


The order of each sequential collaboration network
(The number of editors in each article)

Scatter plot of articles with number of IP users / number of total editors



Scatter plot of articles with number of IP users / number of total editors

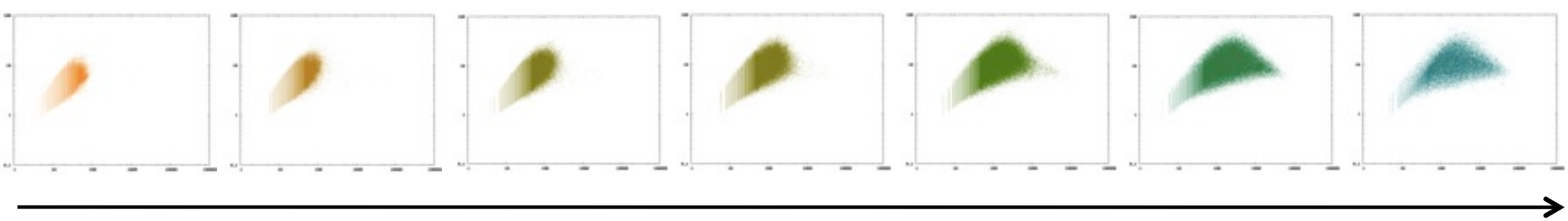


The order of each sequential collaboration network
(The number of editors in each article)

Result of Analysis 2: Distribution of account and IP users

- Top and right area of the “triangle” in scatter plot consist of articles which ratios of users is high.

- As a result, both the average path length and order of network can be large in these areas.



$P_{IP} = 0.0$ → $P_{IP} = 0.6$

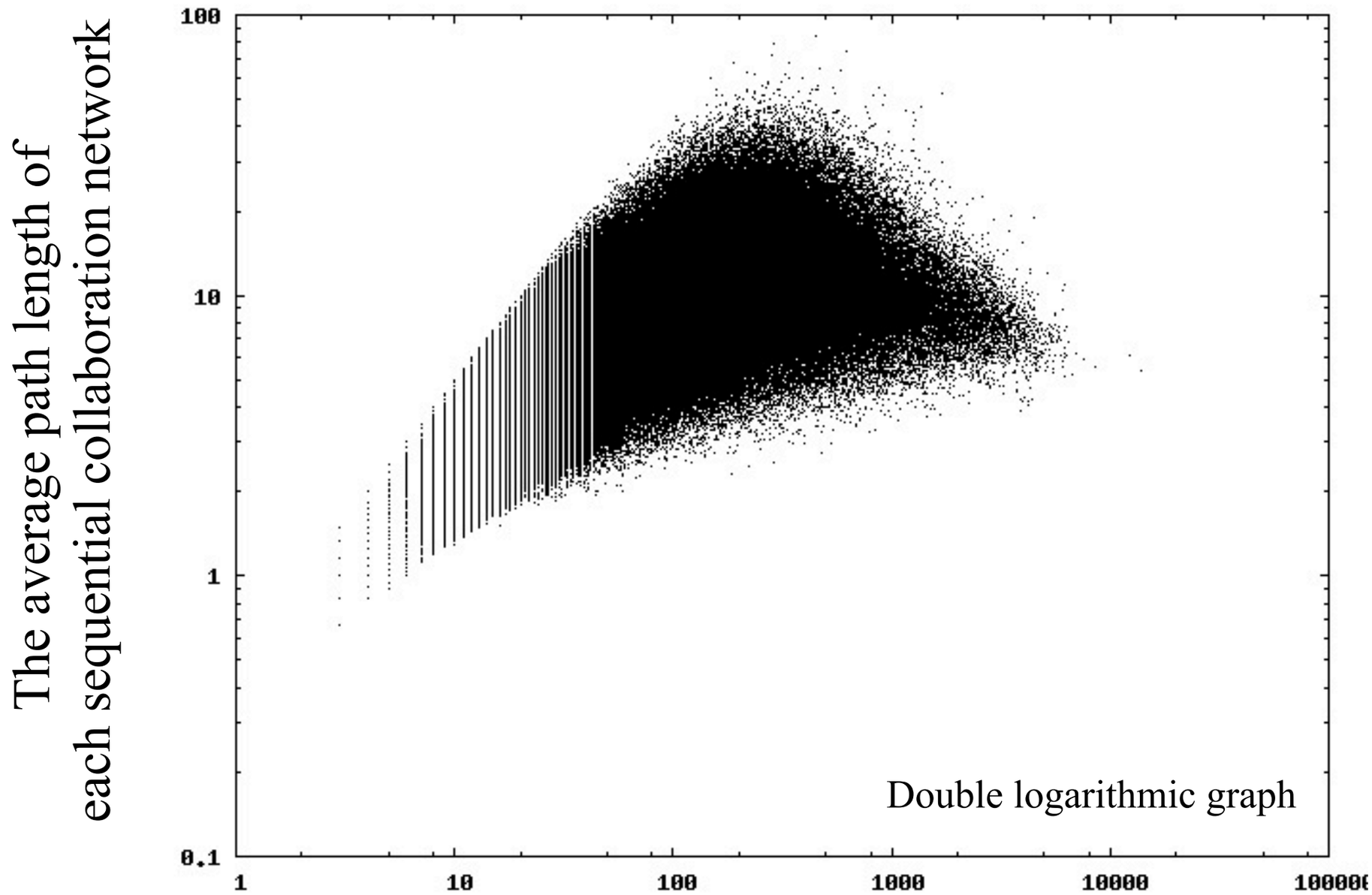
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles

■Analysis 3: Distribution of Featured Articles

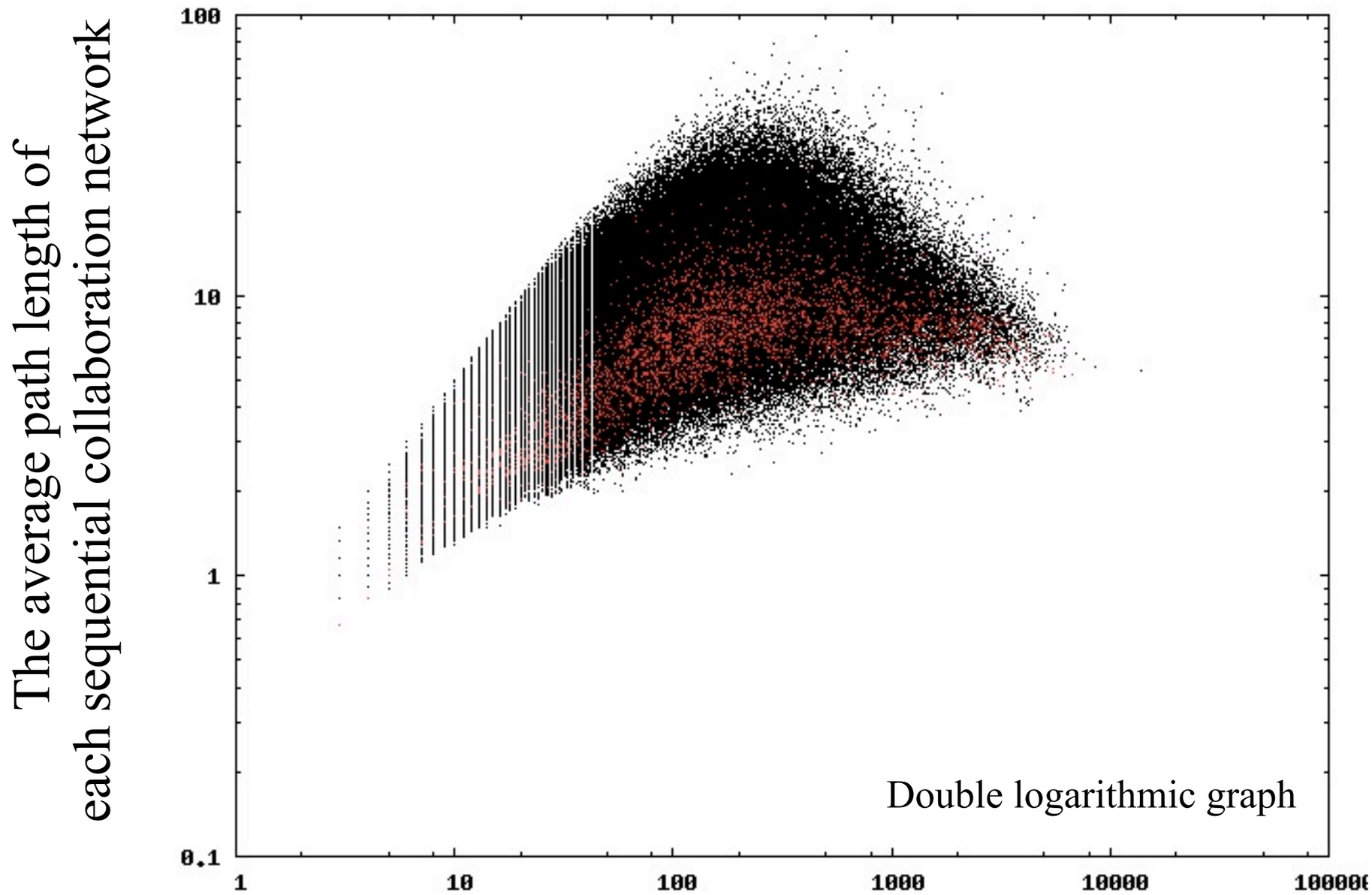
3,372 featured articles / 3,732,033 articles
In English Wikipedia

Scatter plot of all articles in English Wikipedia



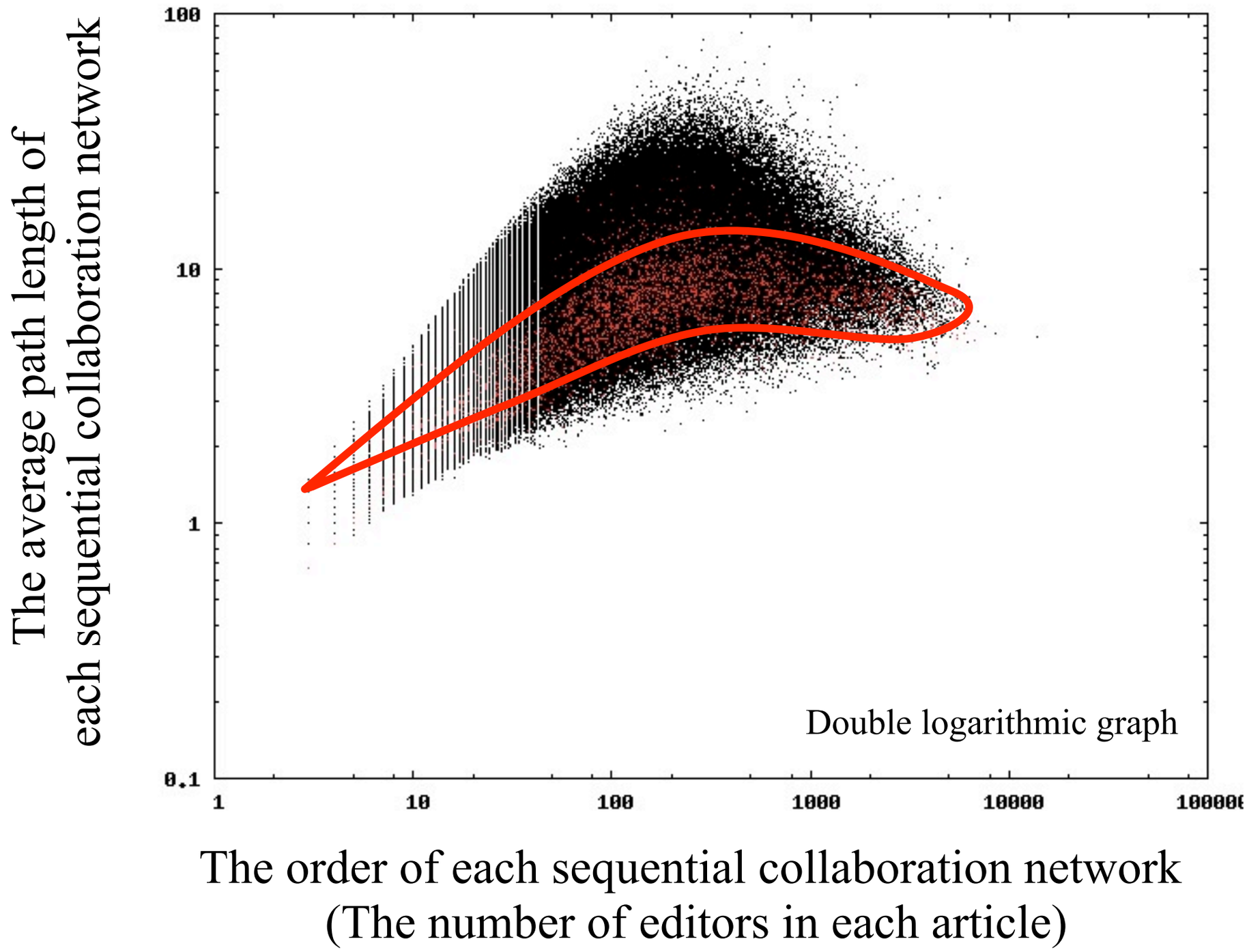
The order of each sequential collaboration network
(The number of editors in each article)

Scatter plot of featured articles on the all articles in English Wikipedia



The order of each sequential collaboration network
(The number of editors in each article)

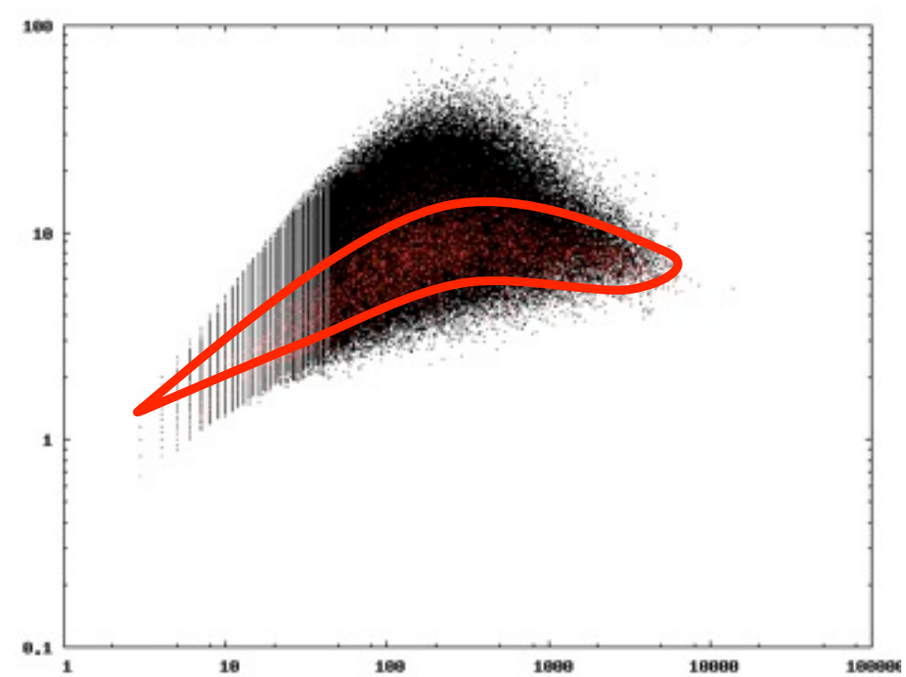
Scatter plot of featured articles on the all articles in English Wikipedia



Result of Analysis 3: Distribution of Featured Articles

- Features articles are located at a certain area in the scatter plot.

- It implies that there would be characteristic patterns of collaboration producing good results.



Editorial Collaboration Networks of Wikipedia Articles in Various Languages

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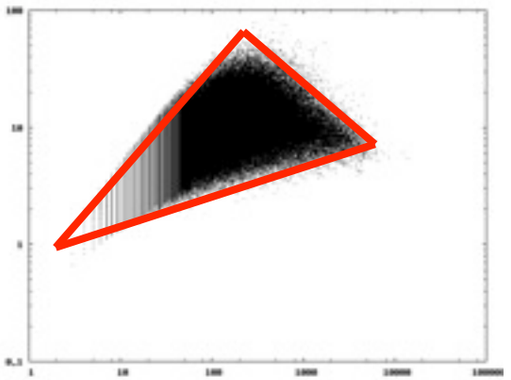
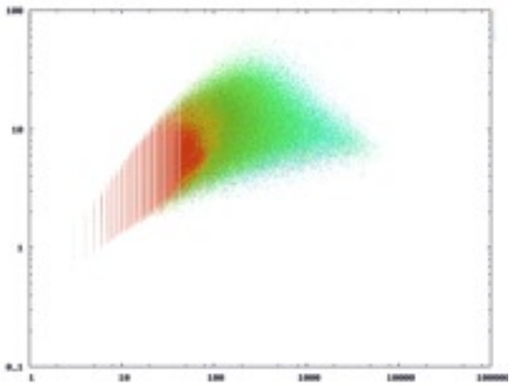
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

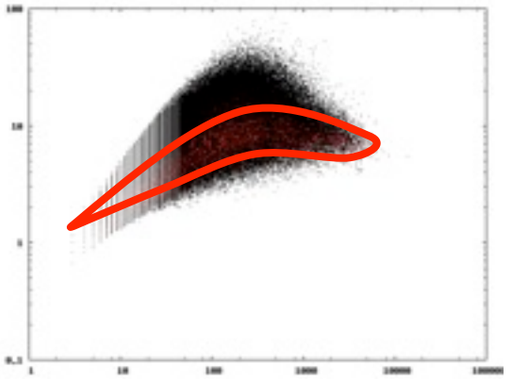
- Scatter plot of all articles commonly exhibits a tilted triangle in all languages, but the height of triangle gets shorter as the number of articles decreases.



- Top and right area of the “triangle” in scatter plot consist of articles which the ratios of IP users are high.



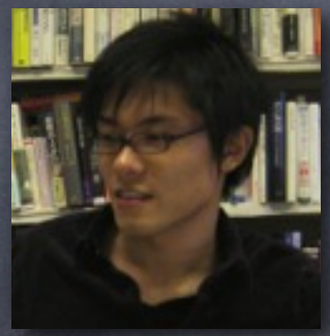
- Features articles are located at a certain area in the scatter plot.



Collaborators

Natsumi Yotsumoto

Former Student, Iba Lab.



Satoshi Itoh

A Former Graduate Student,
Iba Lab.

Ko Matsuzuka

Student, Iba Lab.

Daiki Muramatsu

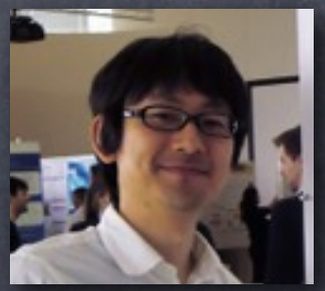
Student, Iba Lab.

Peter Gloor

Research Scientist, MIT CCI

Bui Hong Ha

Former Student, Iba Lab.



Keiichi Nemoto

Visiting Scholar, MIT CCI
Fuji Xerox

Papers & Presentations

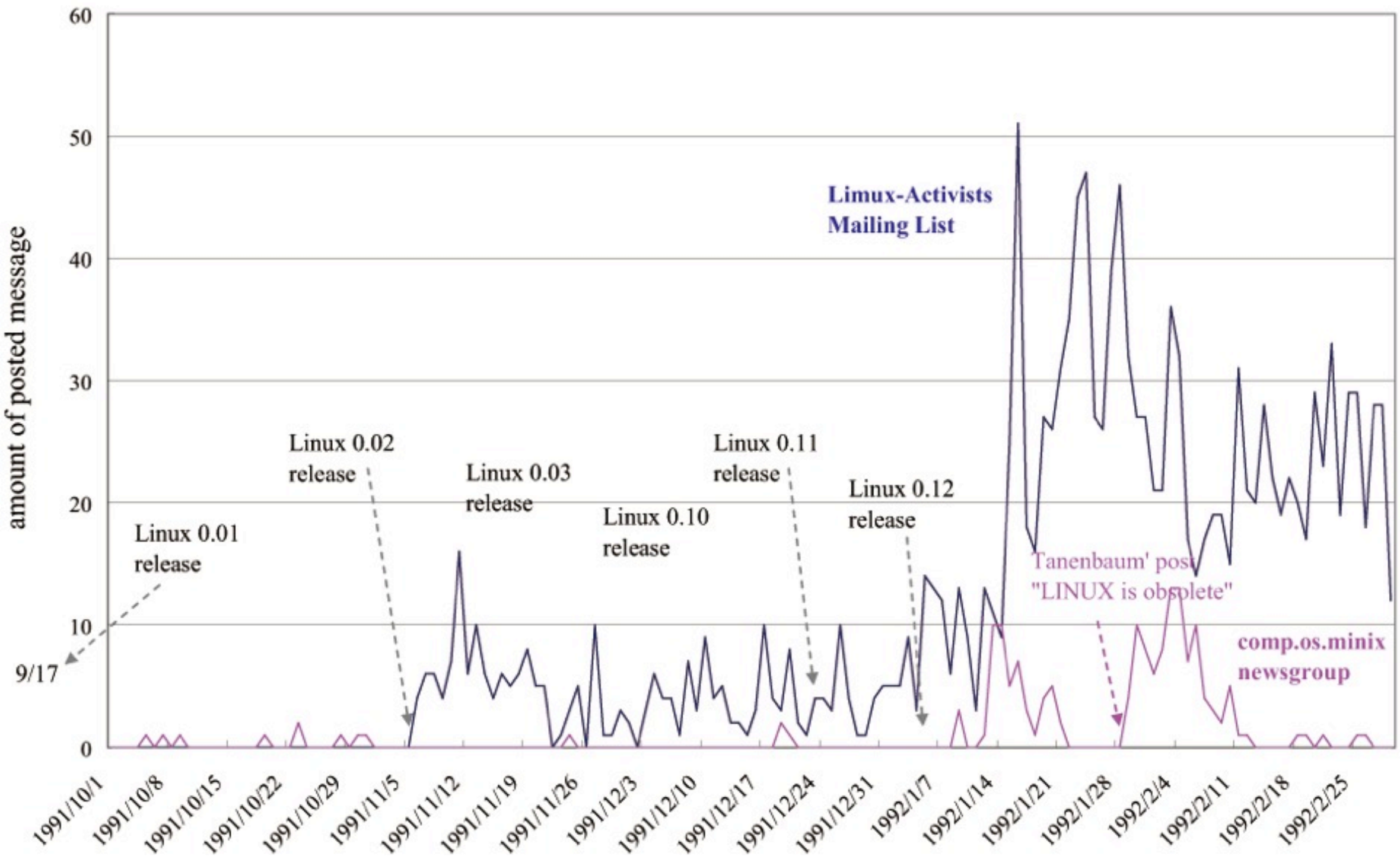
- S. Itoh and T. Iba, "Analyzing Collaboration Network of Editors in Japanese Wikipedia", International Workshop and Conference on Network Science '09, 2009
- S. Itoh, Y. Yamazaki, T. Iba, "Analyzing How Editors Write Articles in Wikipedia", Applications of Physics in Financial Analysis (APFA7), Tokyo, Japan, 2009
- T. Iba & S. Itoh, "Sequential Collaboration Network of Open Collaboration", International Workshop and Conference on Network Science '09, 2009
- S. Itoh & T, Iba, "Analyzing Collaboration Network of Editors in Japanese Wikipedia", International Workshop and Conference on Network Science '09, 2009
- T. Iba, K. Nemoto, B. Peters, & P. A. Gloor, "Analyzing the Creative Editing Behavior of Wikipedia Editors Through Dynamic Social Network Analysis", Procedia - Social and Behavioral Sciences, Vol.2, Issue 4, 2010, pp.6441-6456
- T. Iba, K. Matsuzuka, D. Muramatsu, "Editorial Collaboration Networks of Wikipedia Articles in Various Languages", 3rd Conference on Collaborative Innovation Networks (COINs2011), 2011
- 伊藤 諭志, 伊藤 貴一, 熊坂 賢次, 井庭 崇, 「マスコラボレーションにおけるコンテンツ形成プロセスの分析」, 人工知能学会 第20回セマンティックウェブとオントロジー研究会, 2009
- 四元 菜つみ, 井庭 崇, 「Wikipedia におけるコラボレーションネットワークの成長」, 情報処理学会 第7回ネットワーク生態学シンポジウム, 2011

Collaboration Networks of Linux

Dynamics as Networks

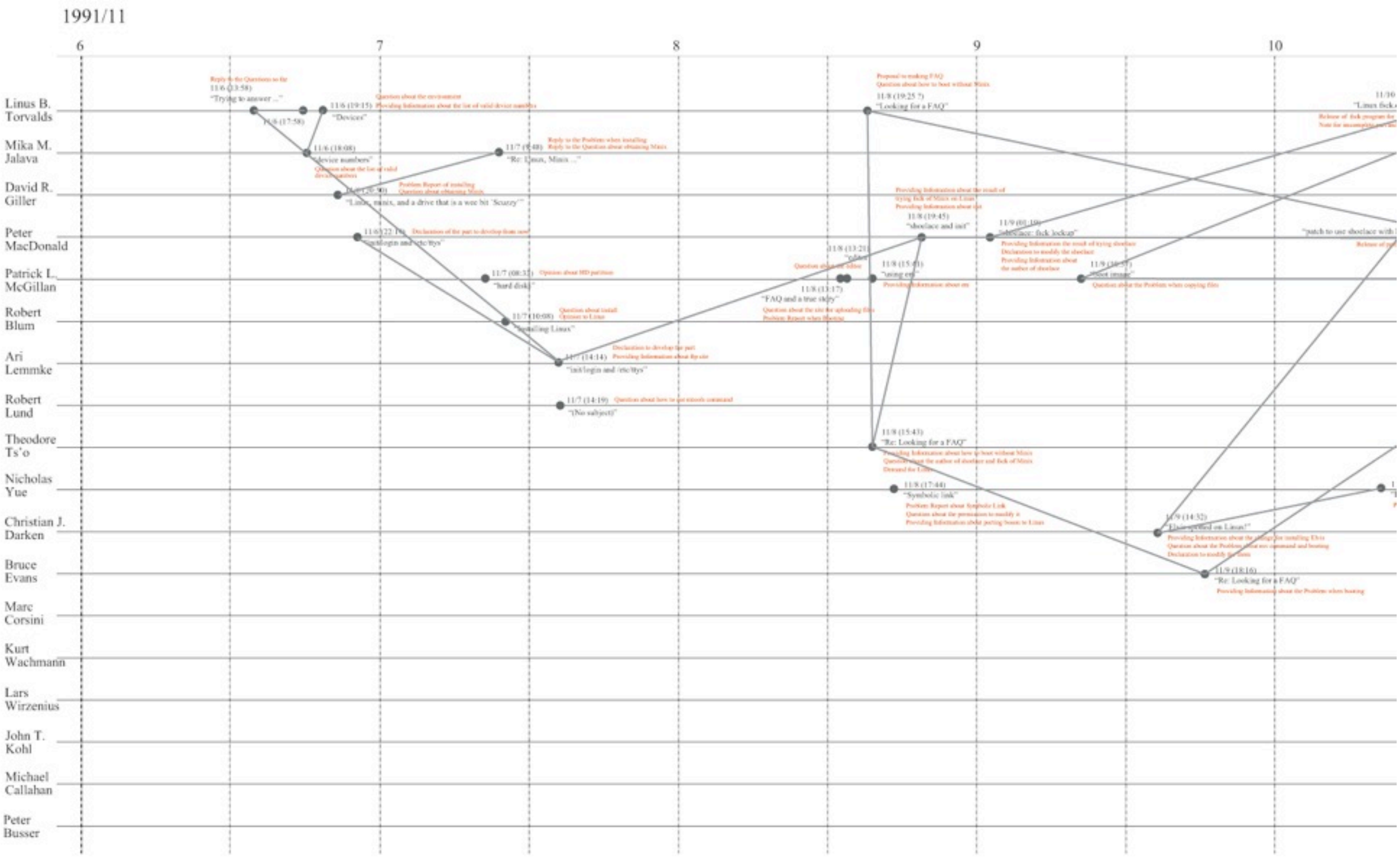
the dynamics of system/phenomena are represented as networks

Linux-Activists Mailing List & comp.os.minix Newsgroup



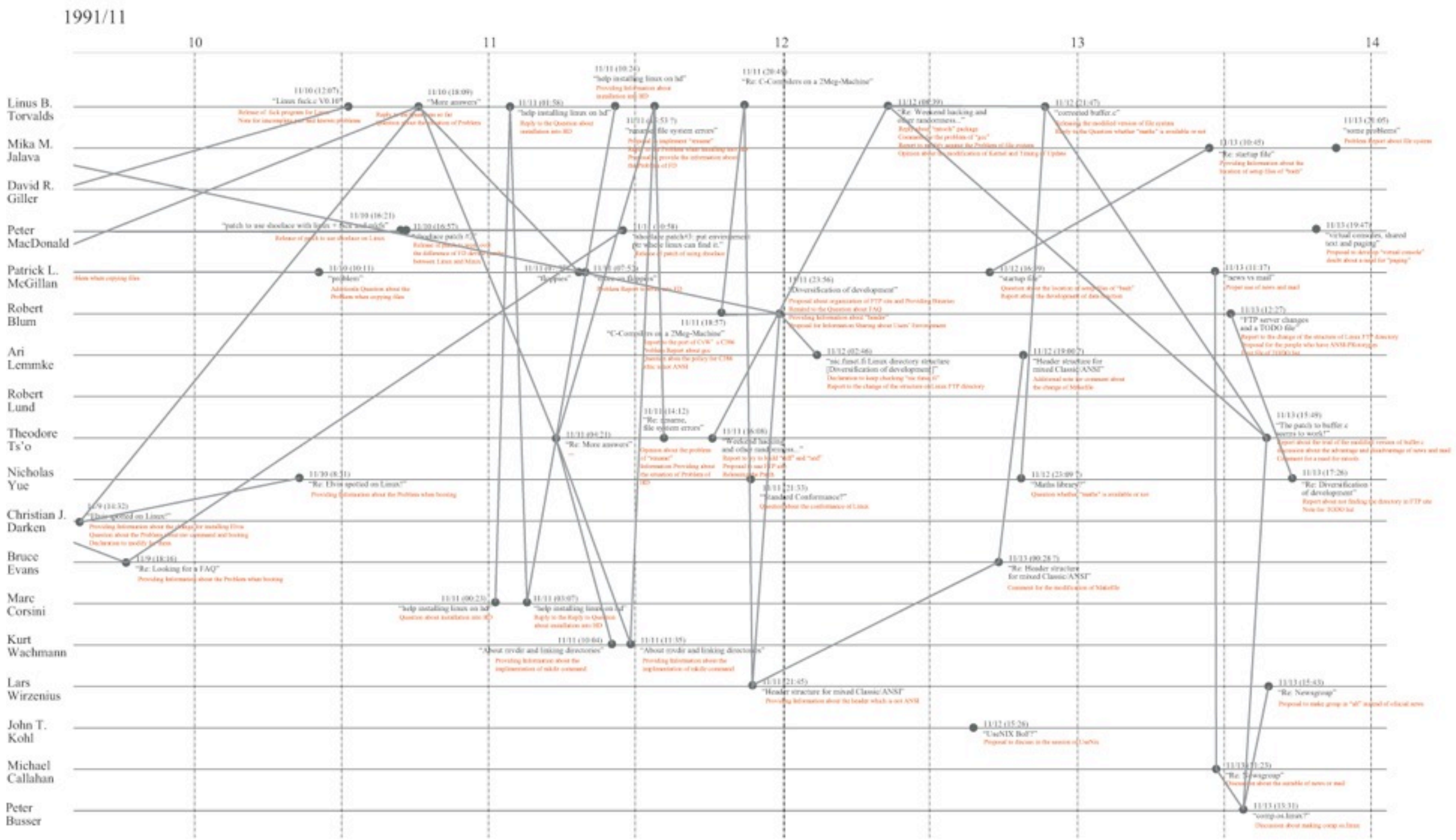
Takashi Iba (2008)

Linux-Activists Mailing List (Nov. 1991)



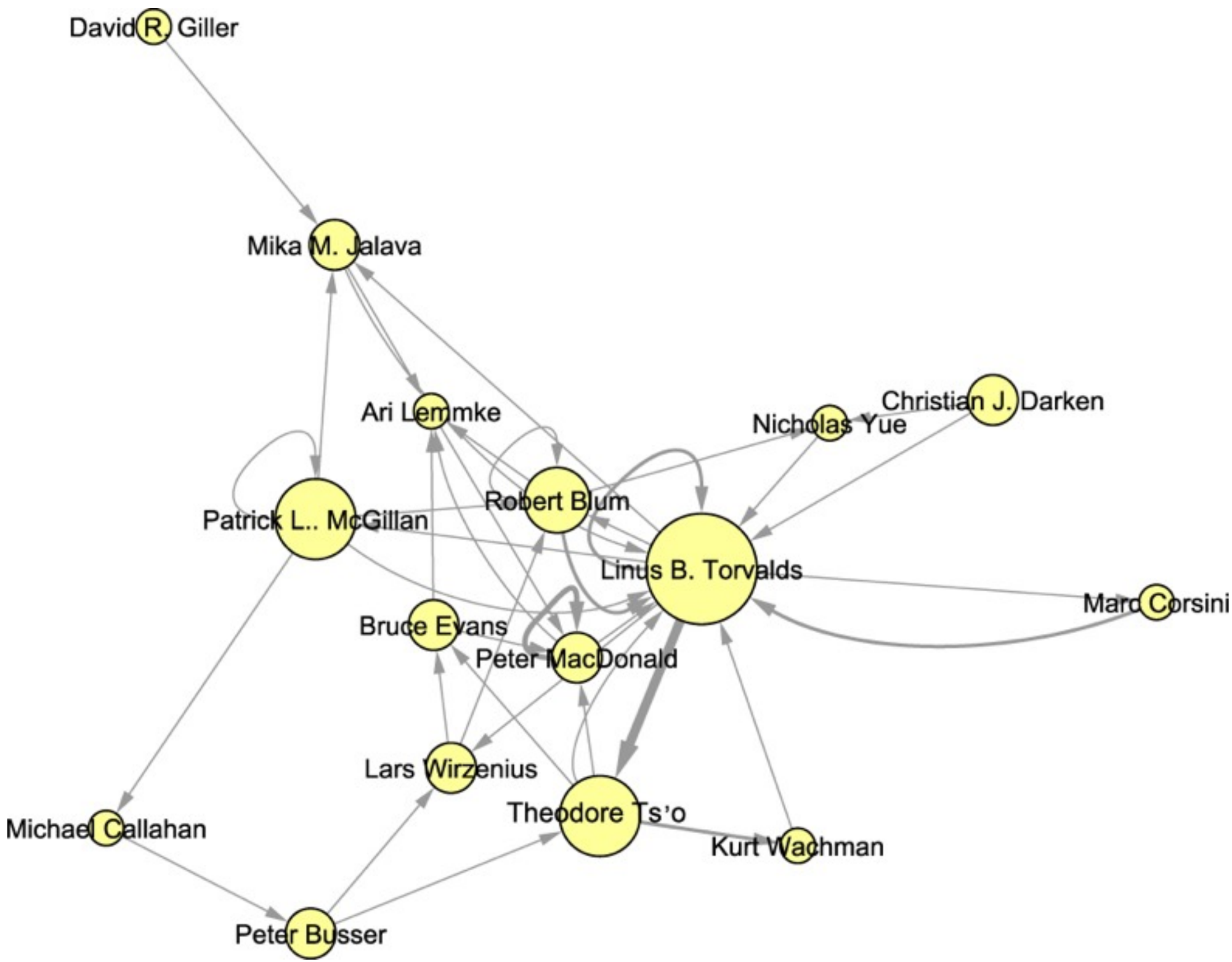
Takashi Iba (2008)

Linux-Activists Mailing List (Nov. 1991)



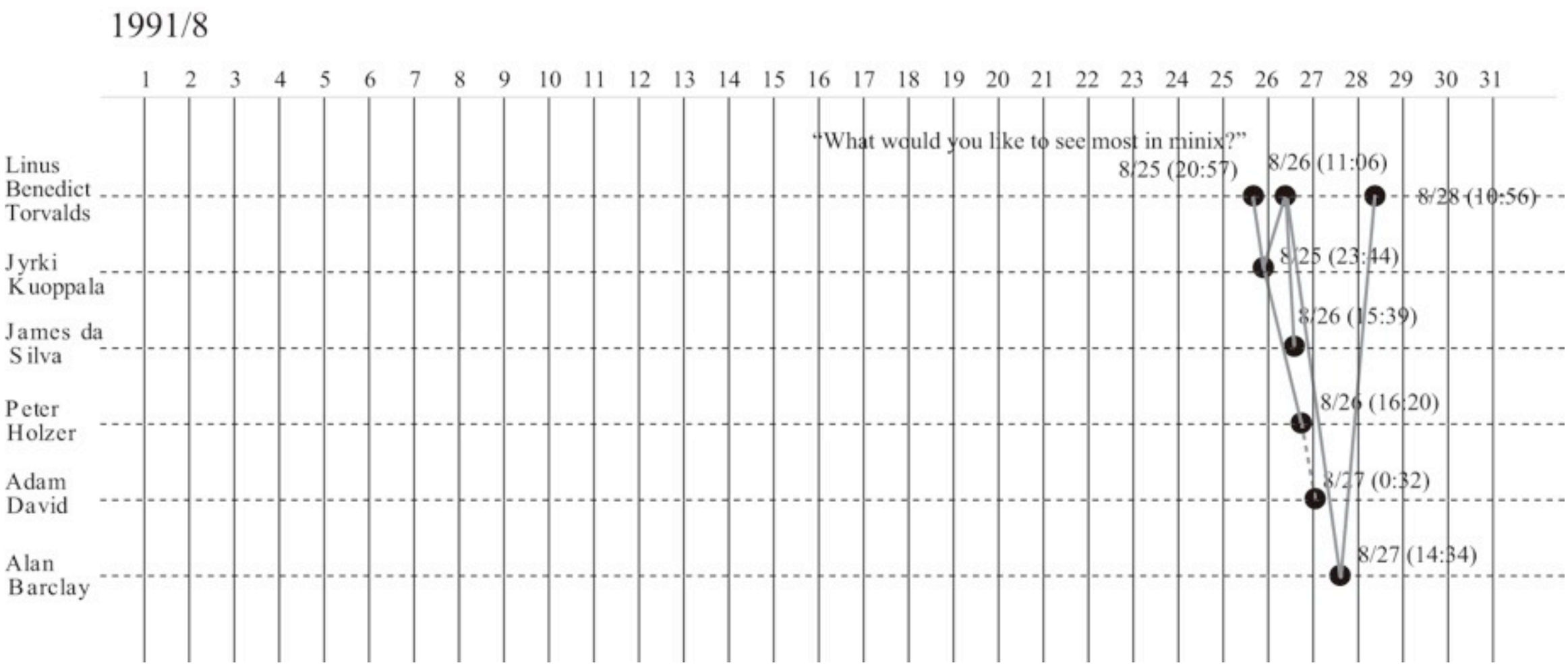
Takashi Iba (2008)

Linux-Activists Mailing List (Nov. 1 – 14, 1991)



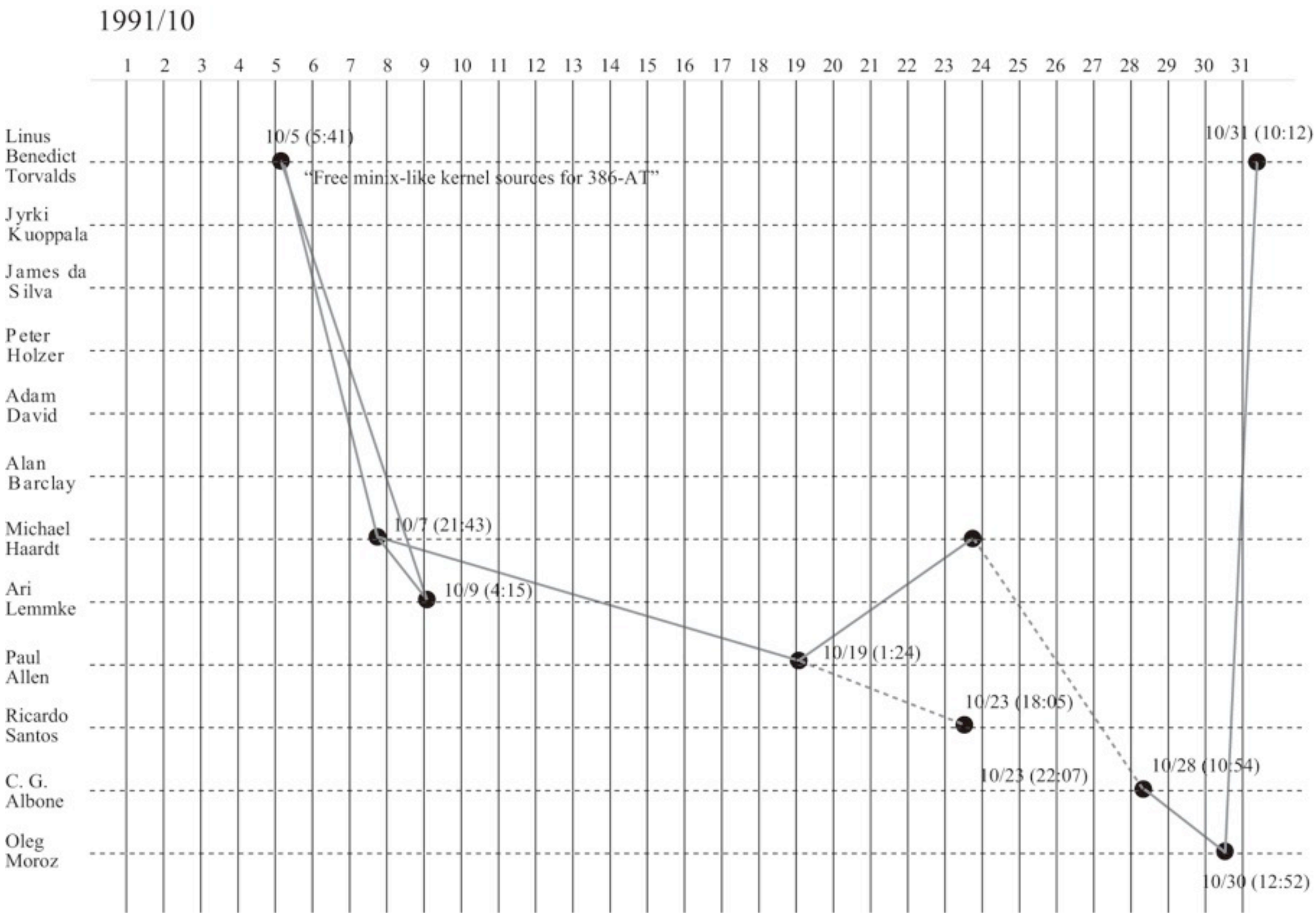
Takashi Iba (2008)

comp.os.minix Newsgroup (Aug. 1991)



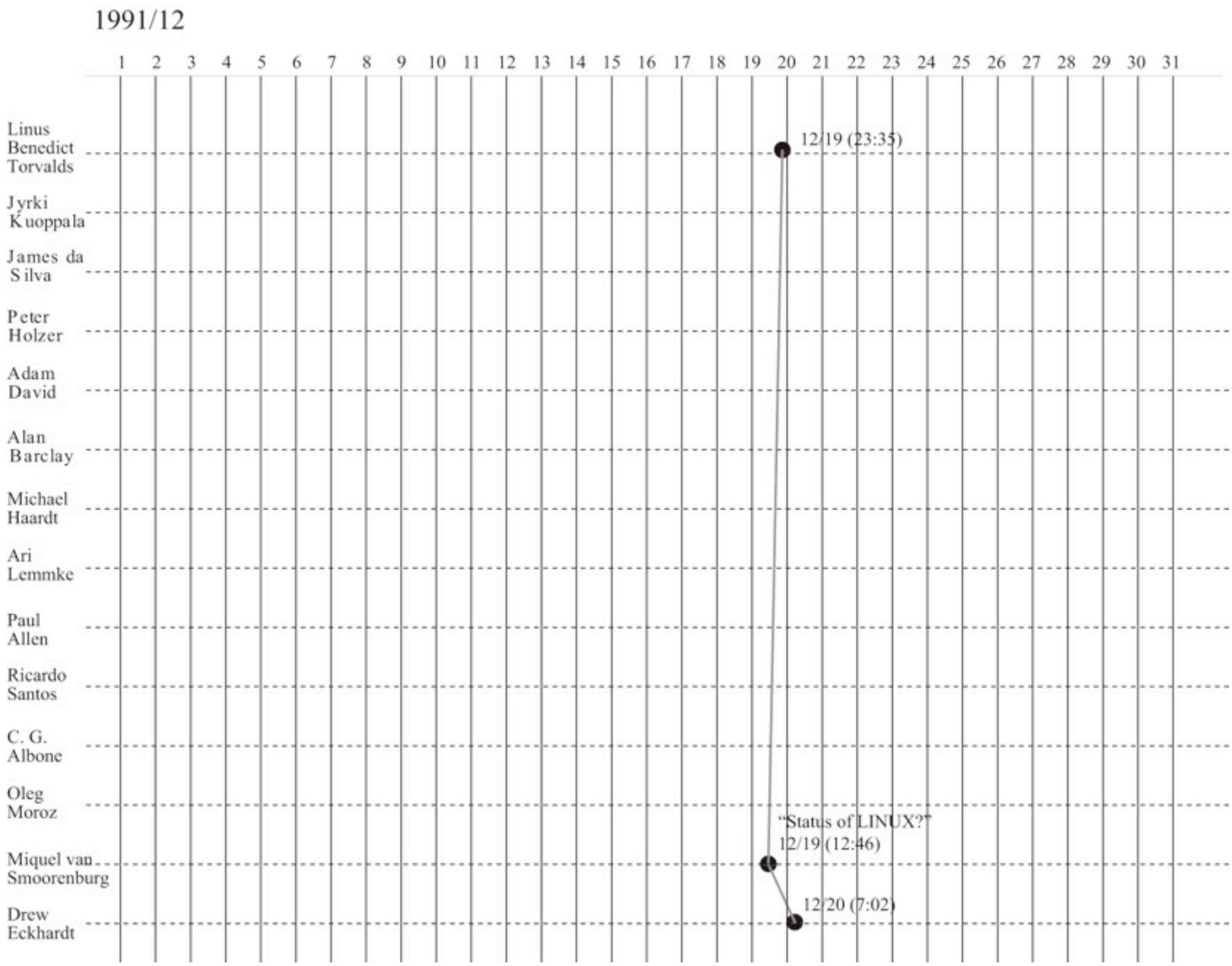
Takashi Iba (2008)

comp.os.minix Newsgroup (Oct. 1991)



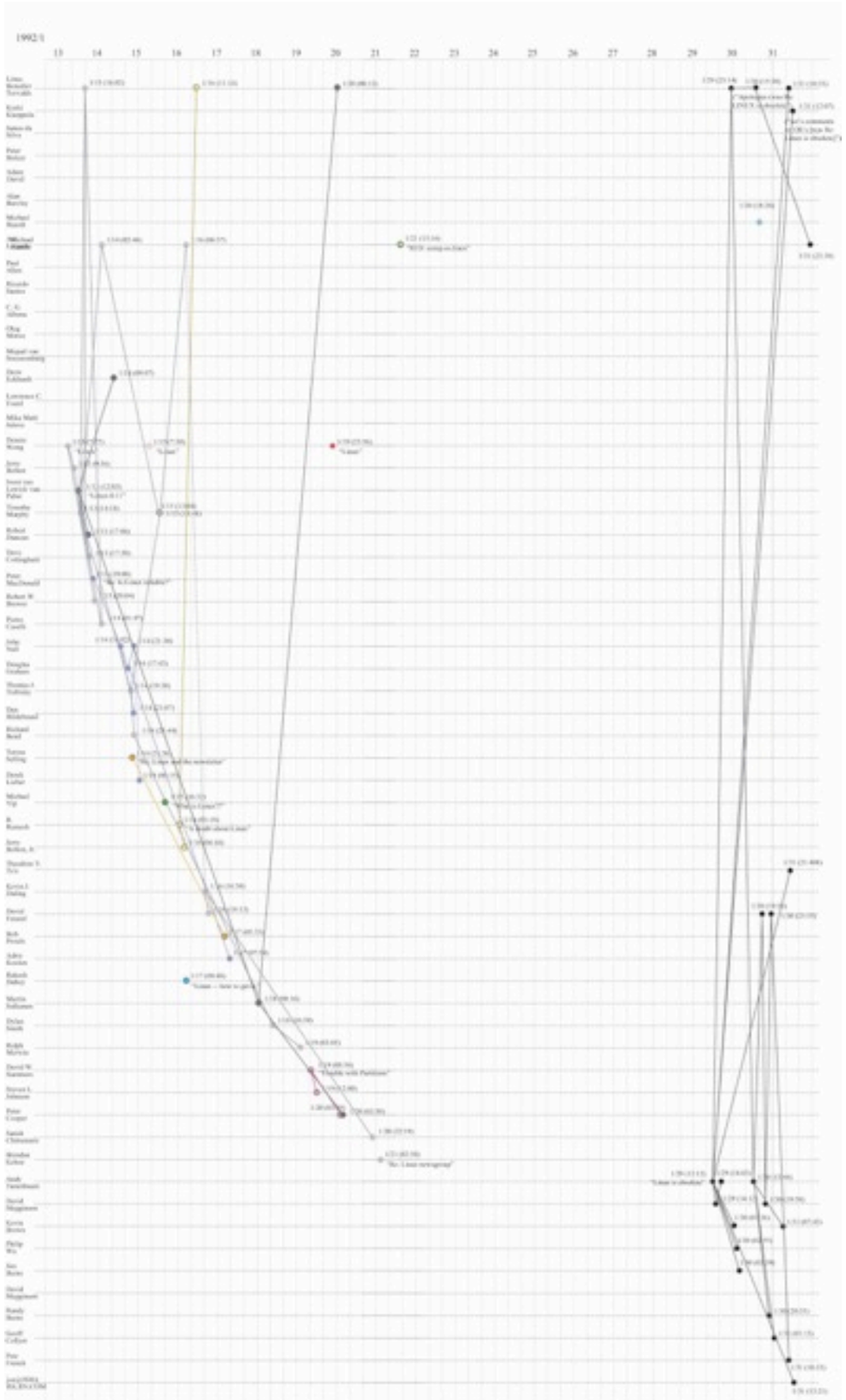
Takashi Iba (2008)

comp.os.minix Newsgroup (Dec. 1991)

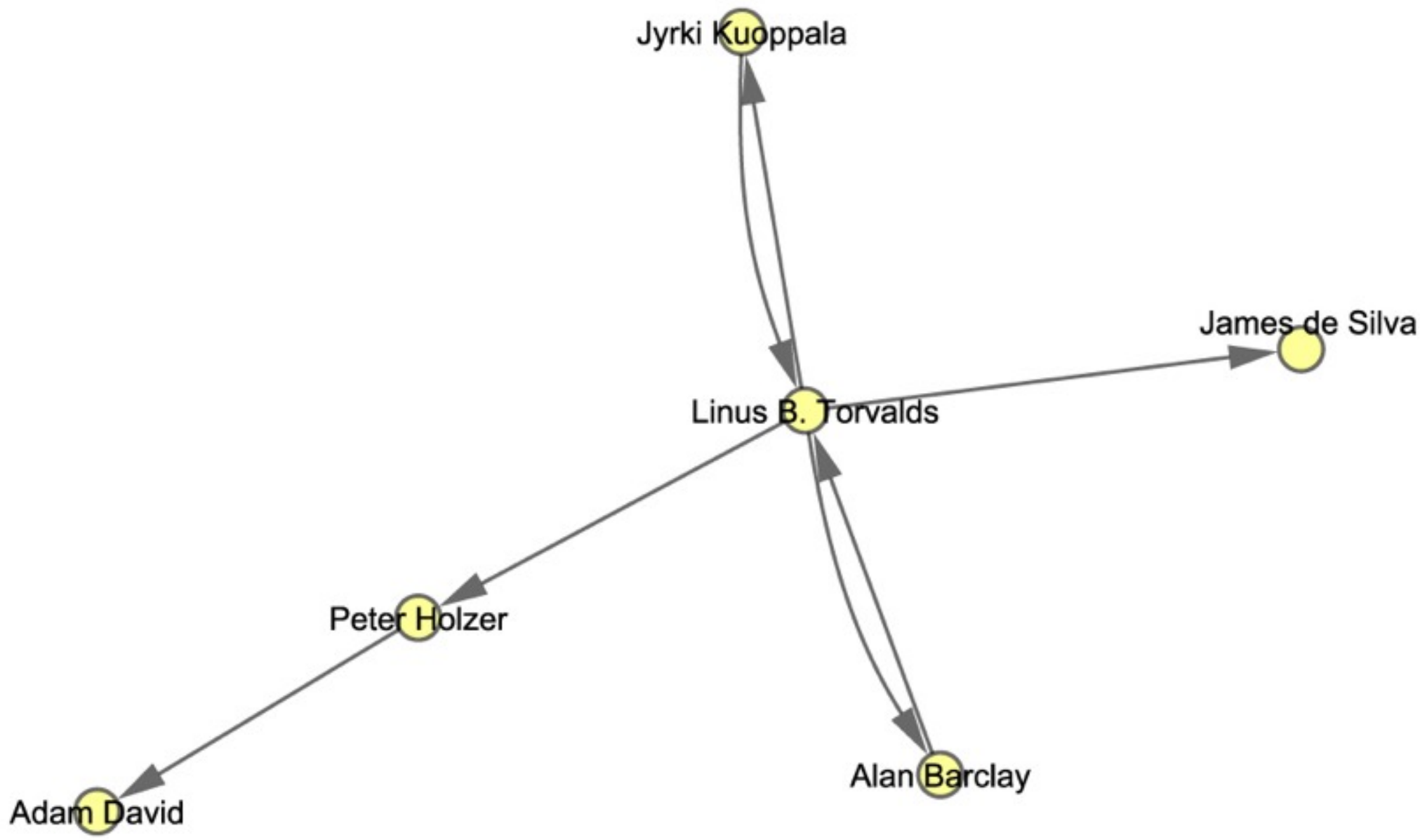


Takashi Iba (2008)

comp.os.minix Newsgroup (Jan. 1992)

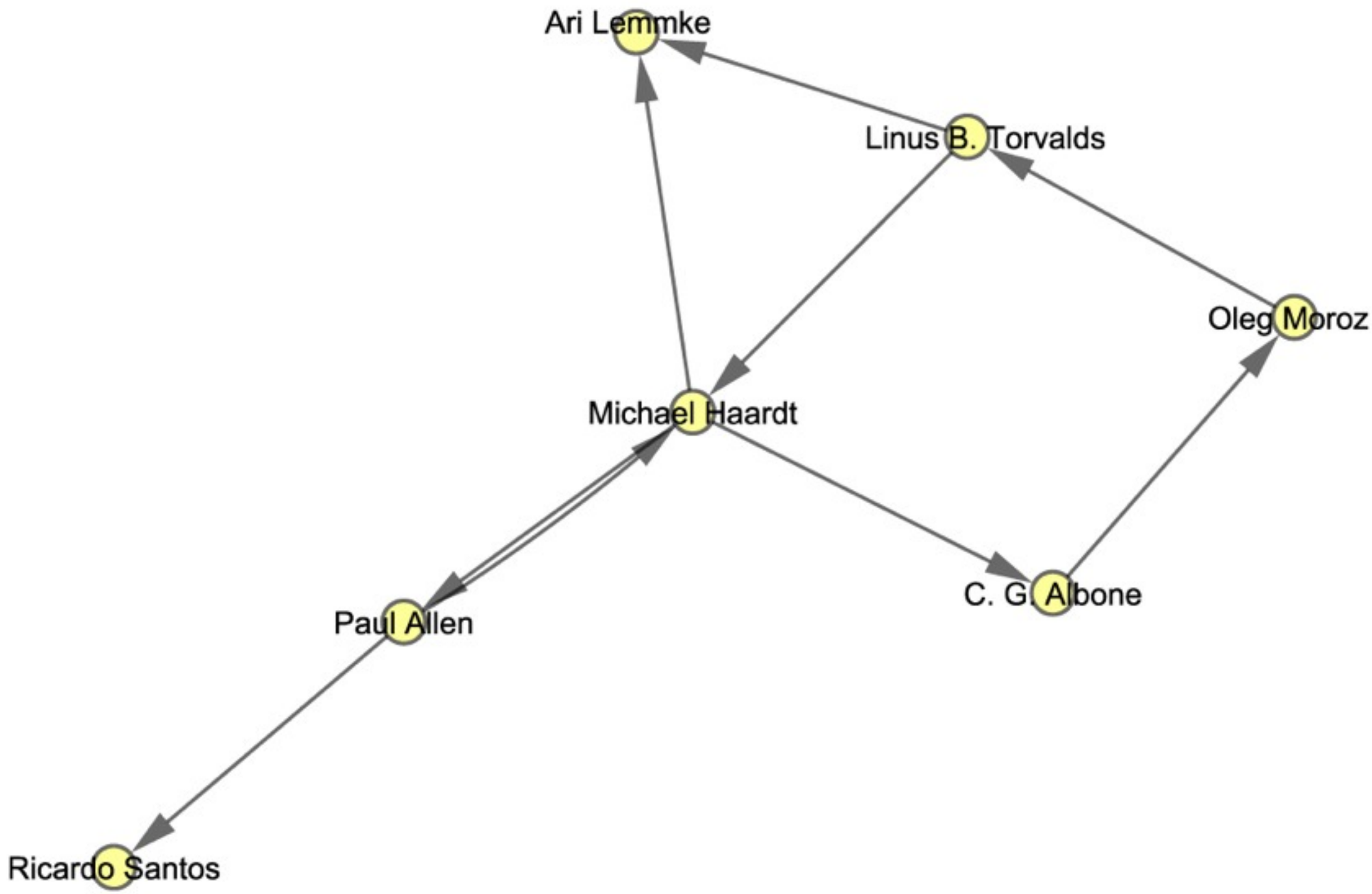


comp.os.minix Newsgroup (Aug. 1991)



Takashi Iba (2008)

comp.os.minix Newsgroup (Oct. 1991)



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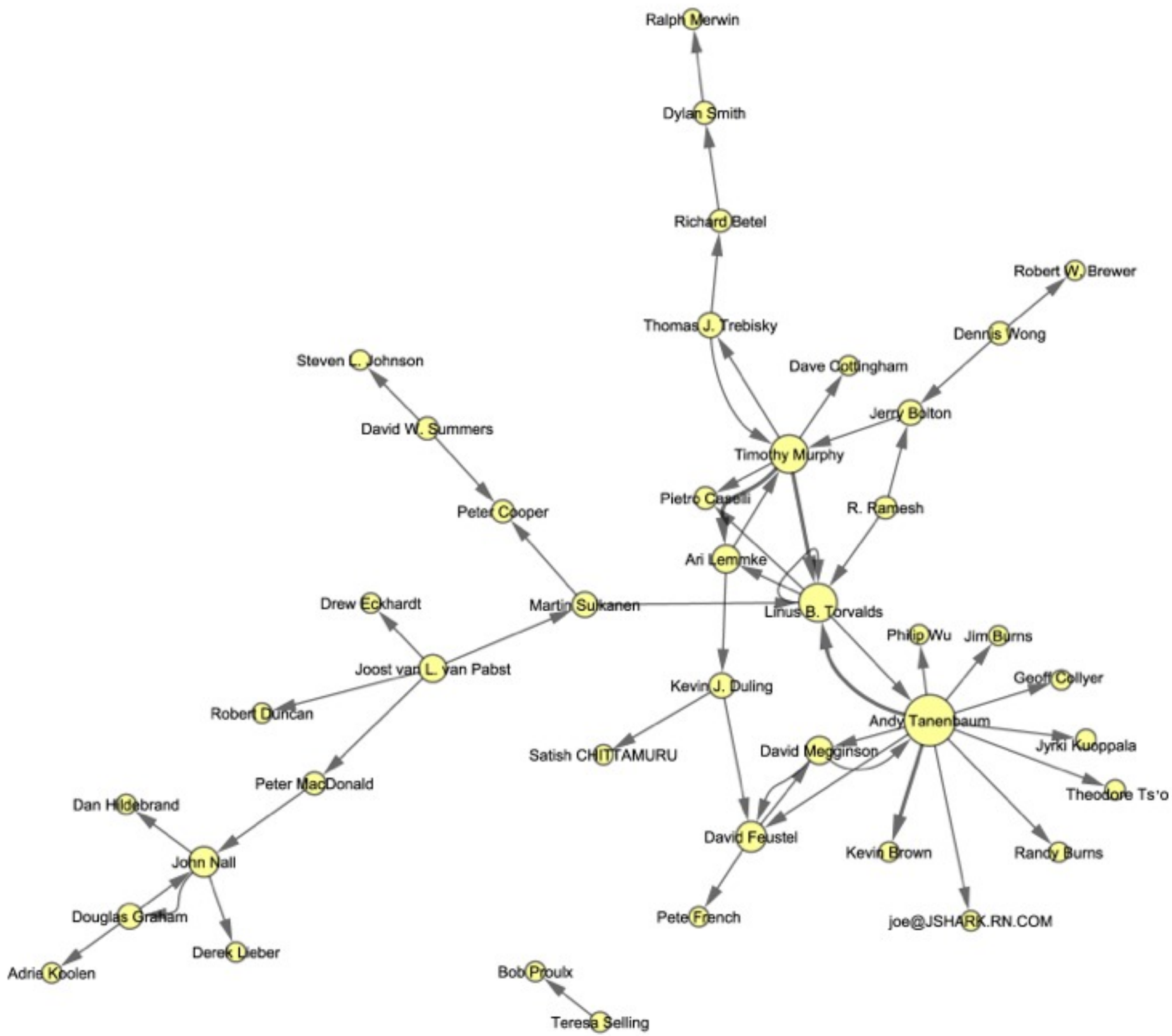
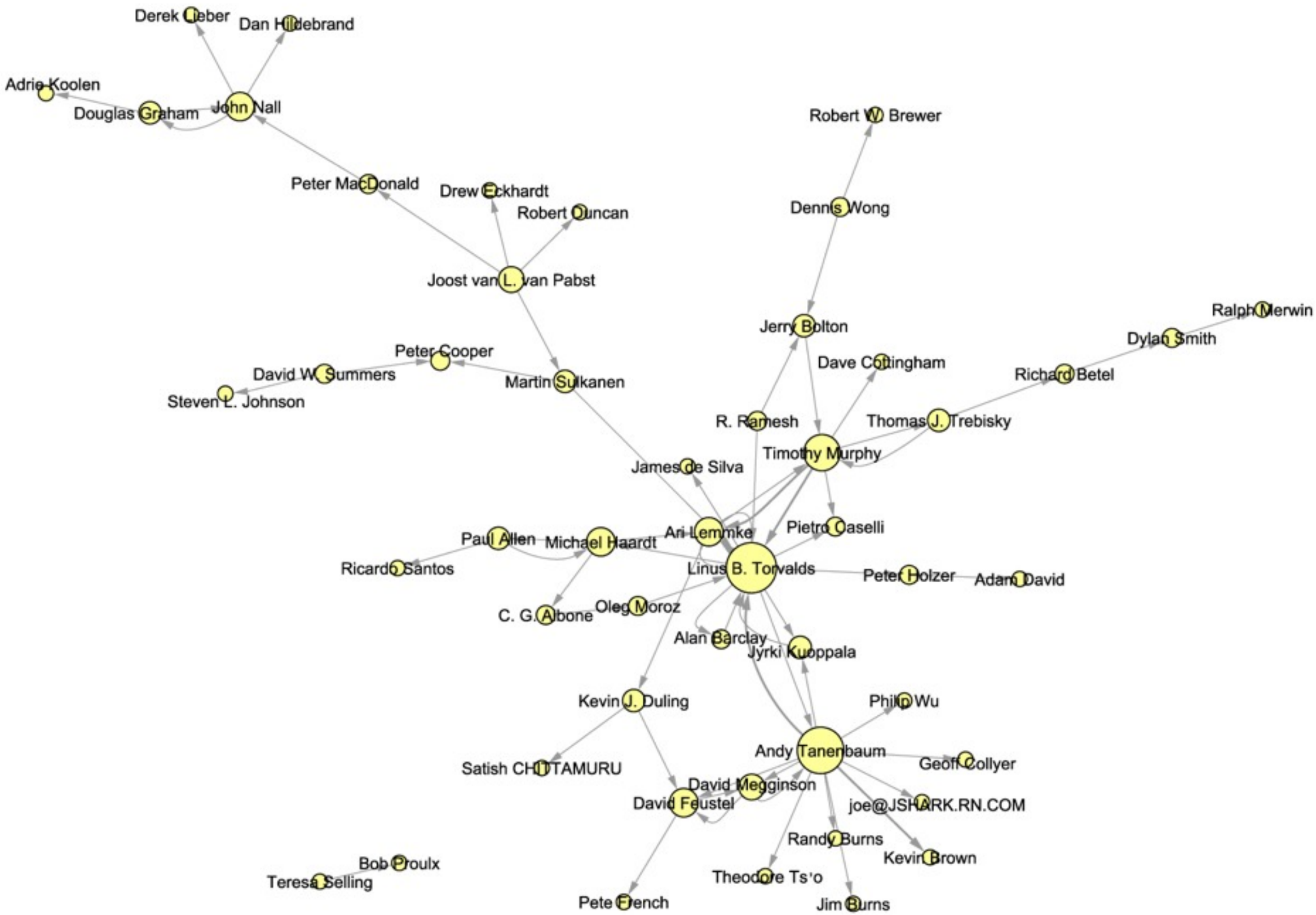


Takashi Iba (2008)

comp.os.minix Newsgroup (Aug. 1991 - Jan. 1992)



Takashi Iba (2008)

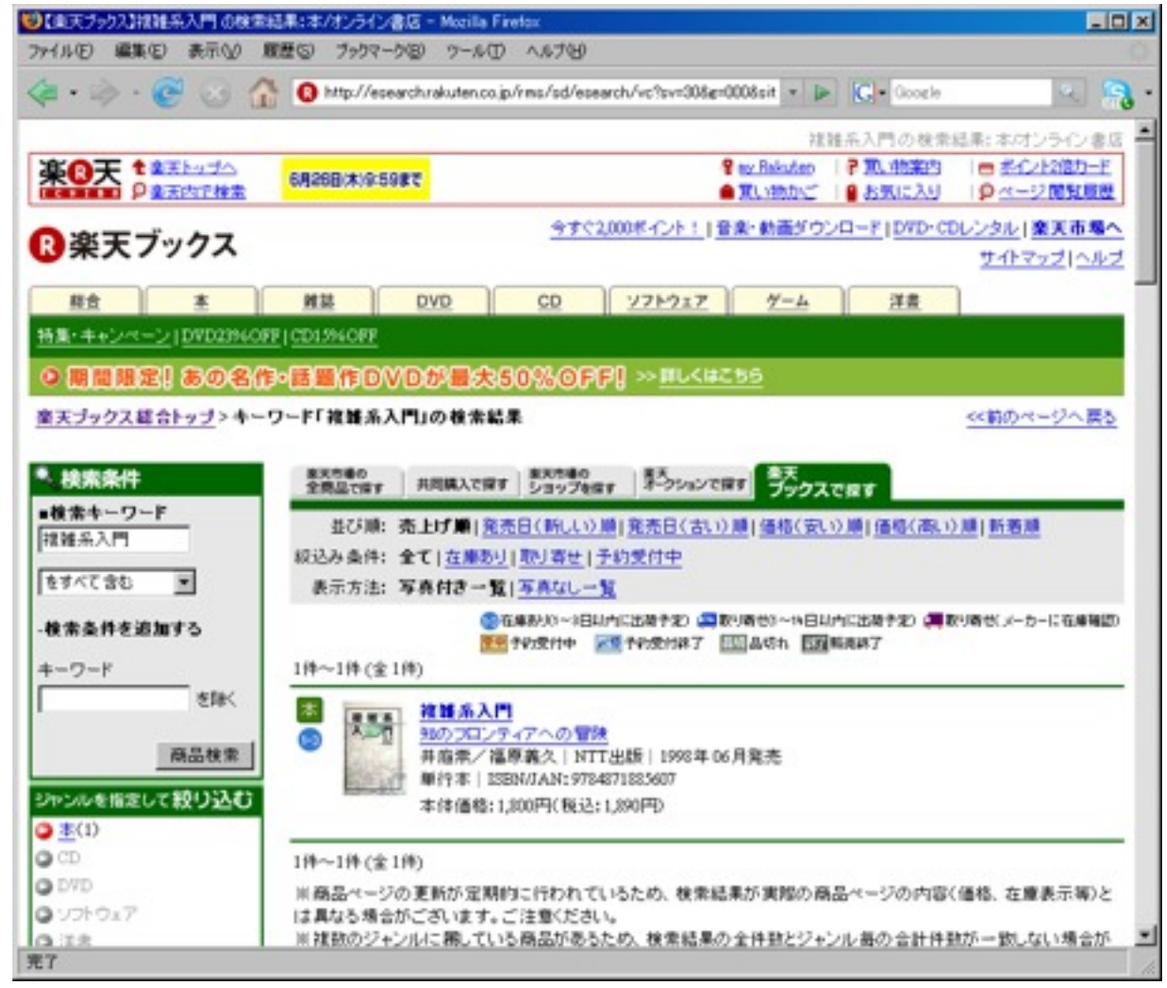
Co-Purchase Networks of Books, CDs, DVDs

Dynamics as Networks

the dynamics of system/phenomena are represented as networks

Rakuten Books: A famous Japanese Online Store

http://books.rakuten.co.jp/



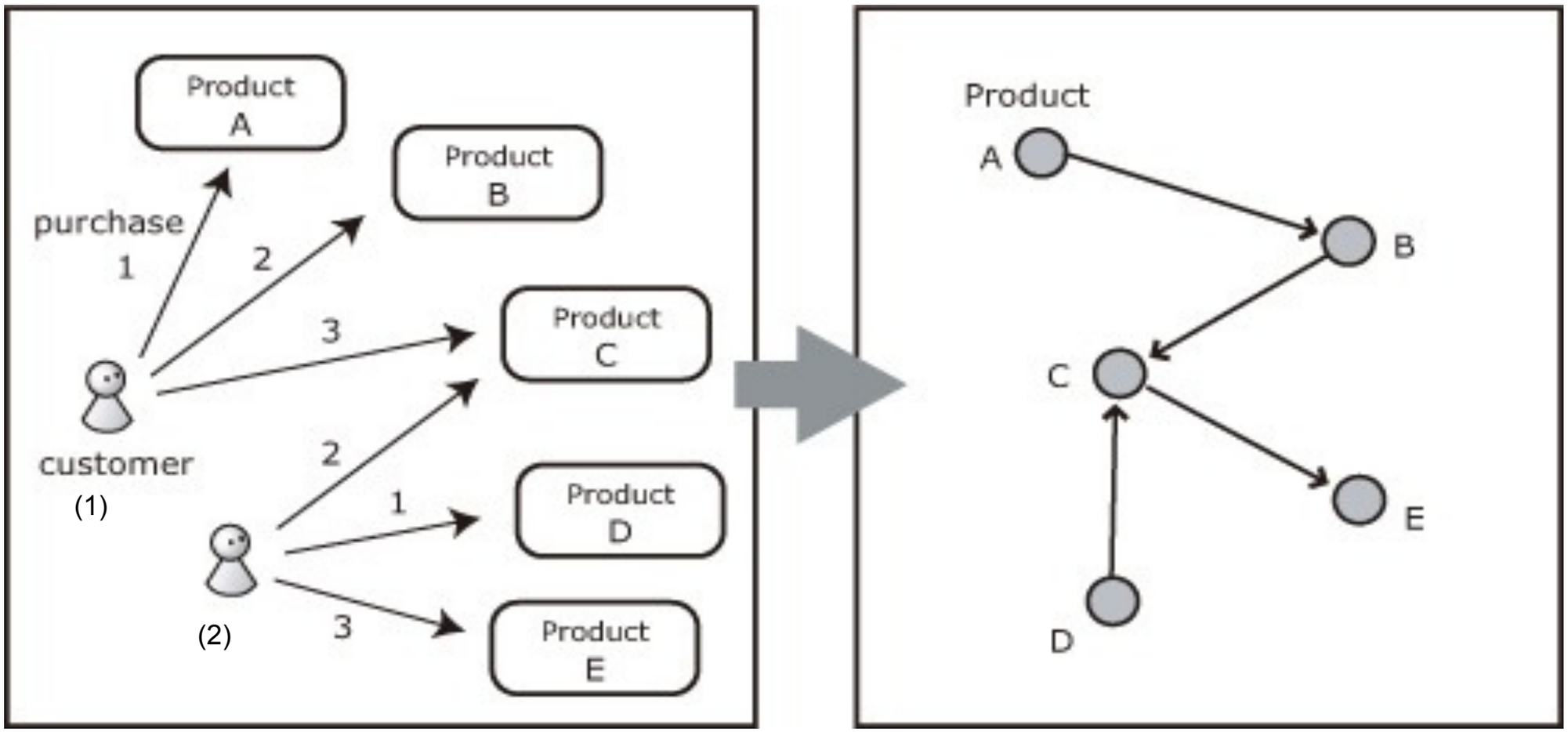
We've got POS data of random-sampled 30,000 customers (2005-2006)

- Customer ID (masked)
- When purchasing
- Which book (CD, DVD) purchasing

* sample customers of books, CDs, and DVDs are different one another.

Building Co-Purchase Network from data

Sequential Connection

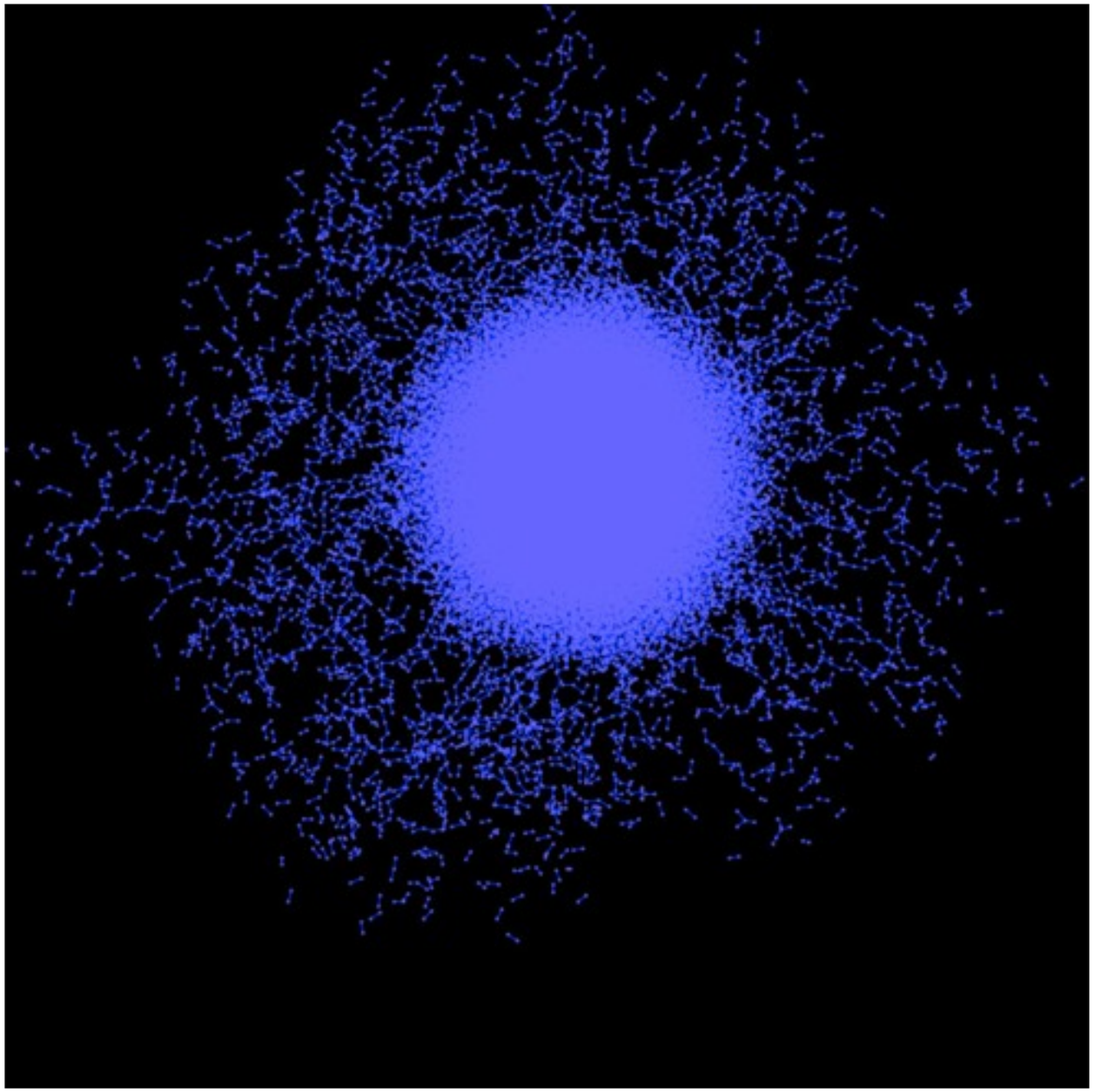


Co-Purchase Network of Books

Sequential Connection

Number of nodes = 68,701

Number of edges = 113,940



* Target Customers = 30,000

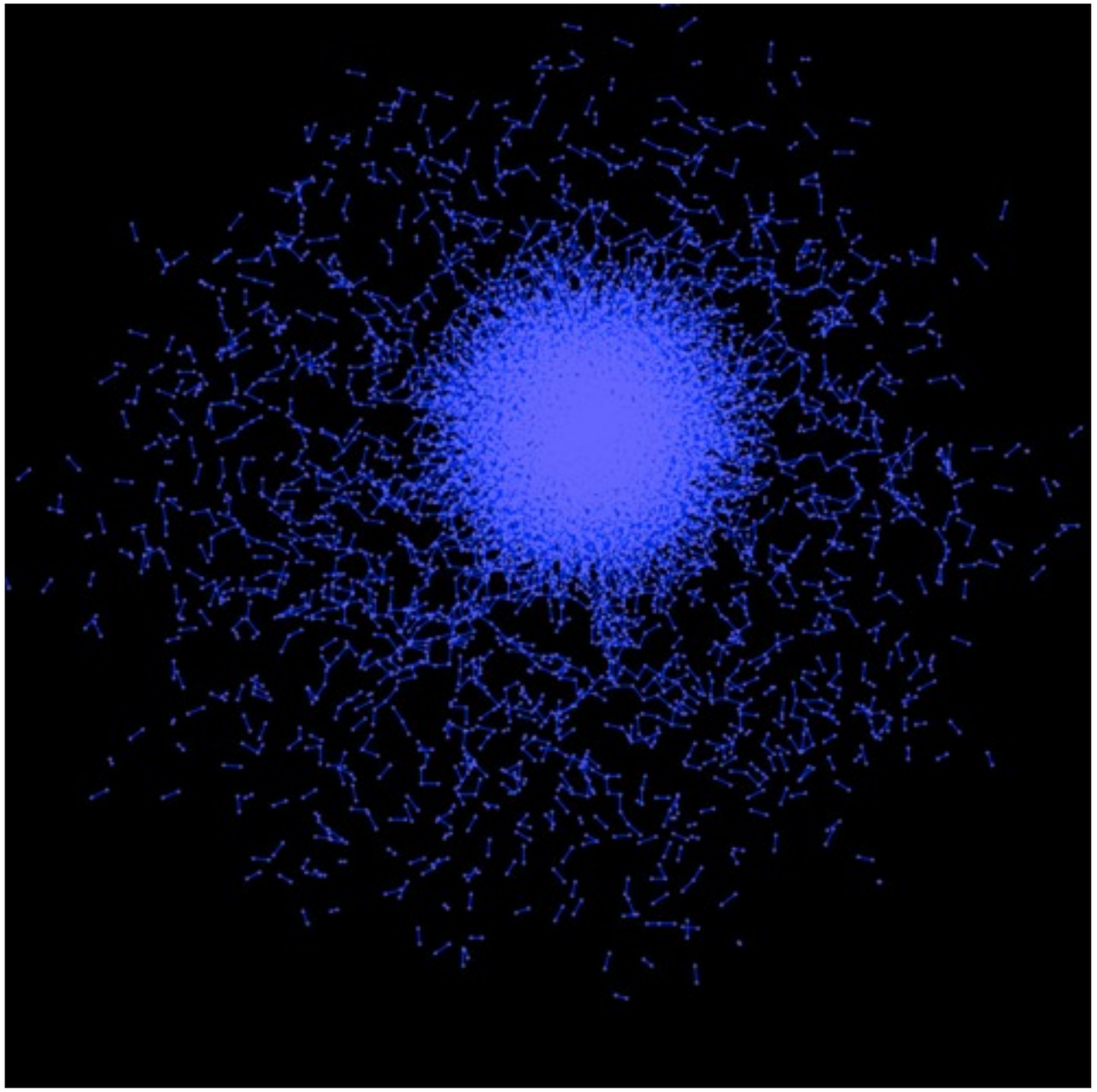
* All Links are Visualized.

Co-Purchase Network of CD

Sequential Connection

Number of nodes = 14,038

Number of edges = 22,727



* Target Customers = 30,000

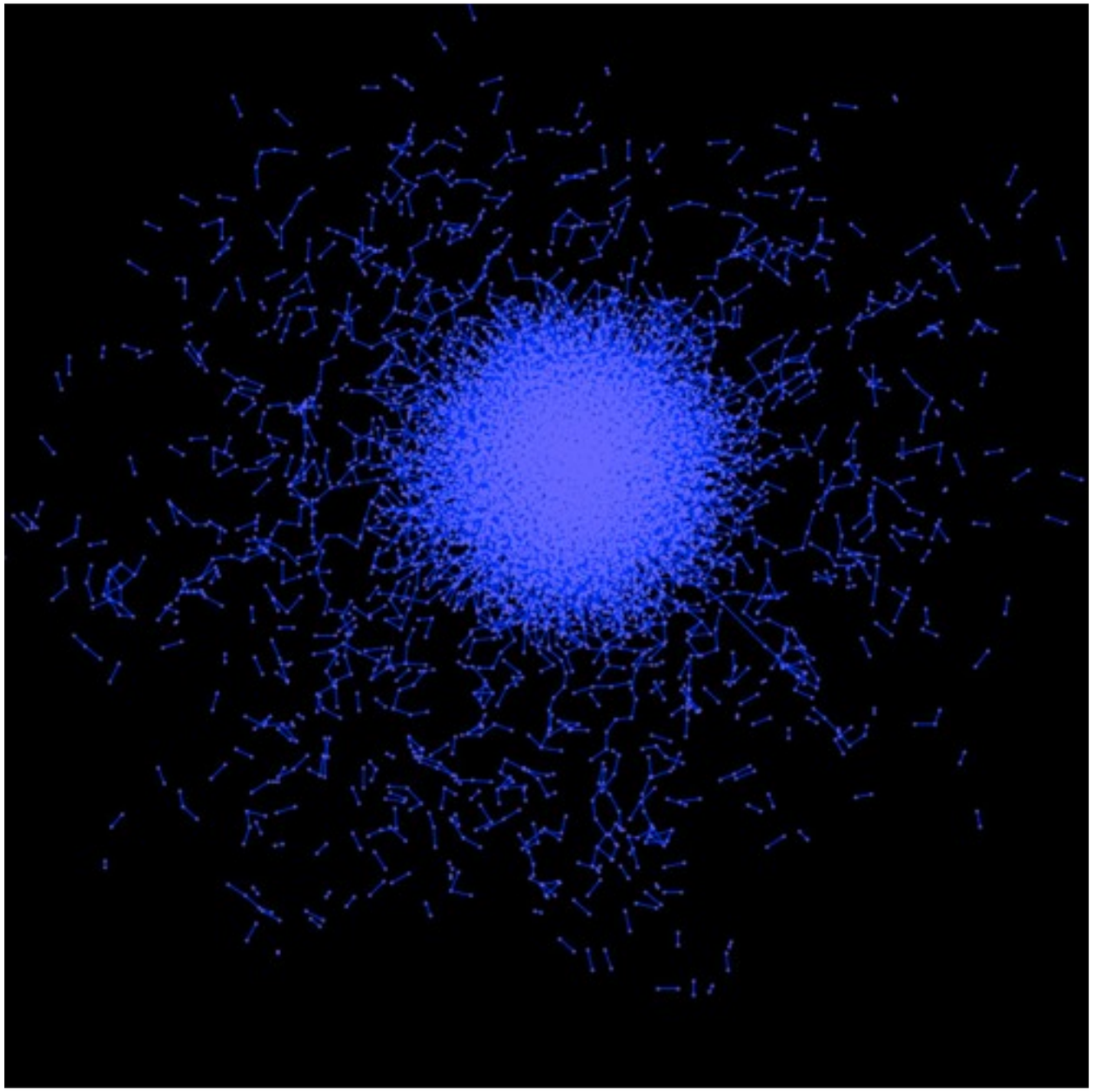
* All Links are Visualized.

Co-Purchase Network of DVD

Sequential Connection

Number of nodes = 10,875

Number of edges = 24,535



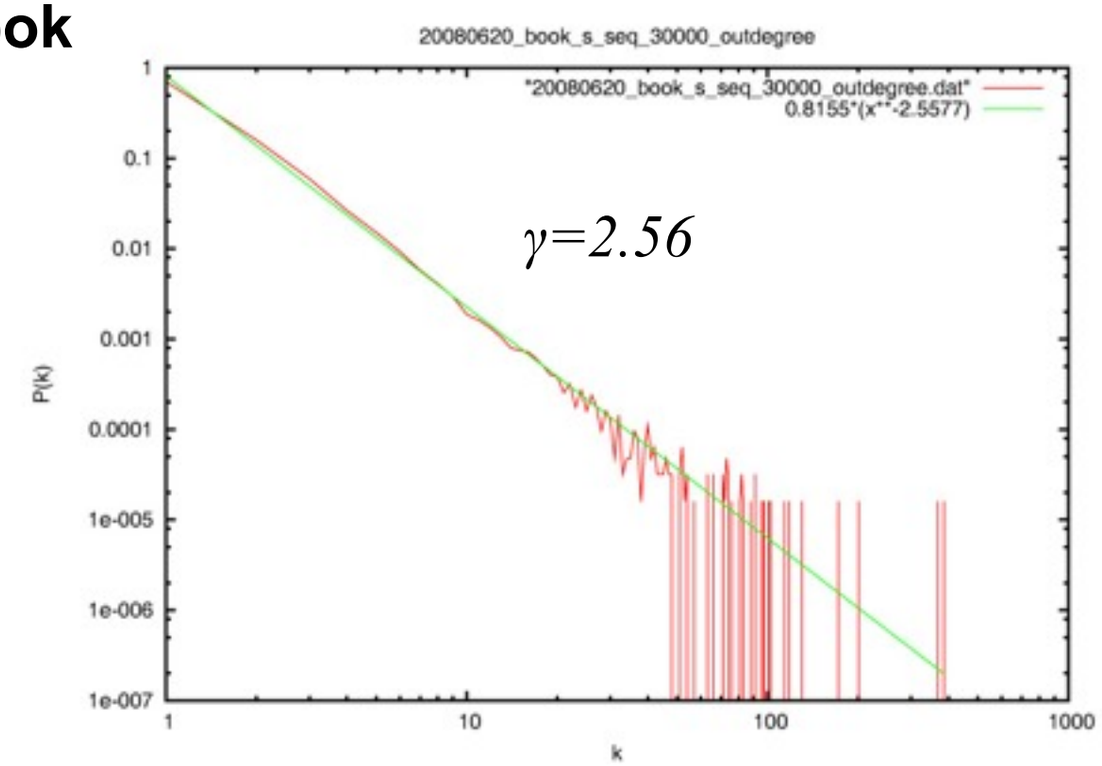
* Target Customers = 30,000

* All Links are Visualized.

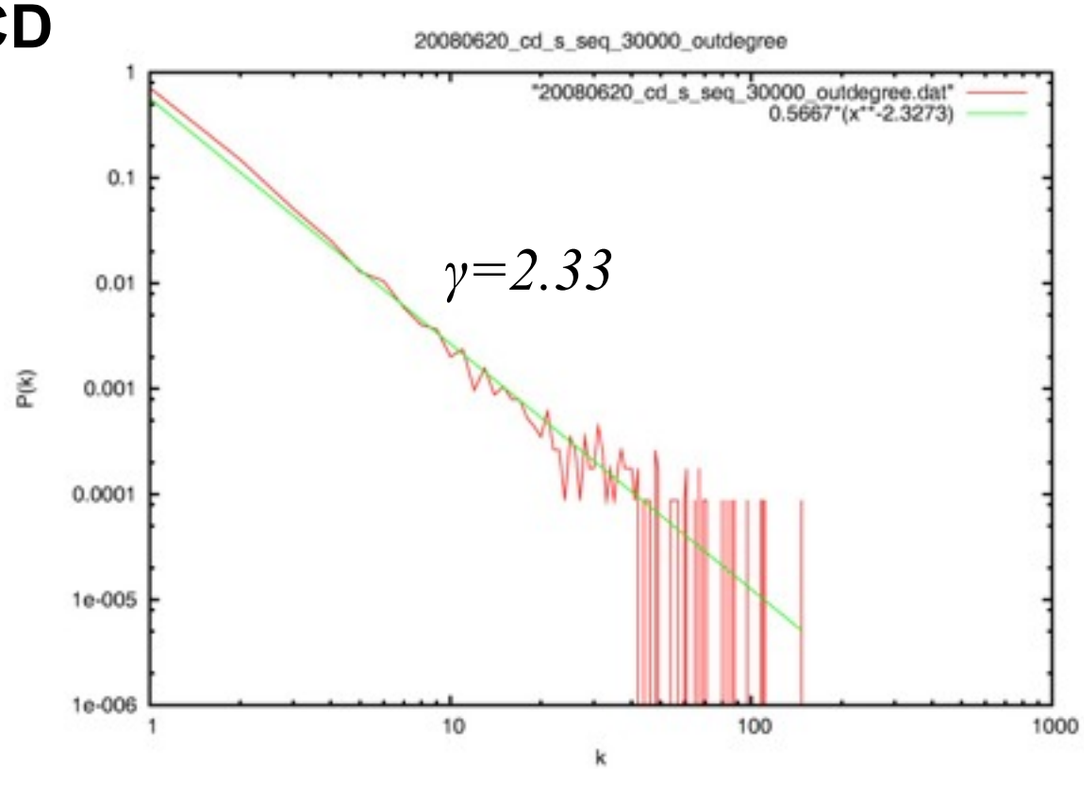
In-Degree Distribution follow Power Law

Sequential Connection

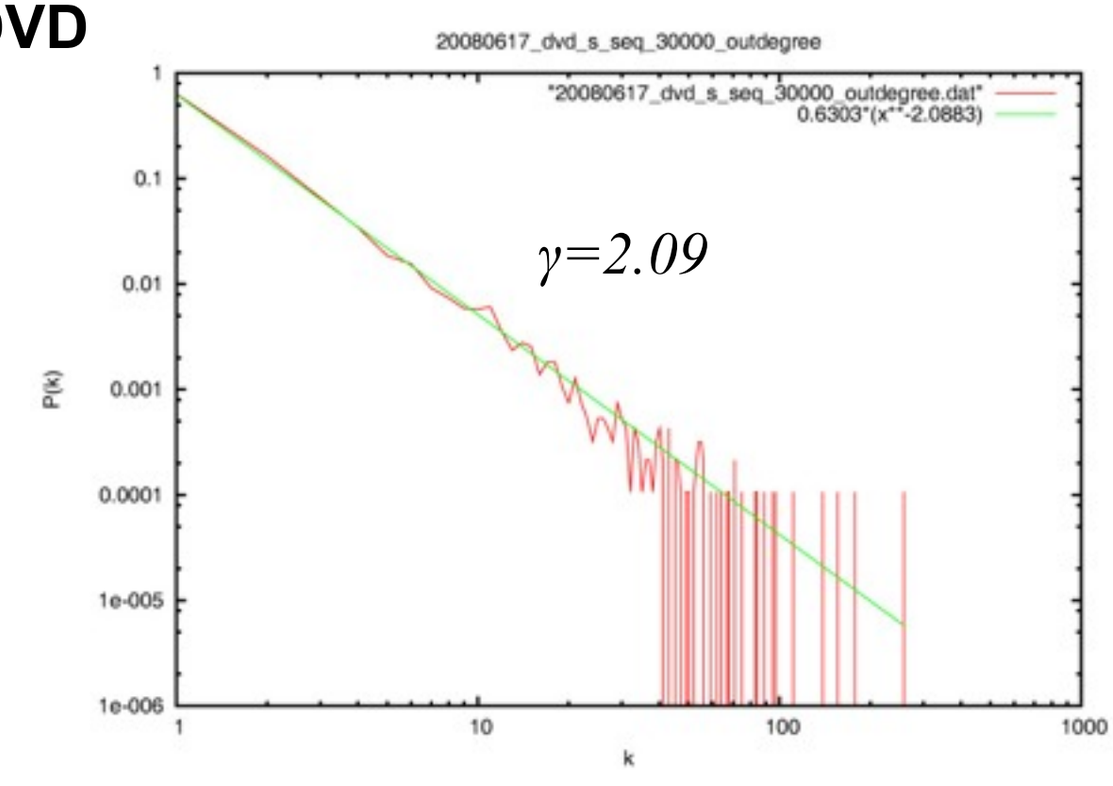
Book



CD



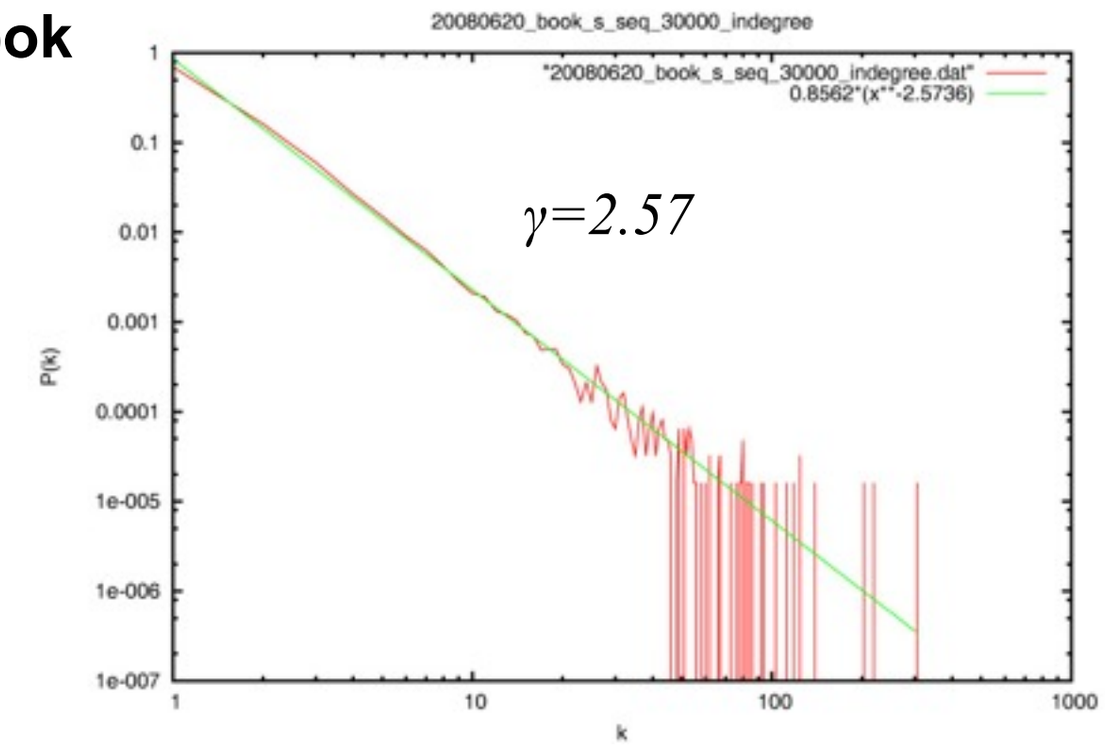
DVD



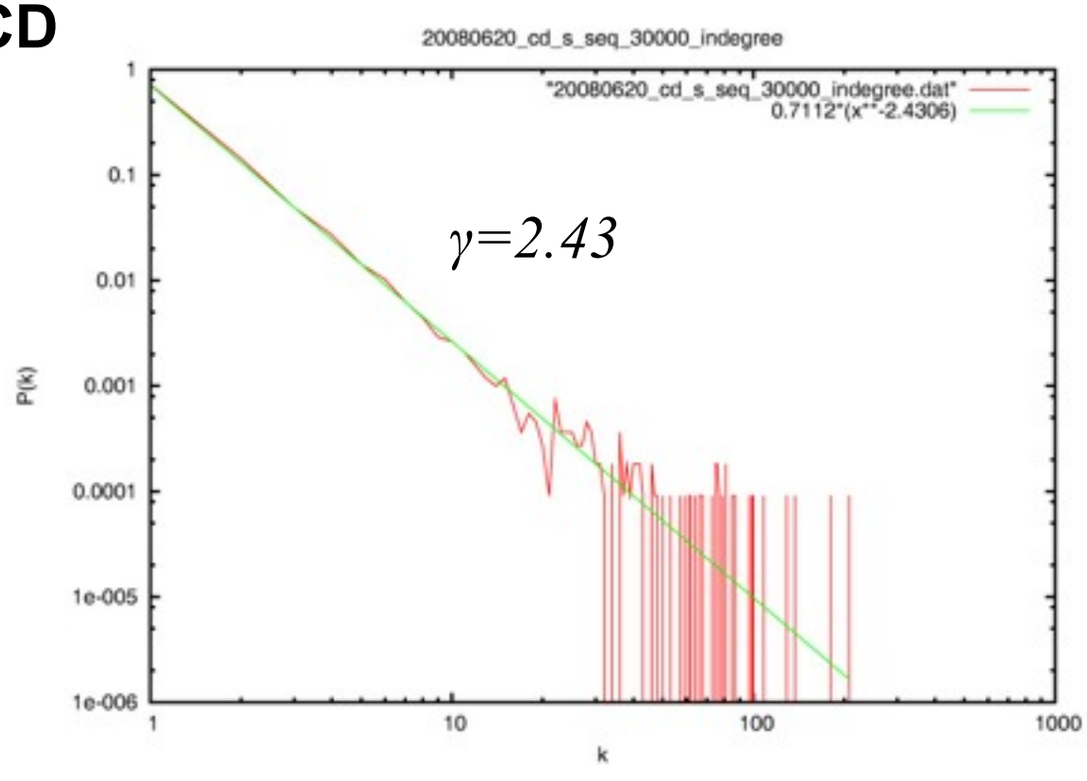
Out-Degree Distribution follow Power Law

Sequential Connection

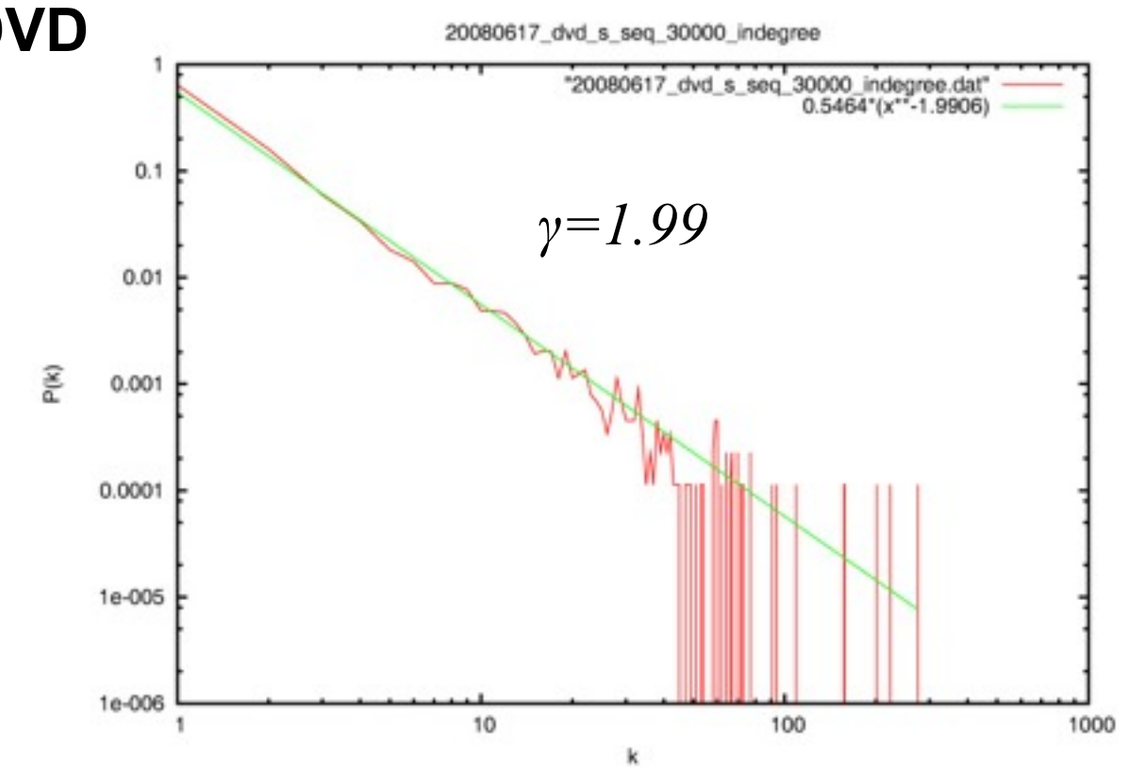
Book



CD



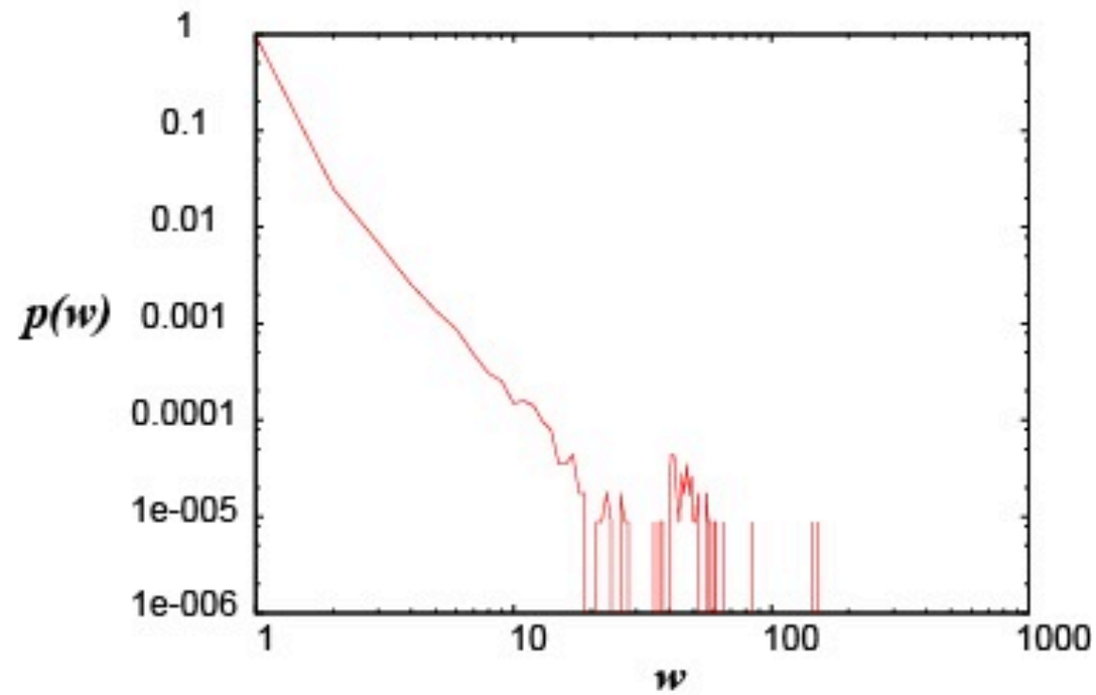
DVD



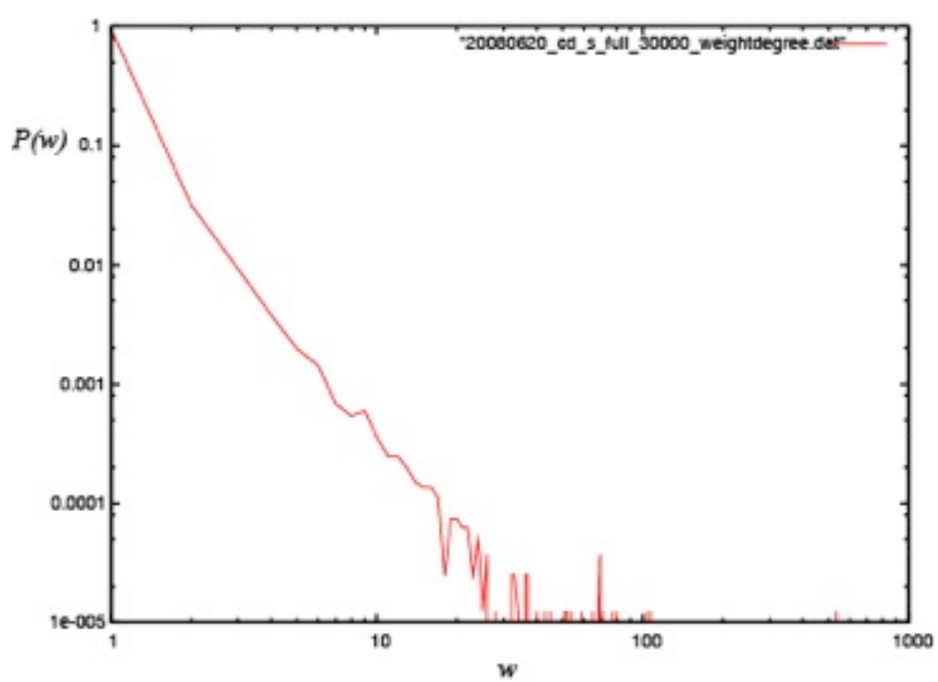
Weight Distribution follow Power Law

Sequential Connection

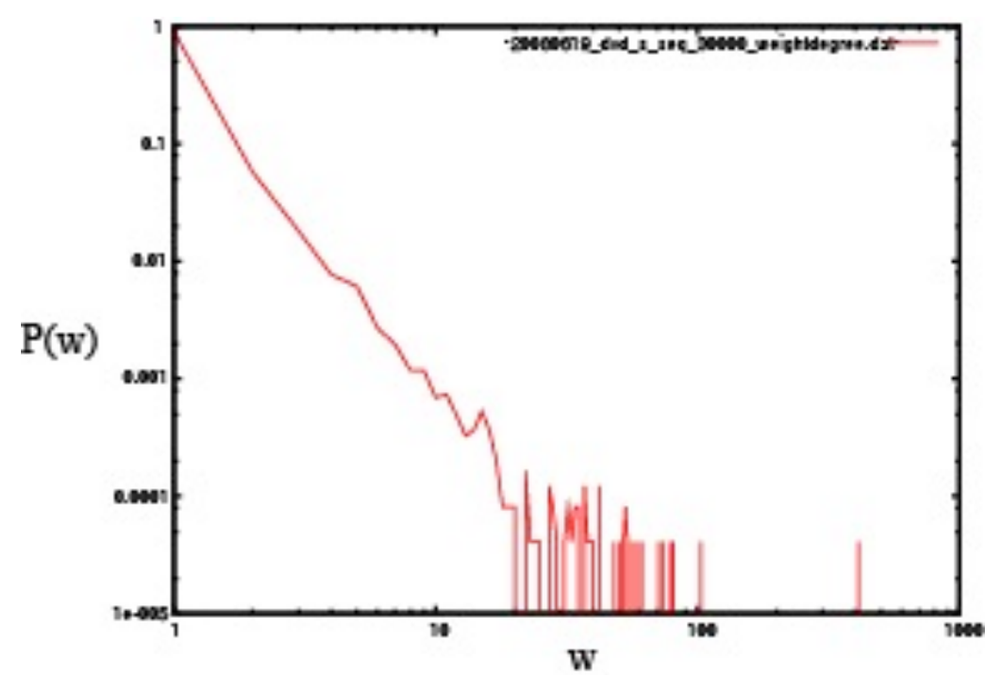
Books



CDs



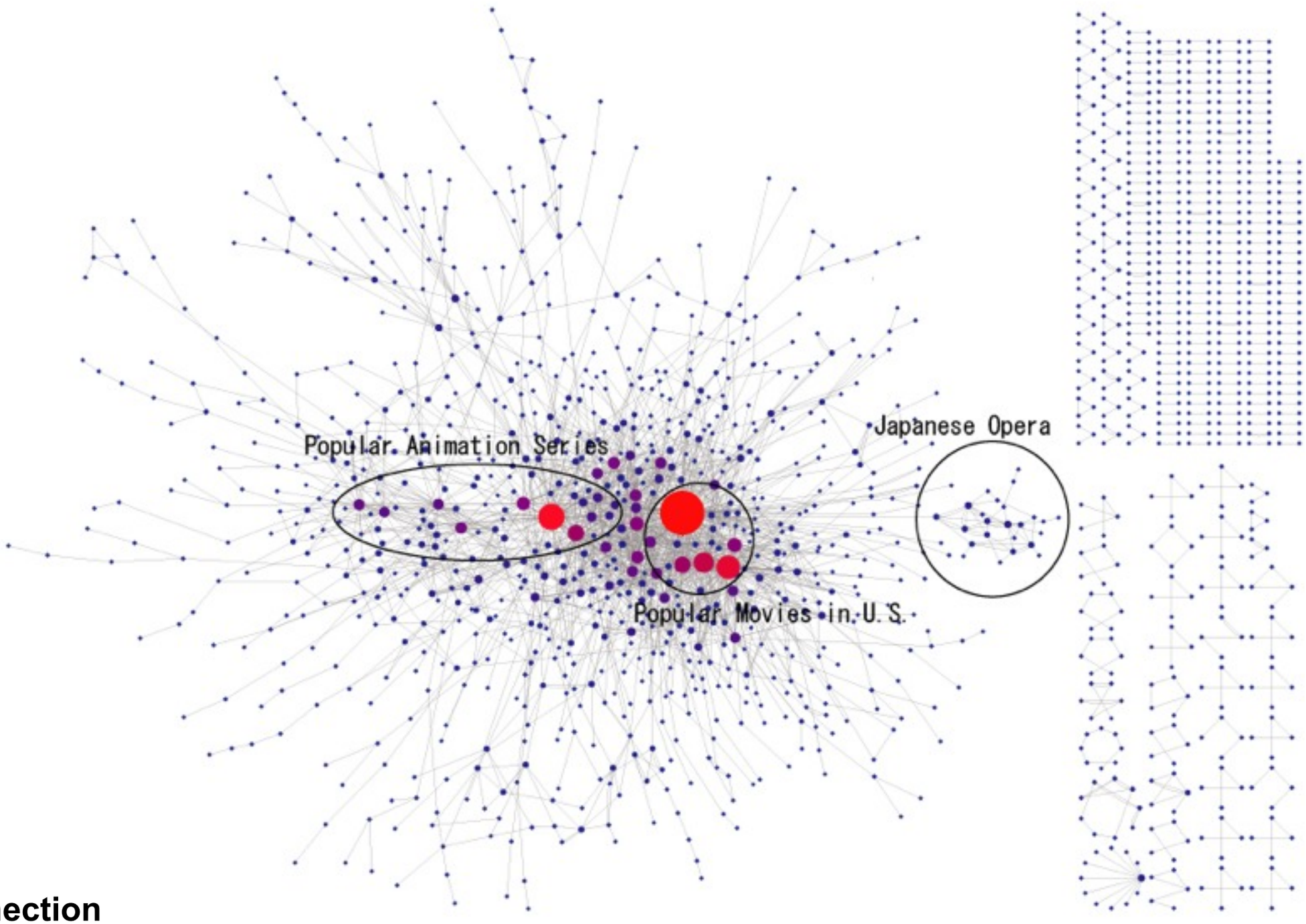
DVDs



Mapping Genre in Co-Purchase Network of DVDs

Sequential Connection

Node Coloring



Sequential Connection

Target Customers = 30,000

Weight > 1

Papers & Presentations

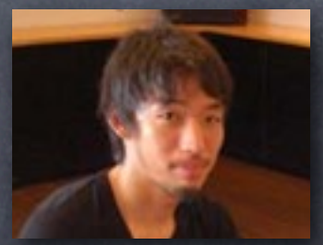
- T. Iba, M. Mori, "Visualizing and Analyzing Networks of Co-Purchased Books, CDs and DVDs", NetSci'08, June, 2008
- Y. Kitayama, M. Yoshida, S. Takami and T. Iba, "Analyzing Co-Purchase Network of Books in Japanese Online Store", poster, NetSci'08, June, 2008
- R. Nishida, M. Mori and T. Iba, "Analyzing Co-Purchase Network of CDs in Japanese Online Store", poster, NetSci'08, June, 2008
- S. Itoh, S. Takami and T. Iba, "Analyzing Co-Purchase Network of DVDs in Japanese Online Store", poster, NetSci'08, June, 2008
- 井庭 崇, 北山 雄樹, 伊藤 諭志, 西田 亮介, 吉田 真理子, 「オンラインストアにおける商品の共購買ネットワークの分析」, 情報処理学会 第5回ネットワーク生態学シンポジウム, 2009

Collaborators

Satoshi Itoh
Former Graduate Student, Iba Lab.

Yuki Kitayama
Former Graduate Student, Iba Lab.

Mariko Yoshida
Former Student, Iba Lab.



Ryosuke Nishida
Former Graduate Student, Iba Lab.

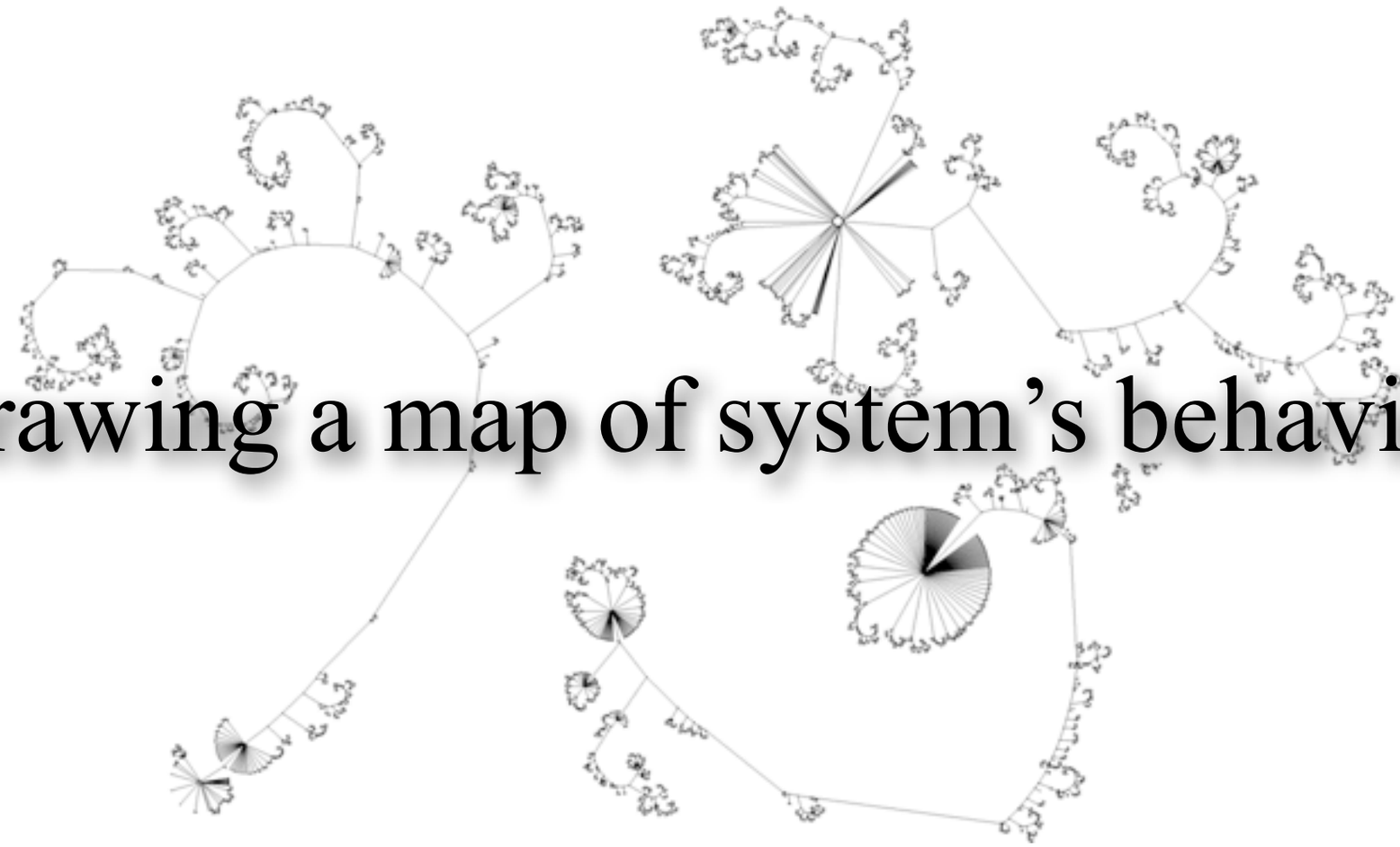
Masaya Mori
Rakuten Institute of Technology

State Networks of Chaotic Dynamical Systems

Dynamics as Networks

the dynamics of system/phenomena are represented as networks

Drawing a map of system's behavior



The networks of state transitions in several discretized chaotic dynamical systems are *scale-free networks*.

It is found in Logistic map, Sine map, Cubic map, General symmetric map, Gaussian map, Sine-circle map.

Fundamentals: Chaos

Highly complex behavior is generated,
although it’s governed by a very simple rule.

Fundamentals: Logistic Map

$$x_{n+1} = 4\mu\, x_n\,(\,1 - x_n\,)$$

a simple population growth model (non-overlapping generations)

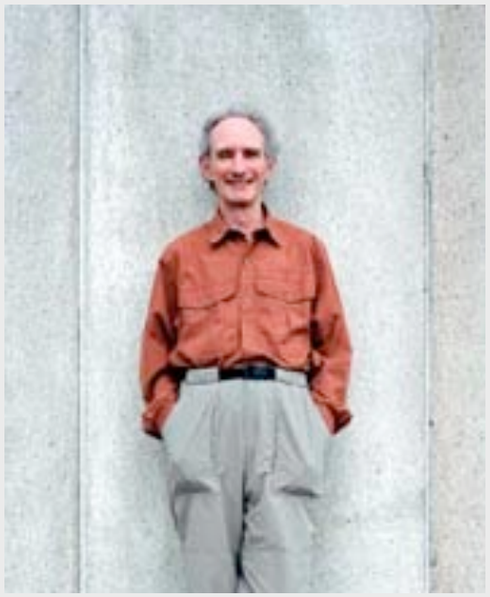
x_n	···	population (capacity)	$0 \le x_n \le 1$	(variable)
μ	···	a rate of growth	$0 \le \mu \le 1$	(constant)

x_0 = *an initial value*

<u>*n = 0*</u> $x_1 = 4\mu\, x_0\,(\,1 - x_0\,)$

<u>*n = 1*</u> $x_2 = 4\mu\, x_1\,(\,1 - x_1\,)$

<u>*n = 2*</u> $x_3 = 4\mu\, x_2\,(\,1 - x_2\,)$

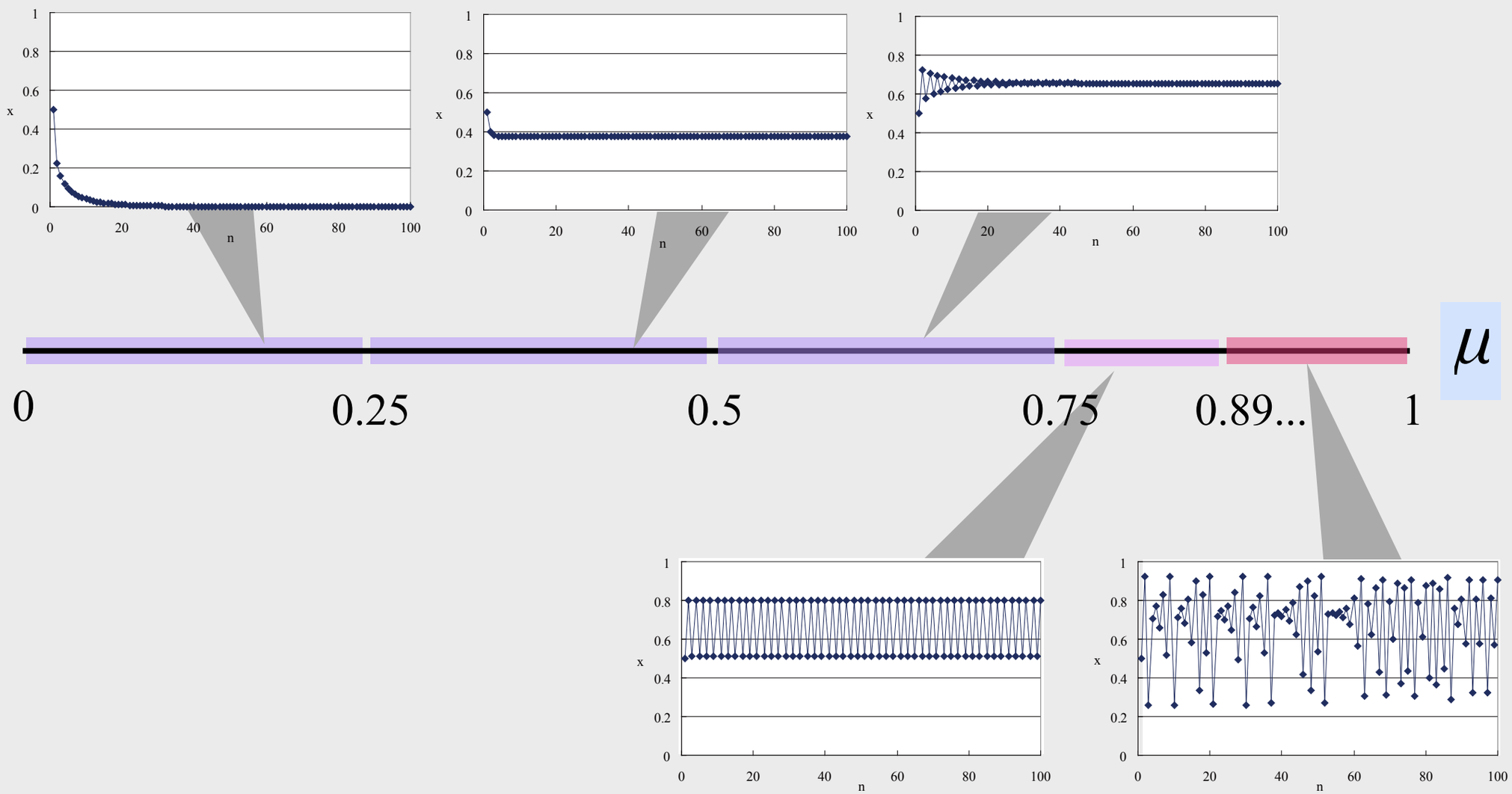


May, R. M. Biological populations with nonoverlapping generations: stable points, stable cycles, and chaos. *Science* **186**, 645–647 (1974).
May, R. M. Simple mathematical models with very complicated dynamics. *Nature* **261**, 459–467 (1976).

Fundamentals: Logistic Map

$$x_{n+1} = 4\mu\, x_n\,(1 - x_n)$$

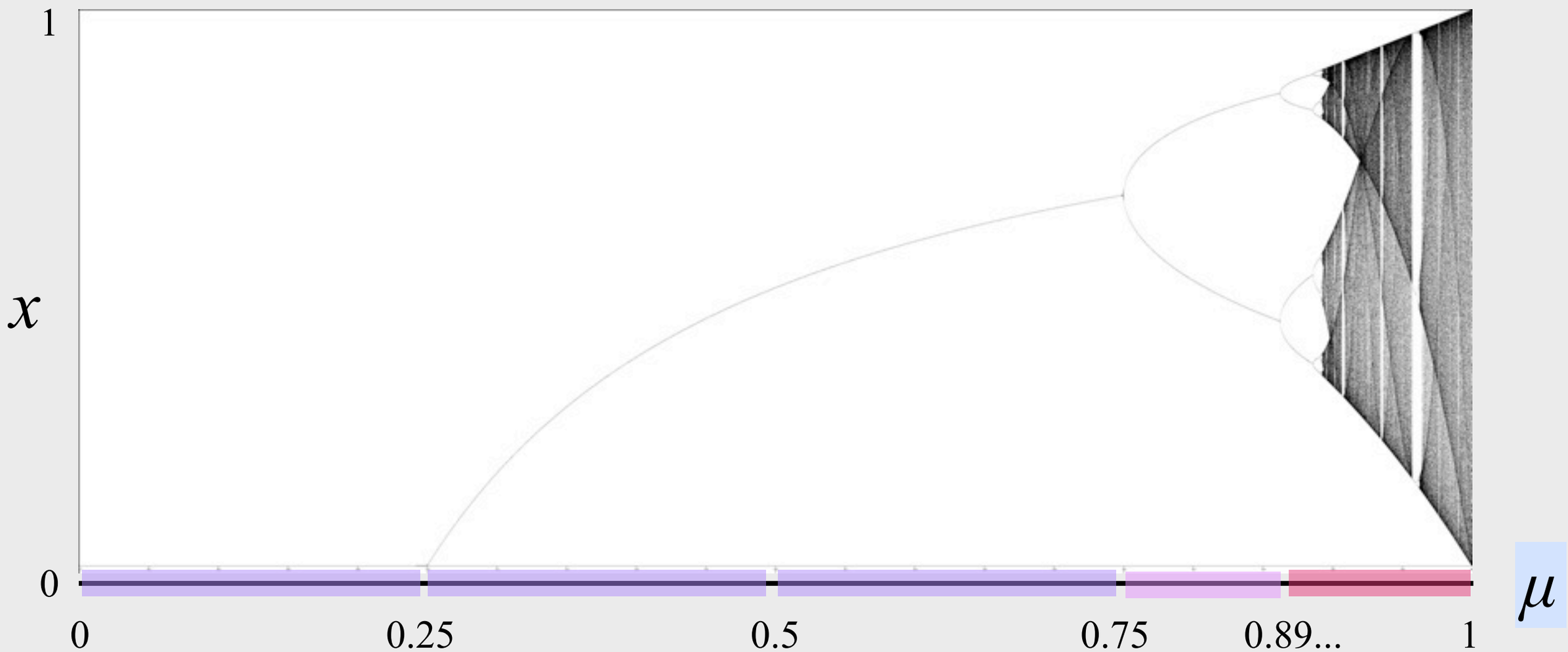
The behavior depends on the value of control parameter μ.



Fundamentals: Logistic Map

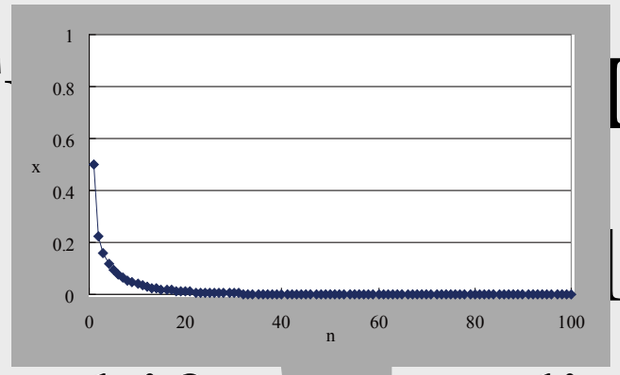
$$x_{n+1} = 4\mu\, x_n\,(1 - x_n)$$

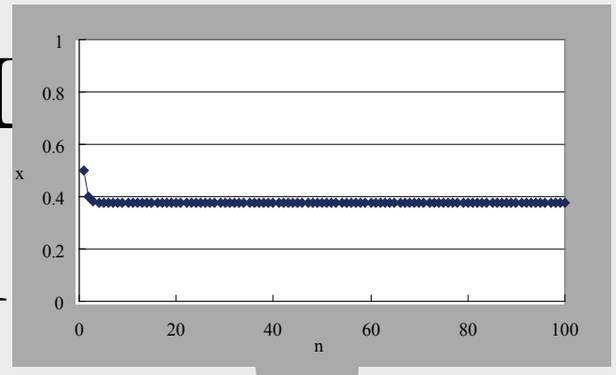
bifurcation diagram

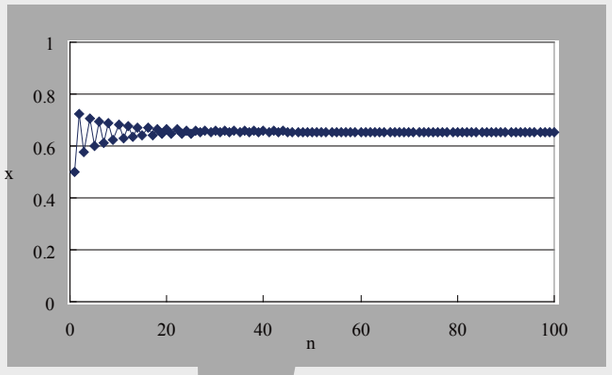


This diagram shows long-term values of x.

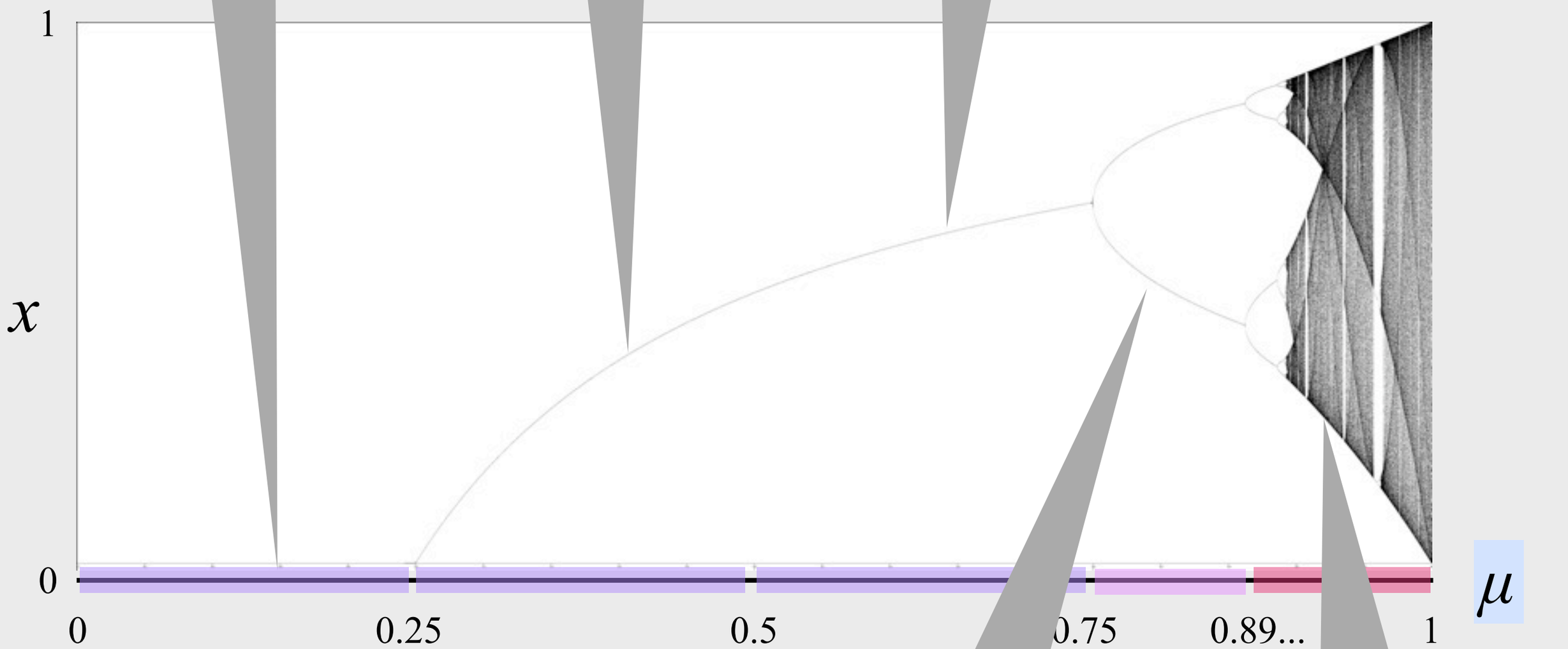
F



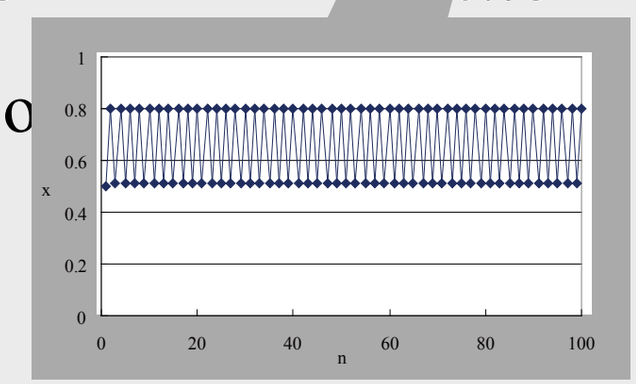


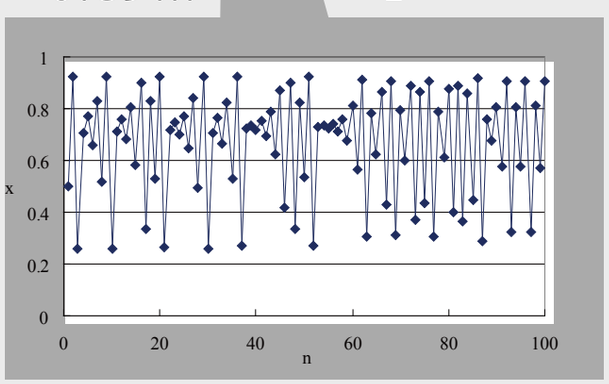


bifurc on diagram



This diagram shows long-term values o





I want a map!

for taking an overview of the whole.

map

1. a. A representation, usually on a plane surface, of a region of the earth or heavens.

b. Something that suggests such a representation, as in clarity of representation.

2. *Mathematics*: The correspondence of elements in one set to elements in the same set or another set.

- The American Heritage Dictionary of the English Language

I want a map of the *map!*

for taking an overview of the whole behavior of the iterated *map*.

Logistic Map

$$x_{n+1} = 4\, x_n\, (\, 1 - x_n \,)$$

map

1. a. A representation, usually on a plane surface, of a region of the earth or heavens.

b. Something that suggests such a representation, as in clarity of representation.

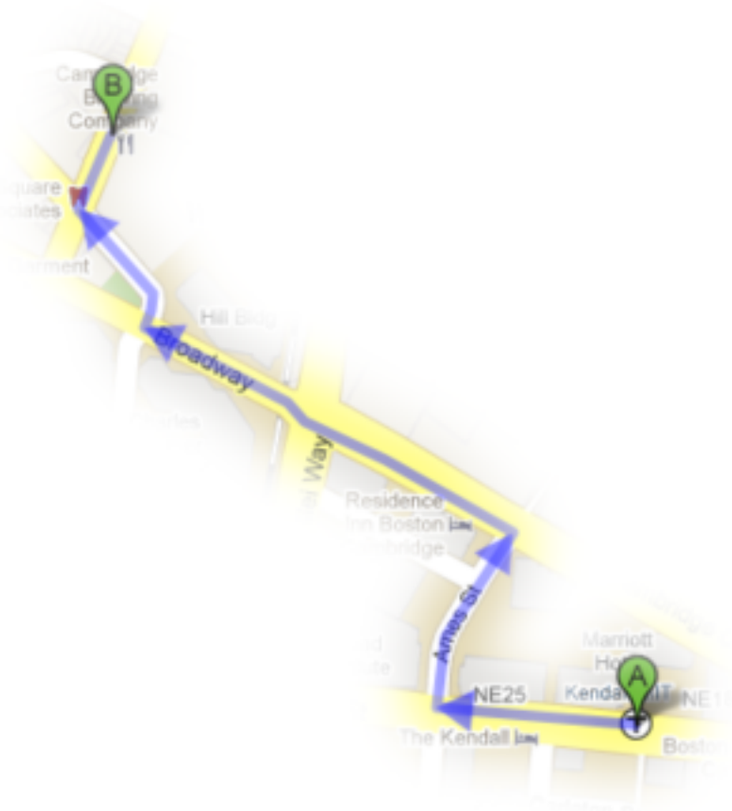
2. *Mathematics*: The correspondence of elements in one set to elements in the same set or another set.

- The American Heritage Dictionary of the English Language

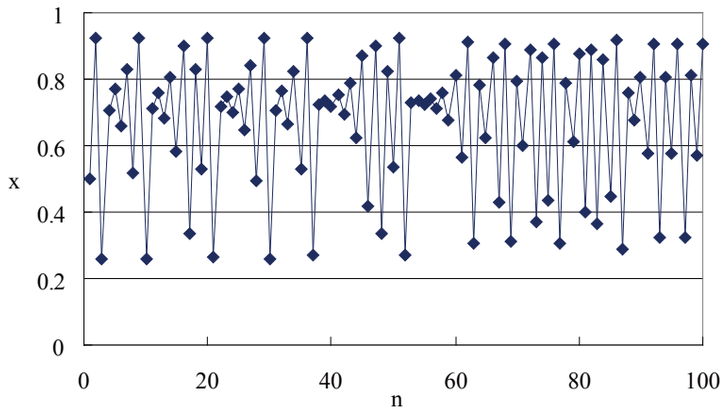
Existing methods are NOT for drawing a map

for taking an overview of the whole

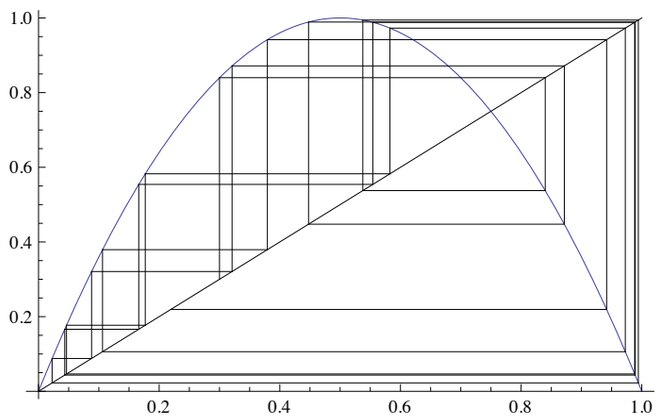
They merely visualize the *trajectory* of ***an instance*** of the system's behavior, giving an initial value.



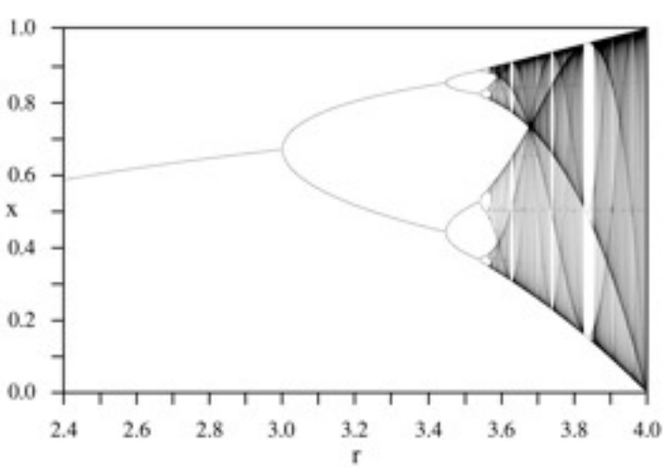
time series



cobweb plot



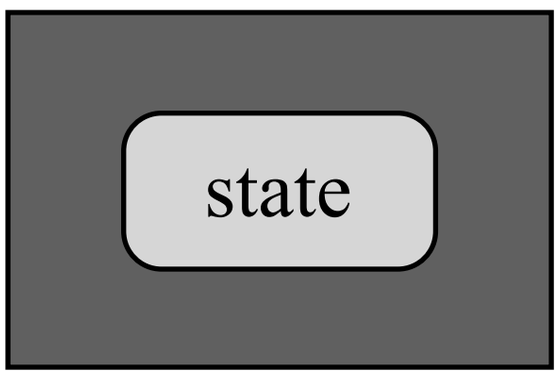
bifurcation diagram

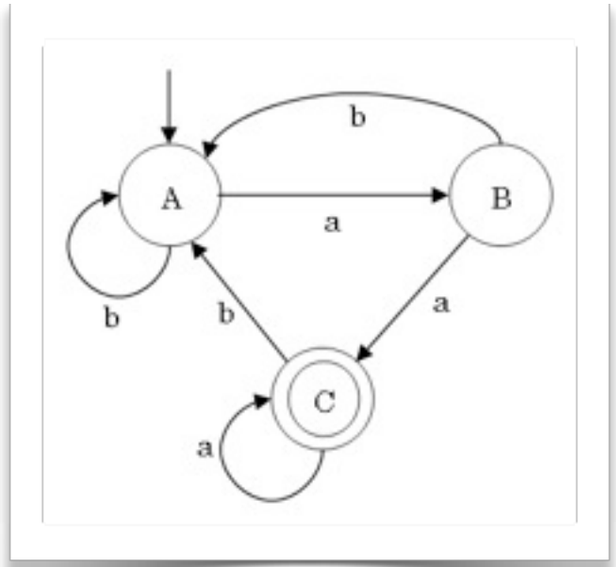


Visualizing State Transitions of a System

for taking an overview of the whole.

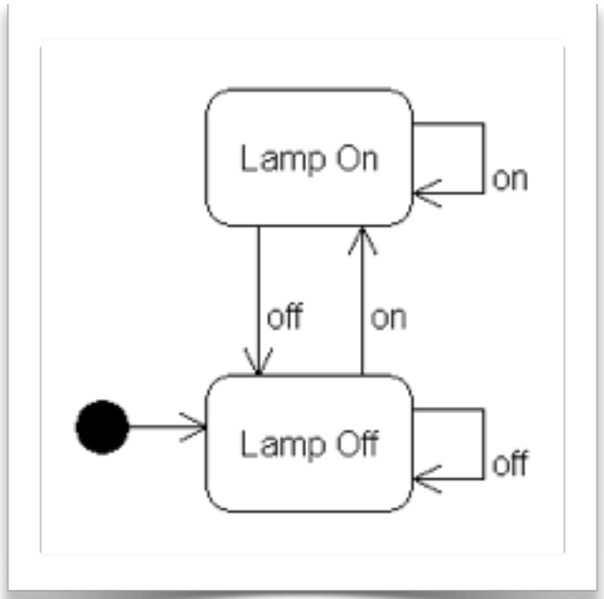
system





$$x_{n+1} = 4\mu\, x_n\, (\, 1 - x_n \,)$$

state: the value of x



map

$$x_{n+1} = 4\mu\, x_n\, (\, 1 - x_n \,)$$

x is treated as *discrete*

which implies the finite number of states

abstraction

target world

$$x_{n+1} = 4\mu\, x_n\, (\, 1 - x_n \,)$$

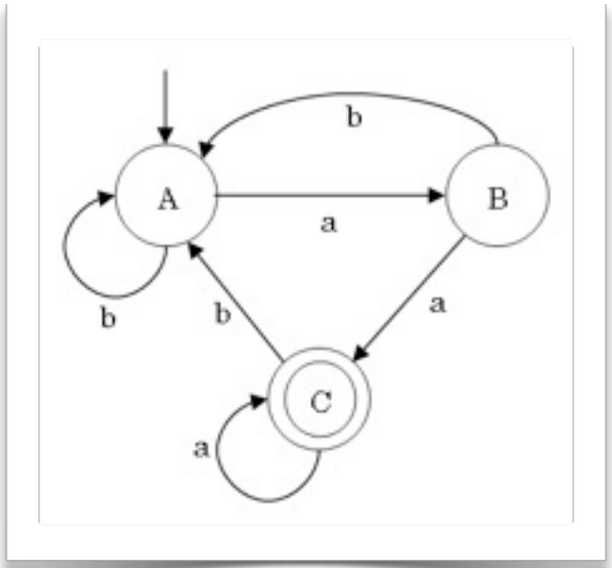
x is a real number

which has no limit to smallness (detail)

$$x_{n+1} = 4\mu\, x_n\,(\,1 - x_n\,)$$

x is treated as *discrete*

which implies the finite number of states



Transit map

Network of stations

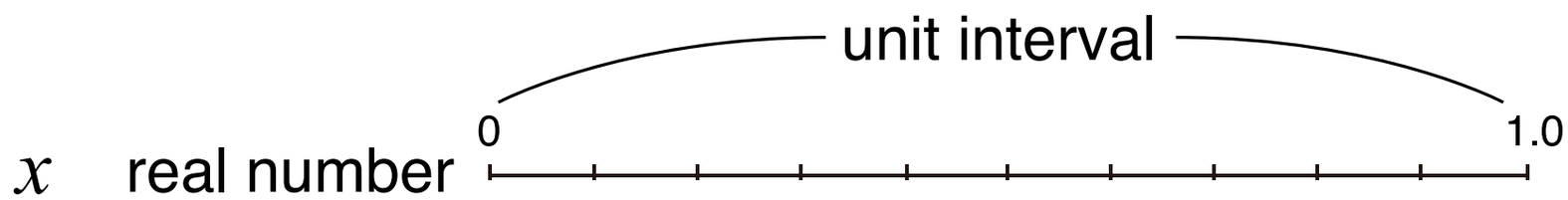
State-transition map

Network of states

Discretizing into the finite number of states

To subdivide a unit interval of variables into a finite number of subintervals

$$f(x) = 4\mu x(1 - x)$$

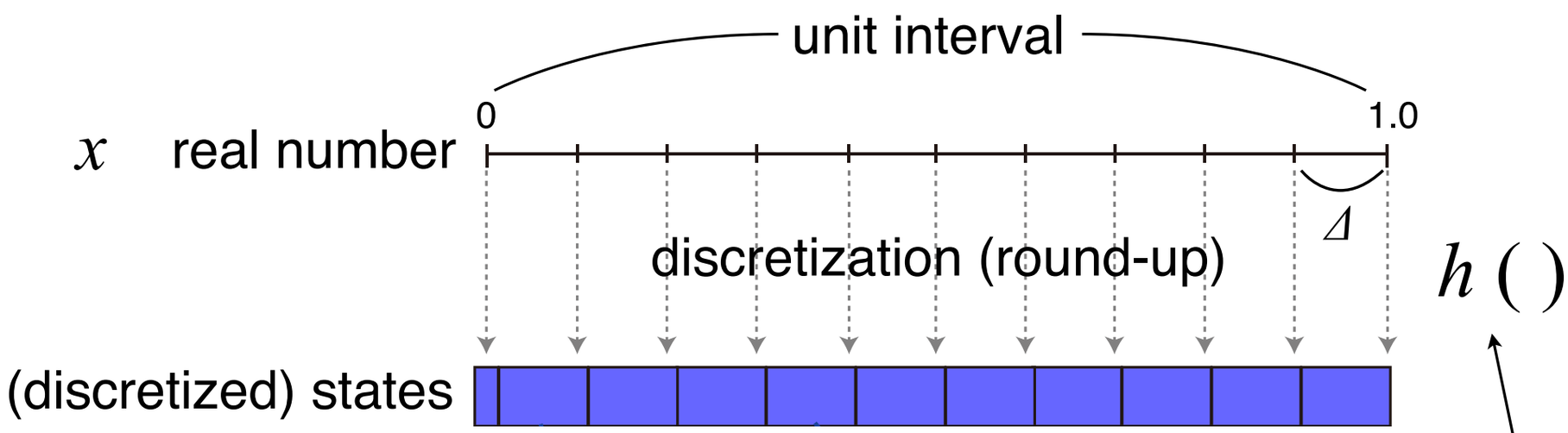


$$x_{n+1} = f(x_n)$$

Discretizing into the finite number of states

To subdivide a unit interval of variables into a finite number of subintervals

$$f(x) = 4\mu x(1 - x)$$



(ex.) *round-up*

$d = 1$: 0.14 0.2

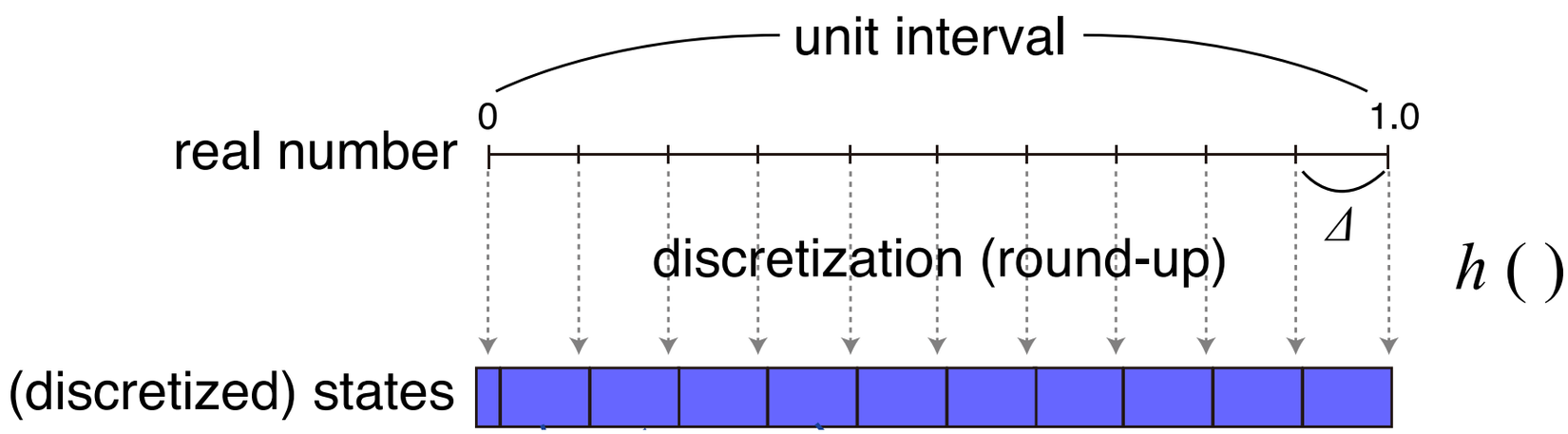
$d = 2$: 0.146 0.15

h( ) is the rounding function:
round-up, round-off, or round-down

The discretization is carried out by rounding the value x to <u>d decimal places</u>.
Therefore, $\Delta = 10^{-d}$

Obtaining the set of state transitions

To apply the map f into all states in order to obtain all state transitions.



(ex.) *round-up*

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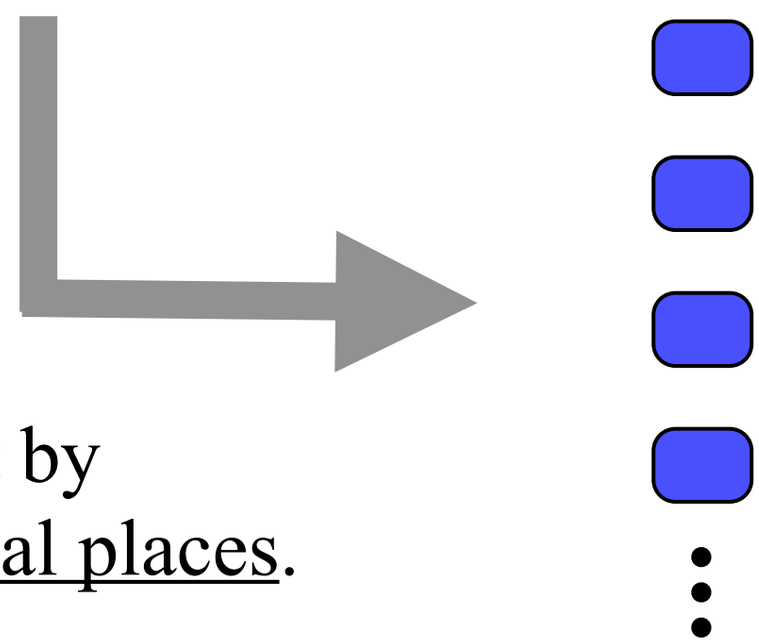

the set of state

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Obtaining the set of state transitions

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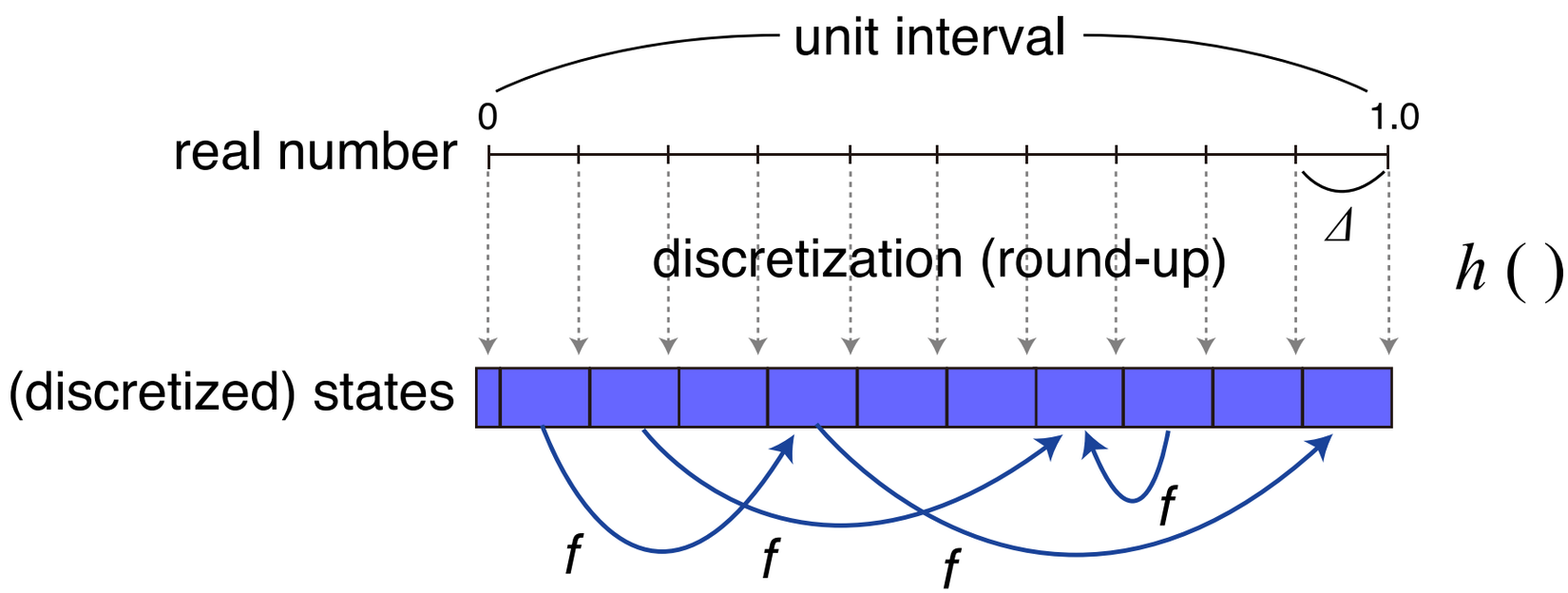
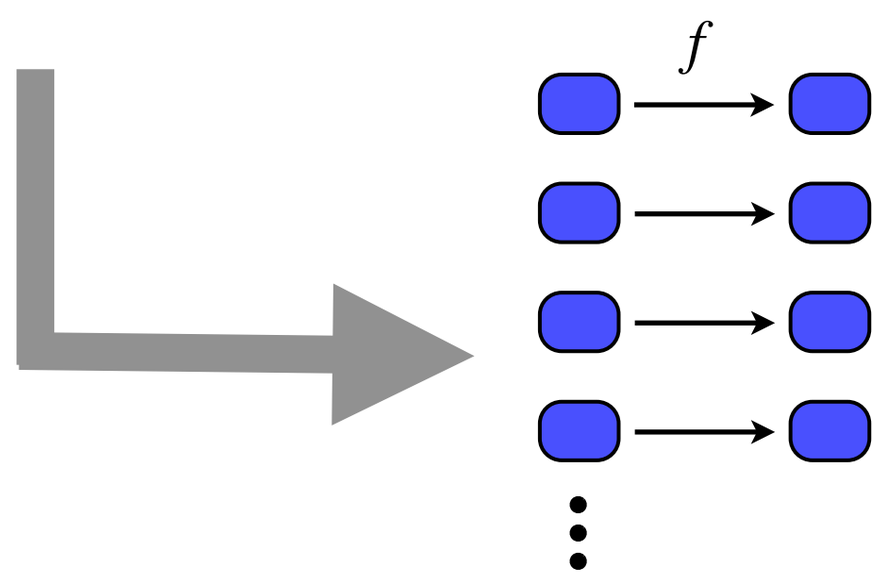




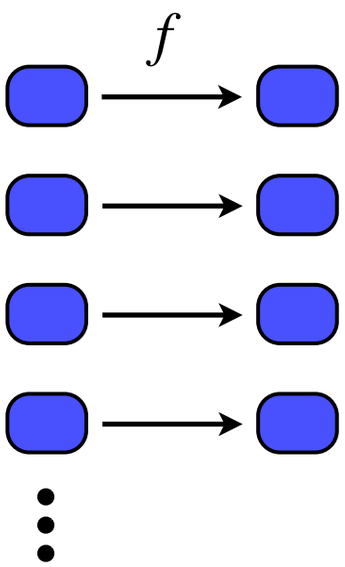
the set of
state transitions

$$x_{n+1} = h\big(f(x_n)\big)$$

where $h(\,)$ is the rounding function, and $f(x) = 4\mu x(1-x)$

Building state-transition networks

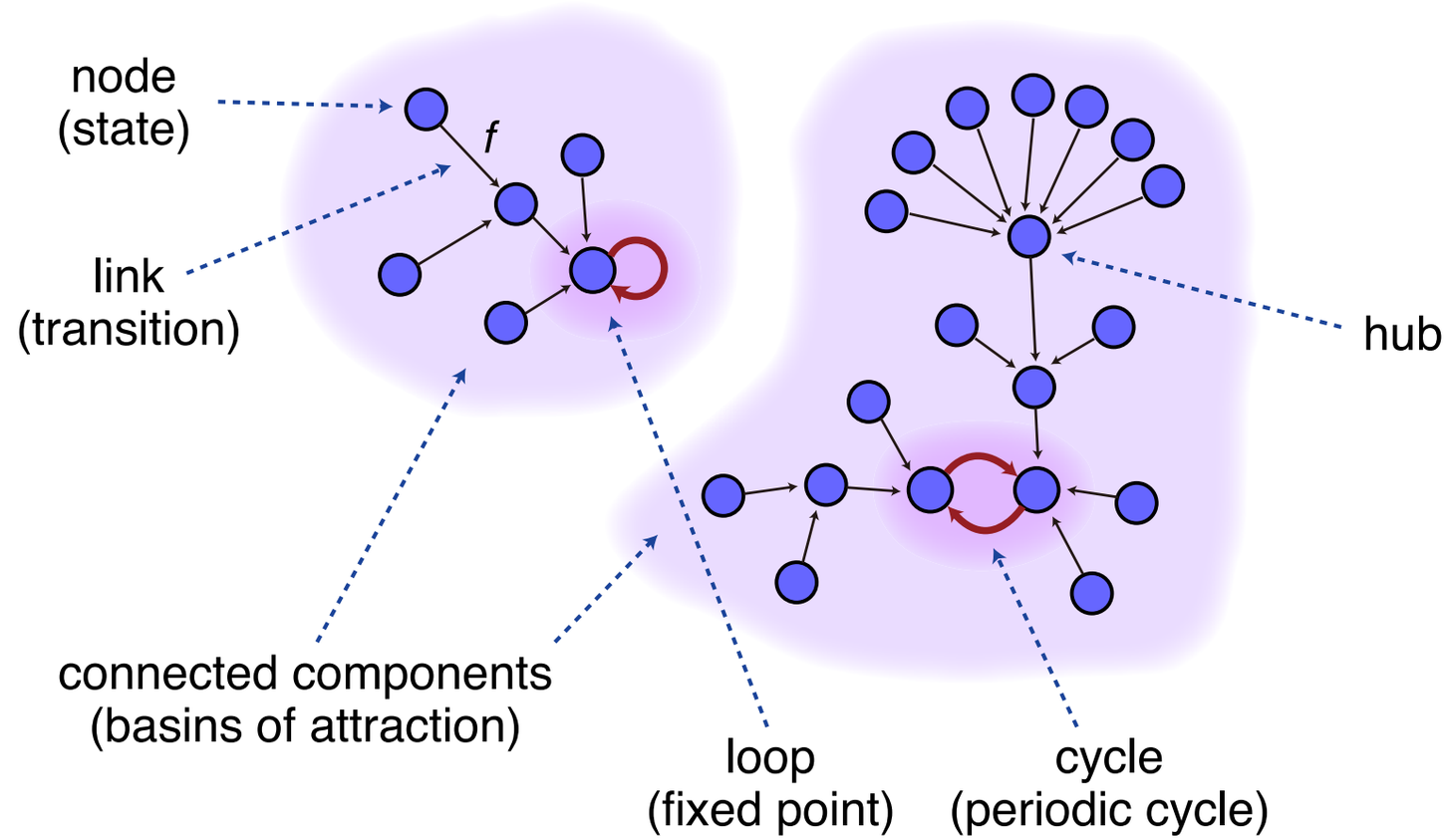
To connect each state into its successive state



the set of state transitions

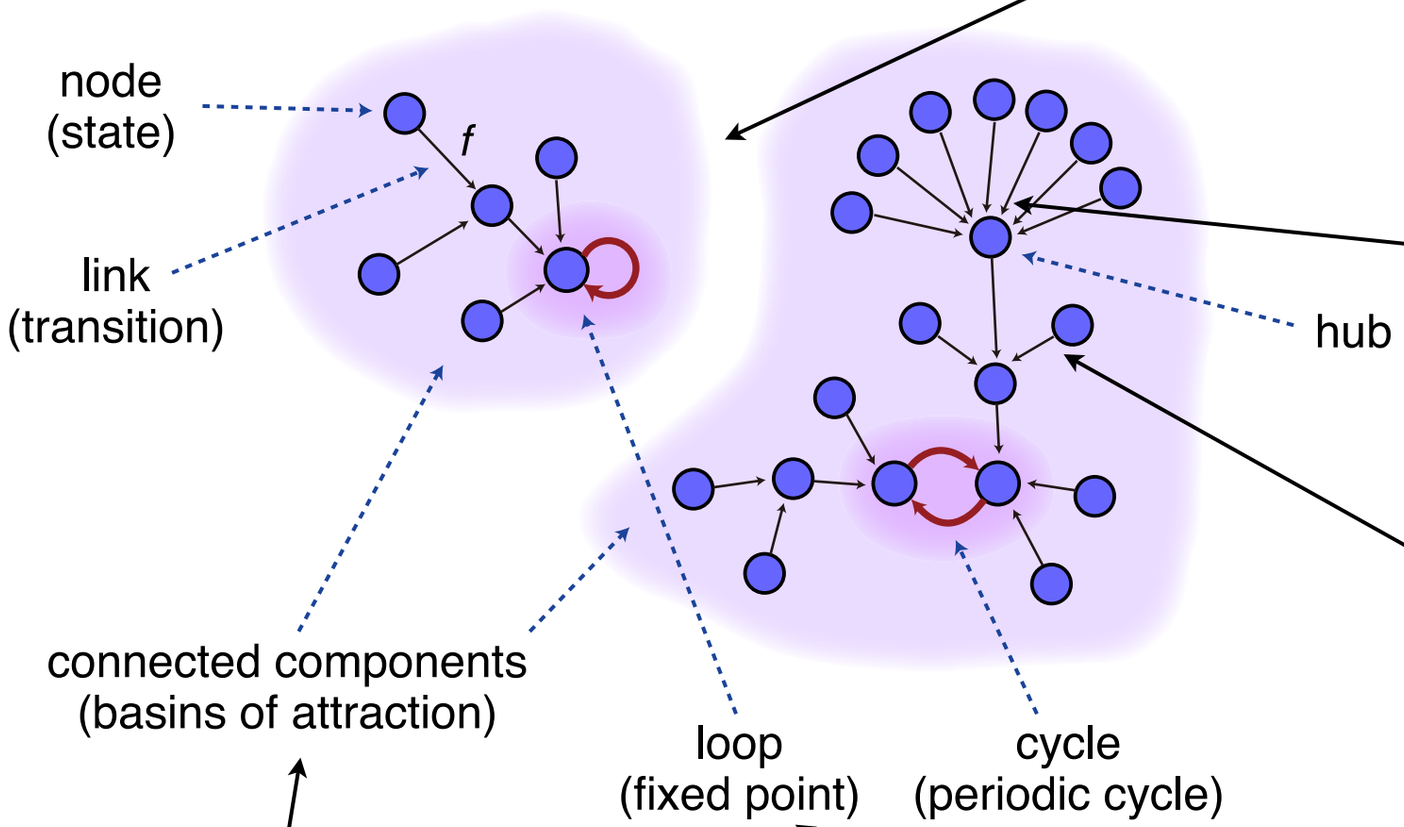
State-transition network is a “map” of the whole behavior of the system, so one can take an overview from bird’s-eye view.

building networks



State-transition networks

The order of the network $N = 1/\Delta + 1$.



The in-degree of each node may exhibit various values.

The out-degree of every node must be always equal to 1, since it is a *deterministic* system.

The whole behavior is often mapped into more than one connected components, which represent *basins of attraction*.

Each connected component must have only one loop or cycle: *fixed point* or *periodic cycle*.

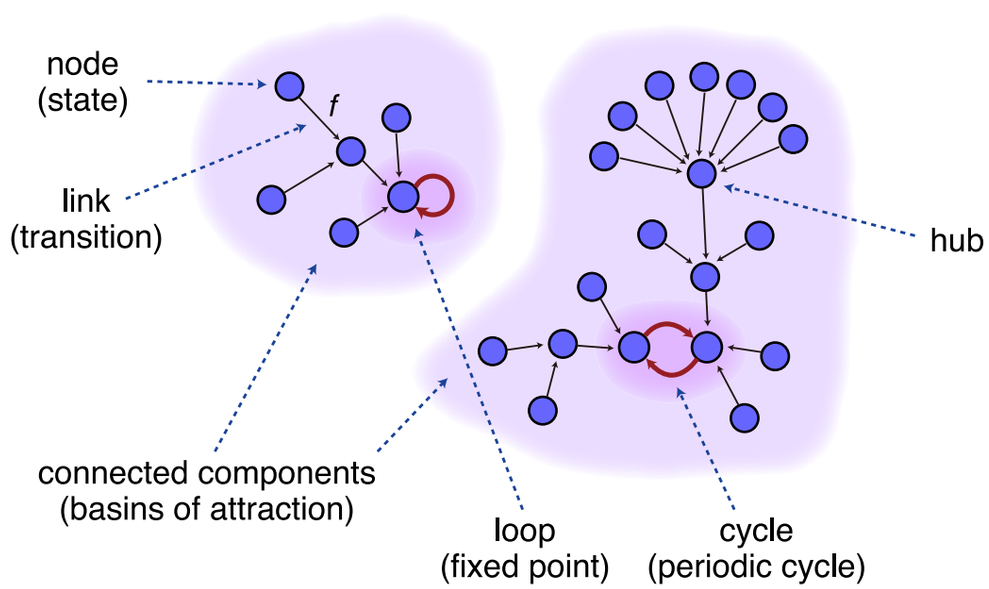
State-transition networks as a “dried-up river”

node: geographical point in the river

link: a connection from a point to another

The direction of a flow in the river is fixed.

The network is *dried-up* river, where there are no water flows on the riverbed.



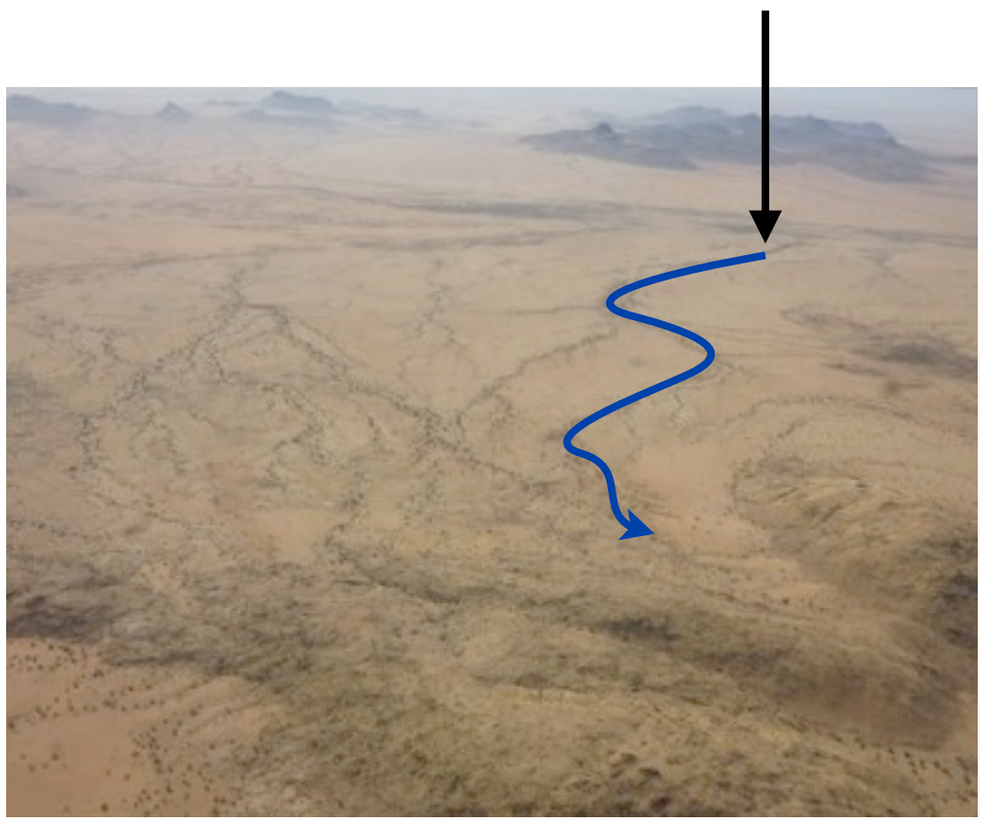
There are many confluences of two or more tributaries.

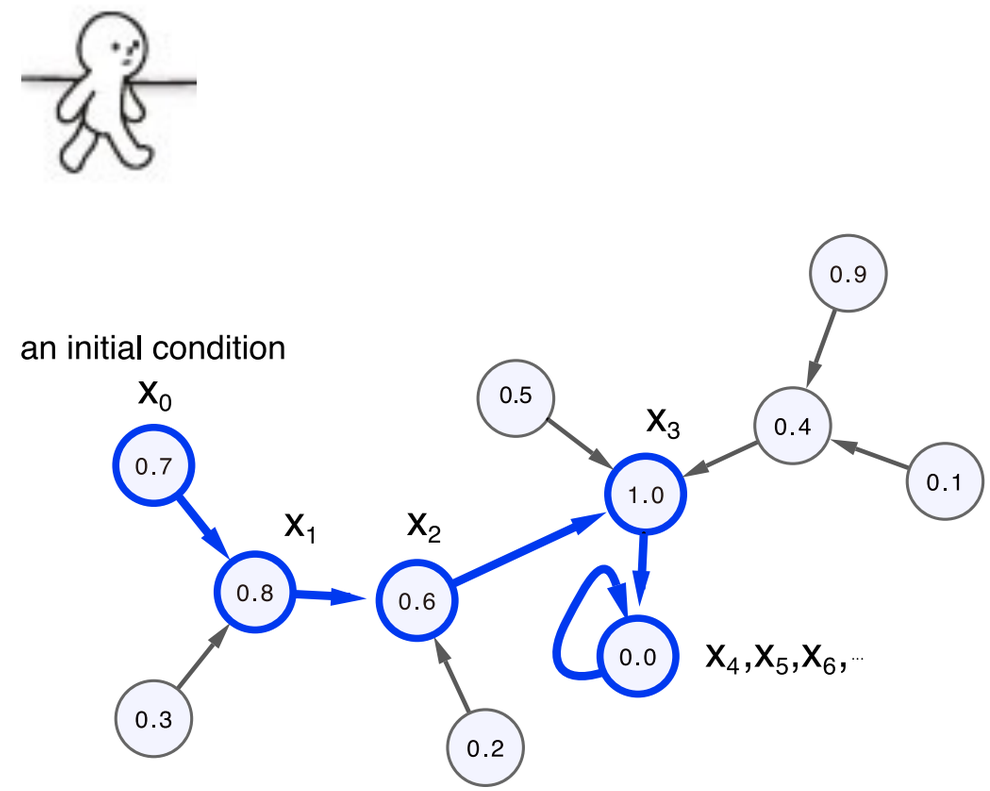
There are no branches of the flow.

Numerical simulation traces an instance

Determining a starting point and discharging water, you will see that the water flow downstream on the river network.

That is a happening you witness when conducting a numerical simulation of system's evolution in time.



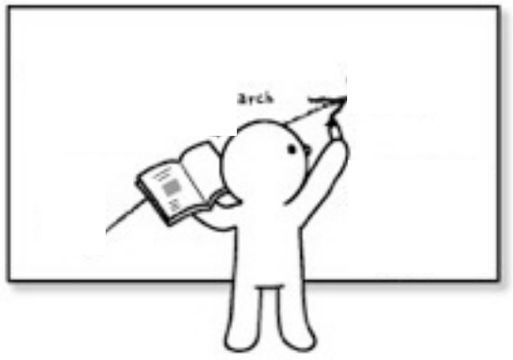


Numerical simulation represents *<u>an instance</u>* of flow on the state-transition networks.

Let's draw a map of the logistic *map!*

for taking an overview of the whole

$$x_{n+1} = 4\mu\, x_n \,(\, 1 - x_n \,)$$



The state-transition networks for the logistic map

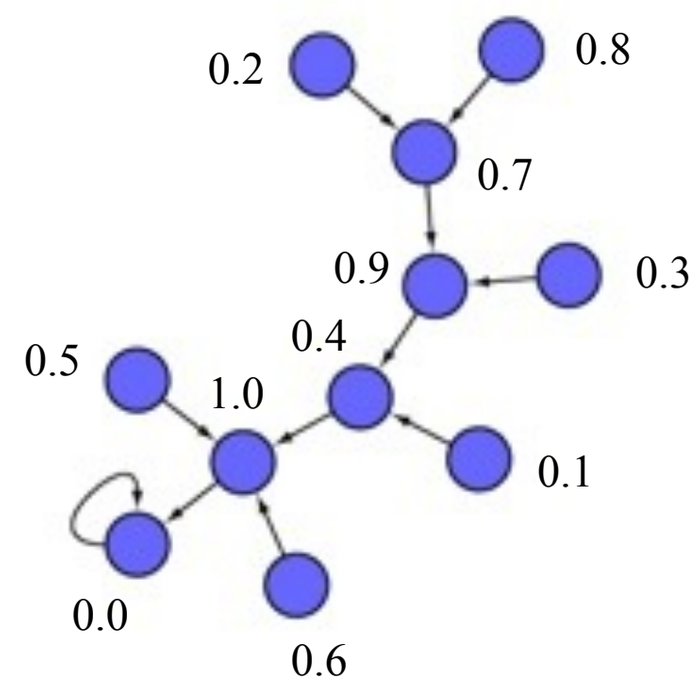
Control Parameter: $\mu = 1$

Round-Up into the decimal place, $d = 1$

Therefore, $x_{n+1} = h(\, 4\, x_n\, (\, 1 - x_n\,)\,)$

Therefore, 11 states

x_n	$\xrightarrow{f}$		$\xrightarrow{h}$	x_{n+1}
0.0	$\longrightarrow$	0.00	$\longrightarrow$	0.0
0.1	$\longrightarrow$	0.36	$\longrightarrow$	0.4
0.2	$\longrightarrow$	0.64	$\longrightarrow$	0.7
0.3	$\longrightarrow$	0.84	$\longrightarrow$	0.9
0.4	$\longrightarrow$	0.96	$\longrightarrow$	1.0
0.5	$\longrightarrow$	1.00	$\longrightarrow$	1.0
0.6	$\longrightarrow$	0.96	$\longrightarrow$	1.0
0.7	$\longrightarrow$	0.84	$\longrightarrow$	0.9
0.8	$\longrightarrow$	0.64	$\longrightarrow$	0.7
0.9	$\longrightarrow$	0.36	$\longrightarrow$	0.4
1.0	$\longrightarrow$	0.00	$\longrightarrow$	0.0



The state-transition networks for the logistic map

Control Parameter: $\mu = 1$

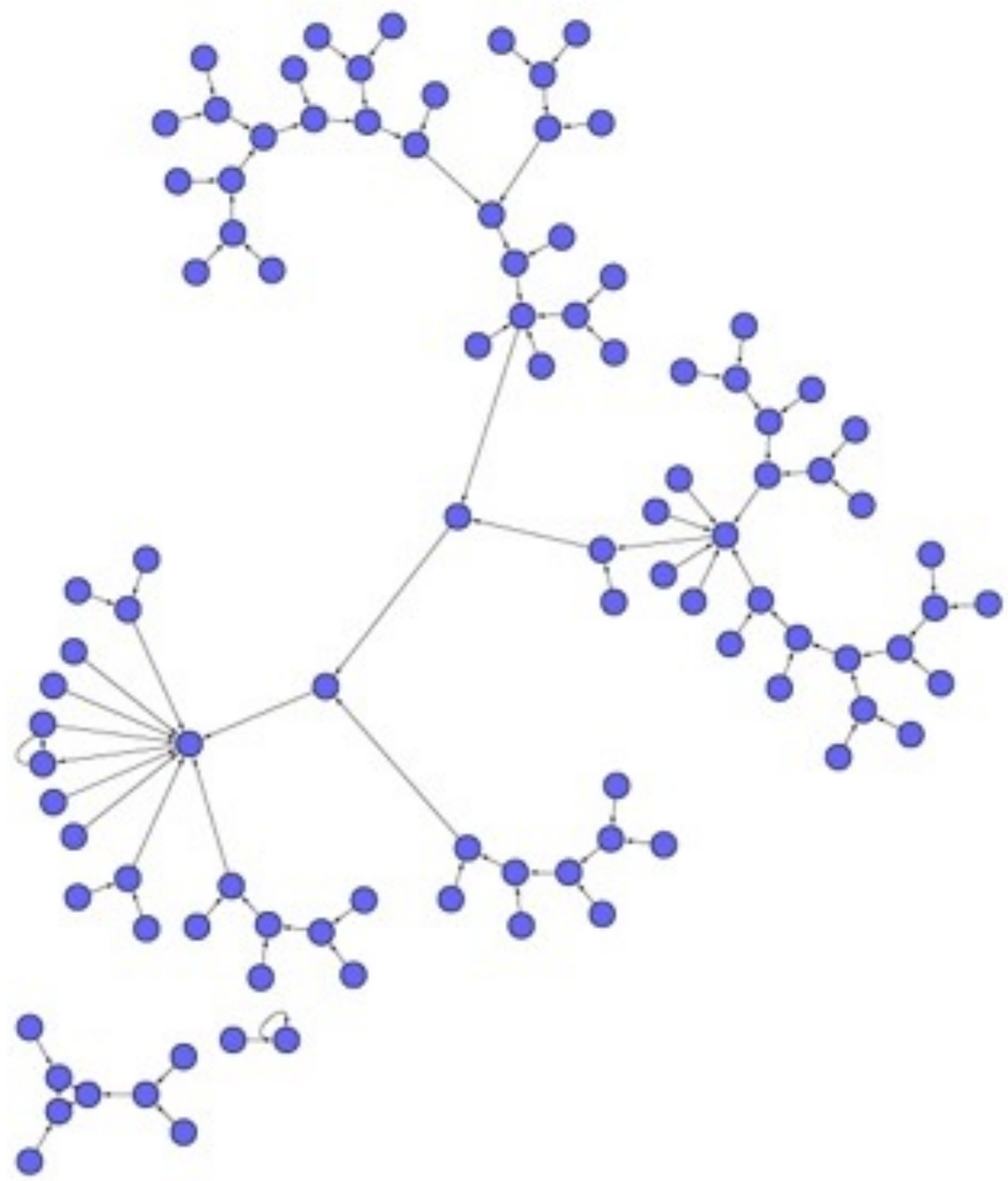
Round-Up into the decimal place, $d = 2$

Therefore, $x_{n+1} = 4\, x_n\, (\, 1 - x_n\,)$

Therefore, 101 states

x_n	$\xrightarrow{f}$		$\xrightarrow{h}$	x_{n+1}
0.00	$\longrightarrow$	0.0000	$\longrightarrow$	0.00
0.01	$\longrightarrow$	0.0396	$\longrightarrow$	0.04
0.02	$\longrightarrow$	0.0784	$\longrightarrow$	0.08
0.03	$\longrightarrow$	0.1164	$\longrightarrow$	0.12
0.04	$\longrightarrow$	0.1536	$\longrightarrow$	0.16
0.05	$\longrightarrow$	0.1900	$\longrightarrow$	0.19
0.06	$\longrightarrow$	0.2256	$\longrightarrow$	0.23
0.07	$\longrightarrow$	0.2604	$\longrightarrow$	0.27
0.08	$\longrightarrow$	0.2944	$\longrightarrow$	0.30
0.09	$\longrightarrow$	0.3276	$\longrightarrow$	0.33
0.10	$\longrightarrow$	0.3600	$\longrightarrow$	0.36
0.11	$\longrightarrow$	0.3916	$\longrightarrow$	0.40
0.12	$\longrightarrow$	0.4224	$\longrightarrow$	0.43
0.13	$\longrightarrow$	0.4524	$\longrightarrow$	0.46
0.14	$\longrightarrow$	0.4816	$\longrightarrow$	0.49
0.15	$\longrightarrow$	0.5100	$\longrightarrow$	0.51
0.16	$\longrightarrow$	0.5376	$\longrightarrow$	0.54

$\vdots$



The state-transition networks for the logistic map

Control Parameter: $\mu = 1$

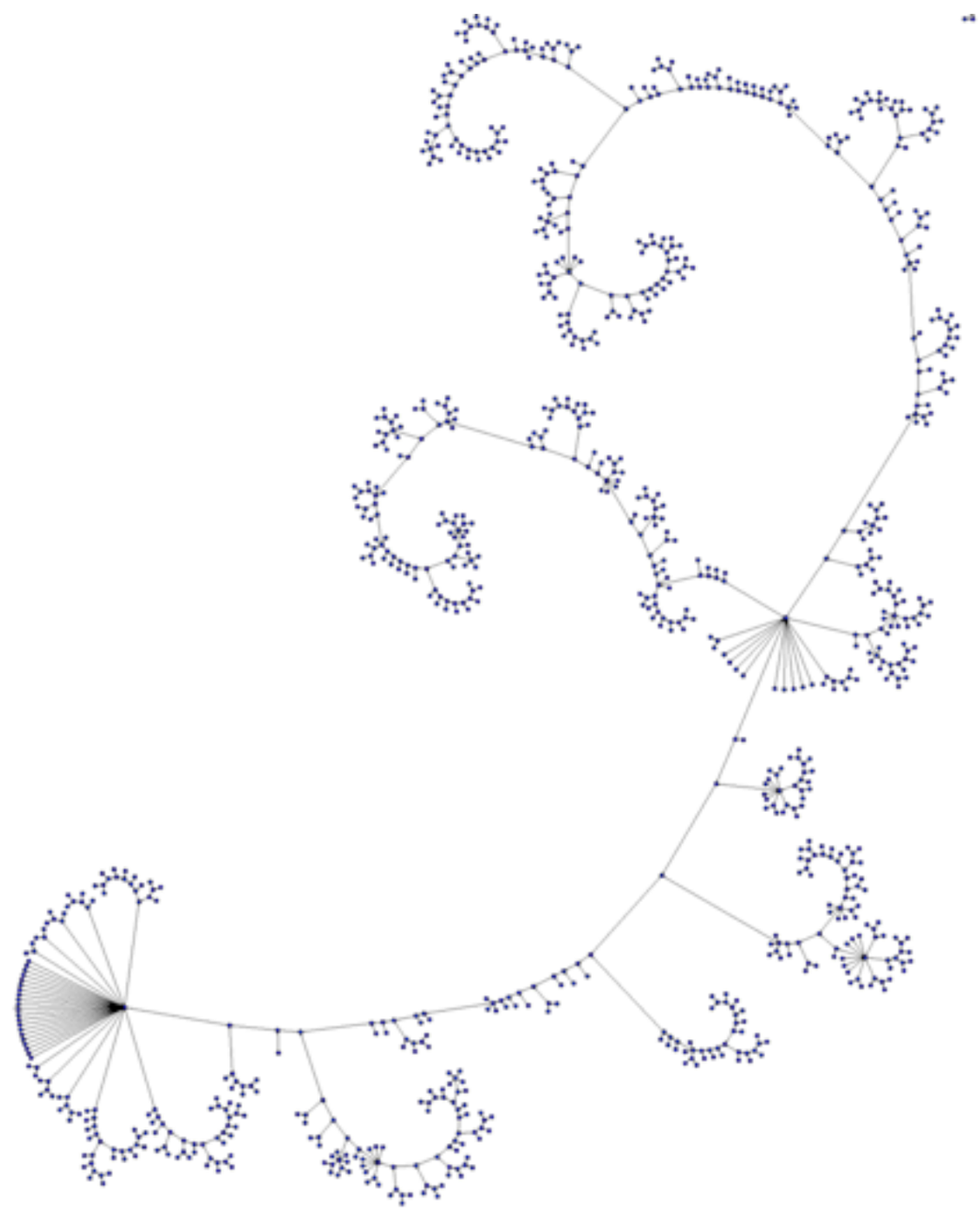
Round-Up into the decimal place, $d = 3$

Therefore, $x_{n+1} = 4\, x_n\, (\, 1 - x_n\,)$

Therefore, 1001 states

x_n	$\xrightarrow{f}$		$\xrightarrow{h}$	x_{n+1}
0.000	$\longrightarrow$	0.000000	$\longrightarrow$	0.000
0.001	$\longrightarrow$	0.003996	$\longrightarrow$	0.004
0.002	$\longrightarrow$	0.007984	$\longrightarrow$	0.008
0.003	$\longrightarrow$	0.011964	$\longrightarrow$	0.012
0.004	$\longrightarrow$	0.015936	$\longrightarrow$	0.016
0.005	$\longrightarrow$	0.019900	$\longrightarrow$	0.020
0.006	$\longrightarrow$	0.023856	$\longrightarrow$	0.024
0.007	$\longrightarrow$	0.027804	$\longrightarrow$	0.028
0.008	$\longrightarrow$	0.031744	$\longrightarrow$	0.032
0.009	$\longrightarrow$	0.035676	$\longrightarrow$	0.036
0.010	$\longrightarrow$	0.039600	$\longrightarrow$	0.040
0.011	$\longrightarrow$	0.043516	$\longrightarrow$	0.044
0.012	$\longrightarrow$	0.047424	$\longrightarrow$	0.048
0.013	$\longrightarrow$	0.051324	$\longrightarrow$	0.052
0.014	$\longrightarrow$	0.055216	$\longrightarrow$	0.056
0.015	$\longrightarrow$	0.059100	$\longrightarrow$	0.060
0.016	$\longrightarrow$	0.062976	$\longrightarrow$	0.063

$\vdots$



The state-transition networks for the logistic map

Control Parameter: $\mu = 1$

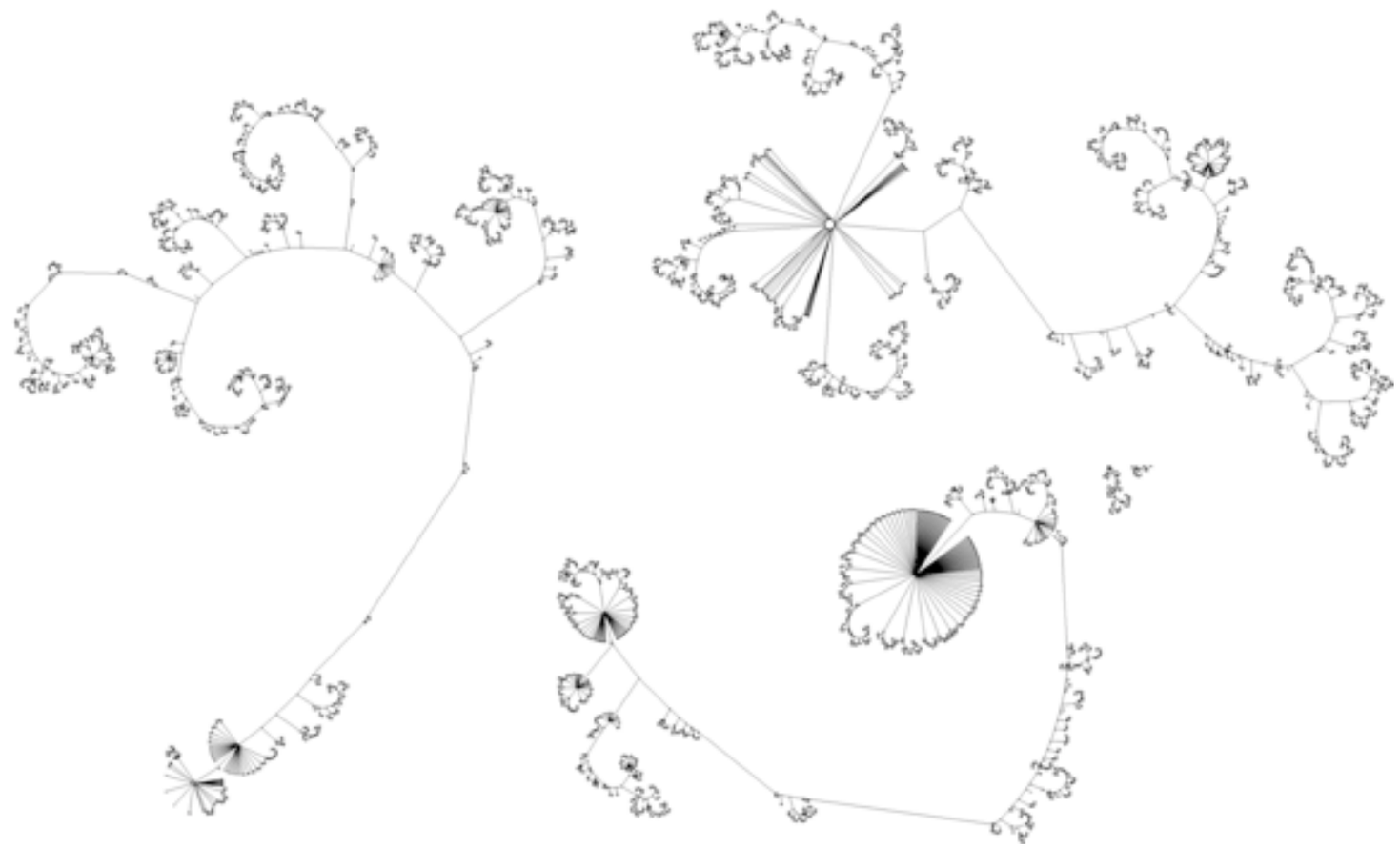
Round-Up into the decimal place, $d = 4$

Therefore, $x_{n+1} = 4\, x_n\, (\, 1 - x_n\,)$

Therefore, 10001 states

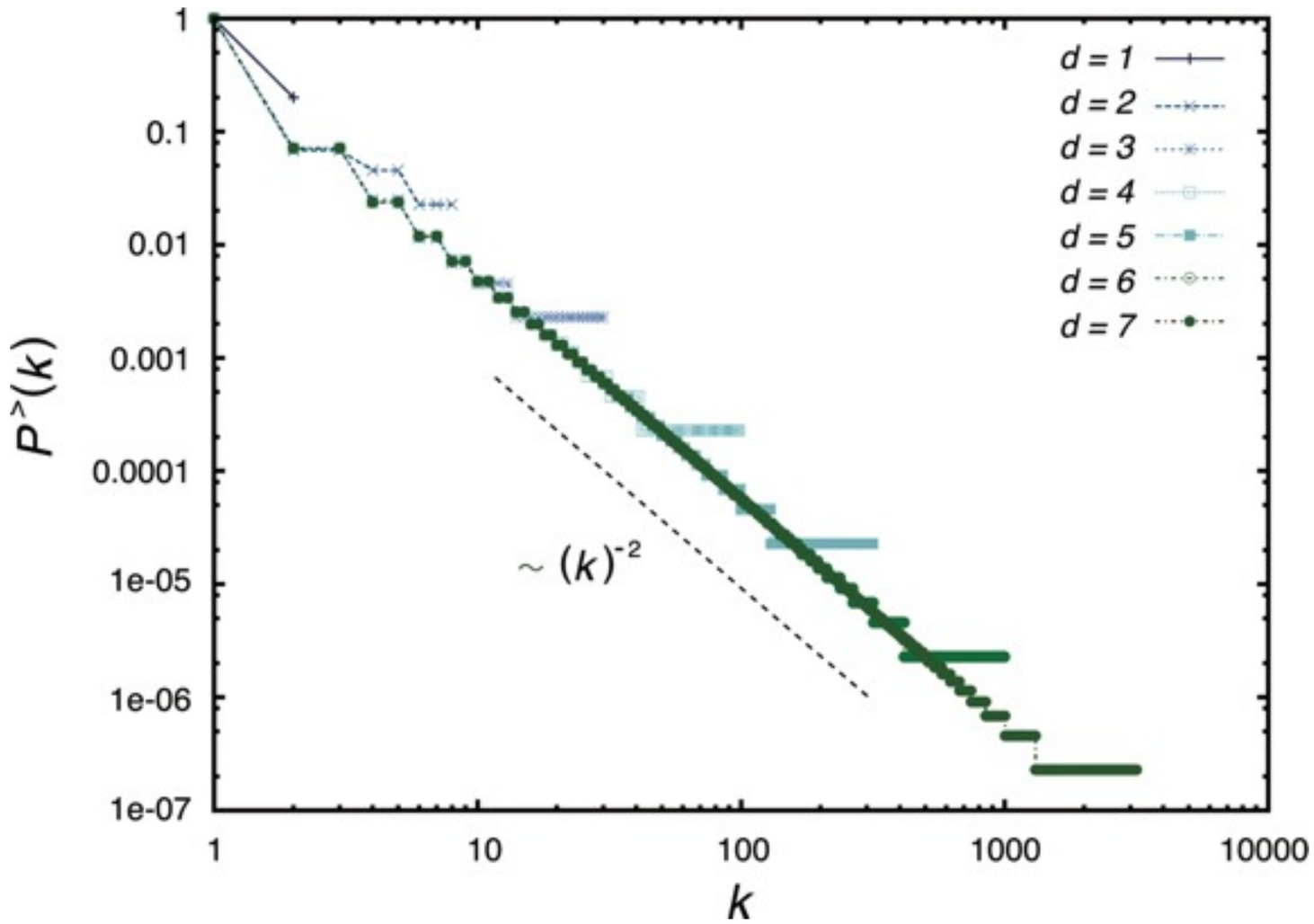
x_n	$\xrightarrow{f}$		$\xrightarrow{h}$	x_{n+1}
0.0000	$\longrightarrow$	0.00000000	$\longrightarrow$	0.0000
0.0001	$\longrightarrow$	0.00039996	$\longrightarrow$	0.0004
0.0002	$\longrightarrow$	0.00079984	$\longrightarrow$	0.0008
0.0003	$\longrightarrow$	0.00119964	$\longrightarrow$	0.0012
0.0004	$\longrightarrow$	0.00159936	$\longrightarrow$	0.0016
0.0005	$\longrightarrow$	0.00199900	$\longrightarrow$	0.0020
0.0006	$\longrightarrow$	0.00239856	$\longrightarrow$	0.0024
0.0007	$\longrightarrow$	0.00279804	$\longrightarrow$	0.0028
0.0008	$\longrightarrow$	0.00319744	$\longrightarrow$	0.0032
0.0009	$\longrightarrow$	0.00359676	$\longrightarrow$	0.0036
0.0010	$\longrightarrow$	0.00399600	$\longrightarrow$	0.0040
0.0011	$\longrightarrow$	0.00439516	$\longrightarrow$	0.0044
0.0012	$\longrightarrow$	0.00479424	$\longrightarrow$	0.0048
0.0013	$\longrightarrow$	0.00519324	$\longrightarrow$	0.0052
0.0014	$\longrightarrow$	0.00559216	$\longrightarrow$	0.0056
0.0015	$\longrightarrow$	0.00599100	$\longrightarrow$	0.0060
0.0016	$\longrightarrow$	0.00638976	$\longrightarrow$	0.0064

$\vdots$



The cumulative in-degree distributions of state-transition networks for the logistic map

with $\mu = 1$ in the case from $d = 1$ to $d = 7$



The dashed line has slope -2.
The networks are scale-free networks with the degree exponent $\gamma = 1$, regardless of the value of d.

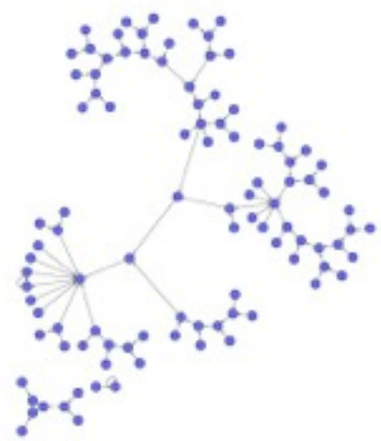
The state-transition networks for the logistic map

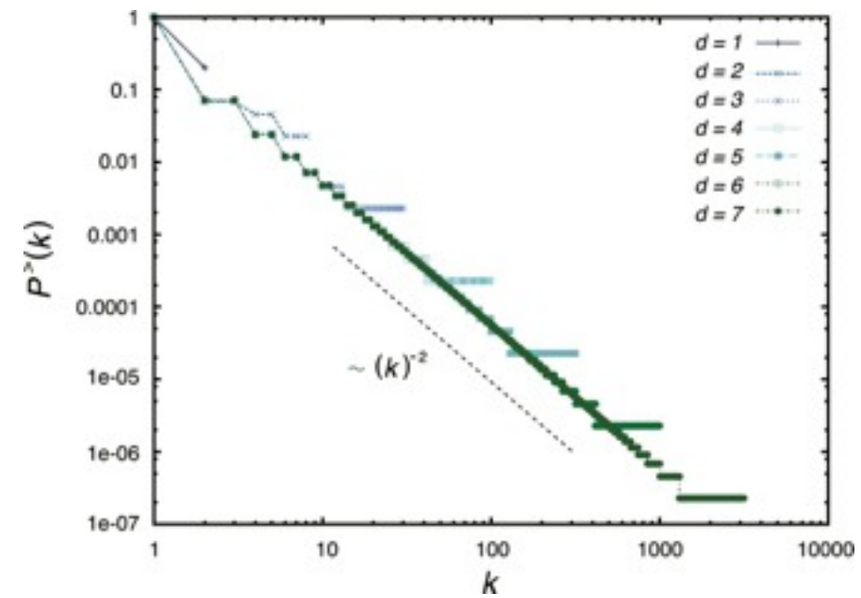
Control Parameter: $\boldsymbol{\mu} = \mathbf{1}$ Therefore, $x_{n+1} = 4\, x_n\, (\, 1 - x_n\,)$

Round-Up into the decimal place, ranging from $d = 1$ to $d = 4$

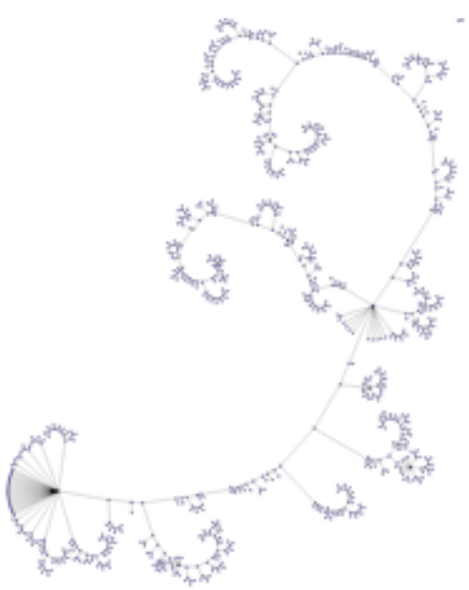
$d = 1$

$d = 2$

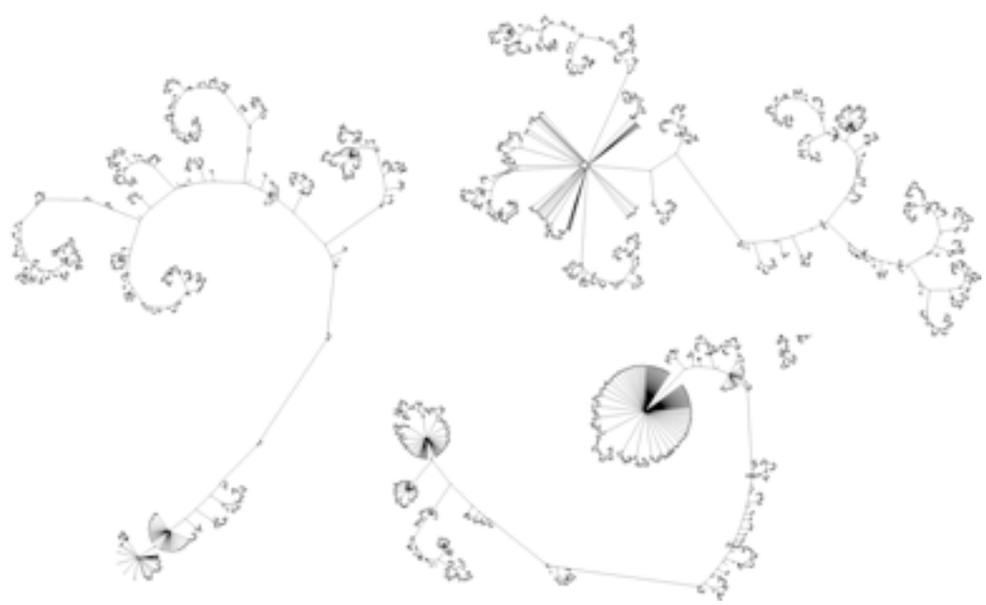




$d = 3$



$d = 4$

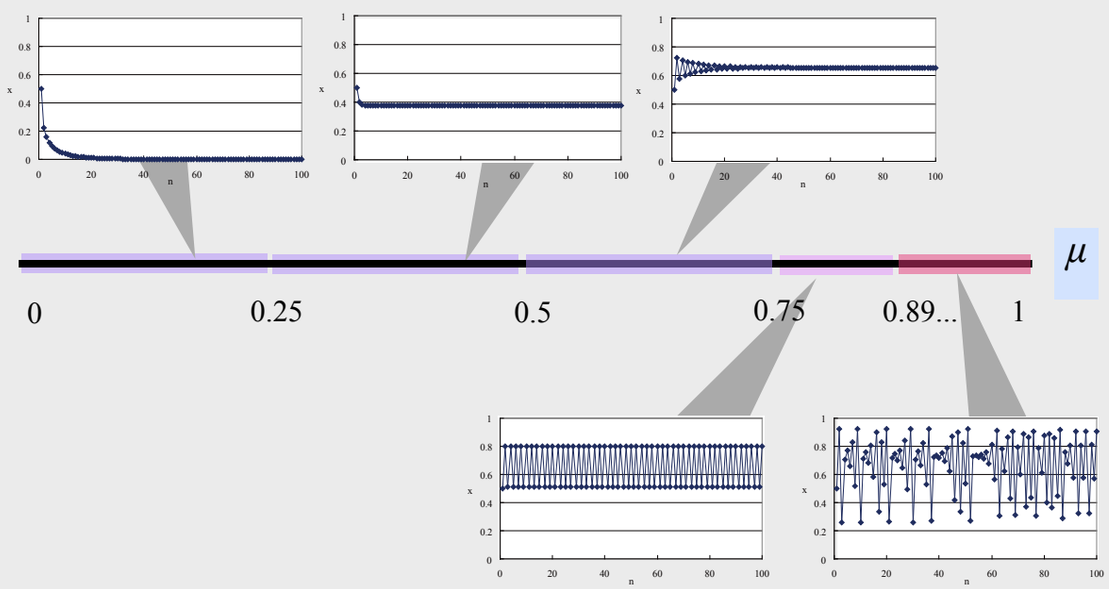


scale-free networks!

Does the *scale-free* property depend on the control parameter μ ?

Does the *scale-free* property depend on the control parameter μ ?

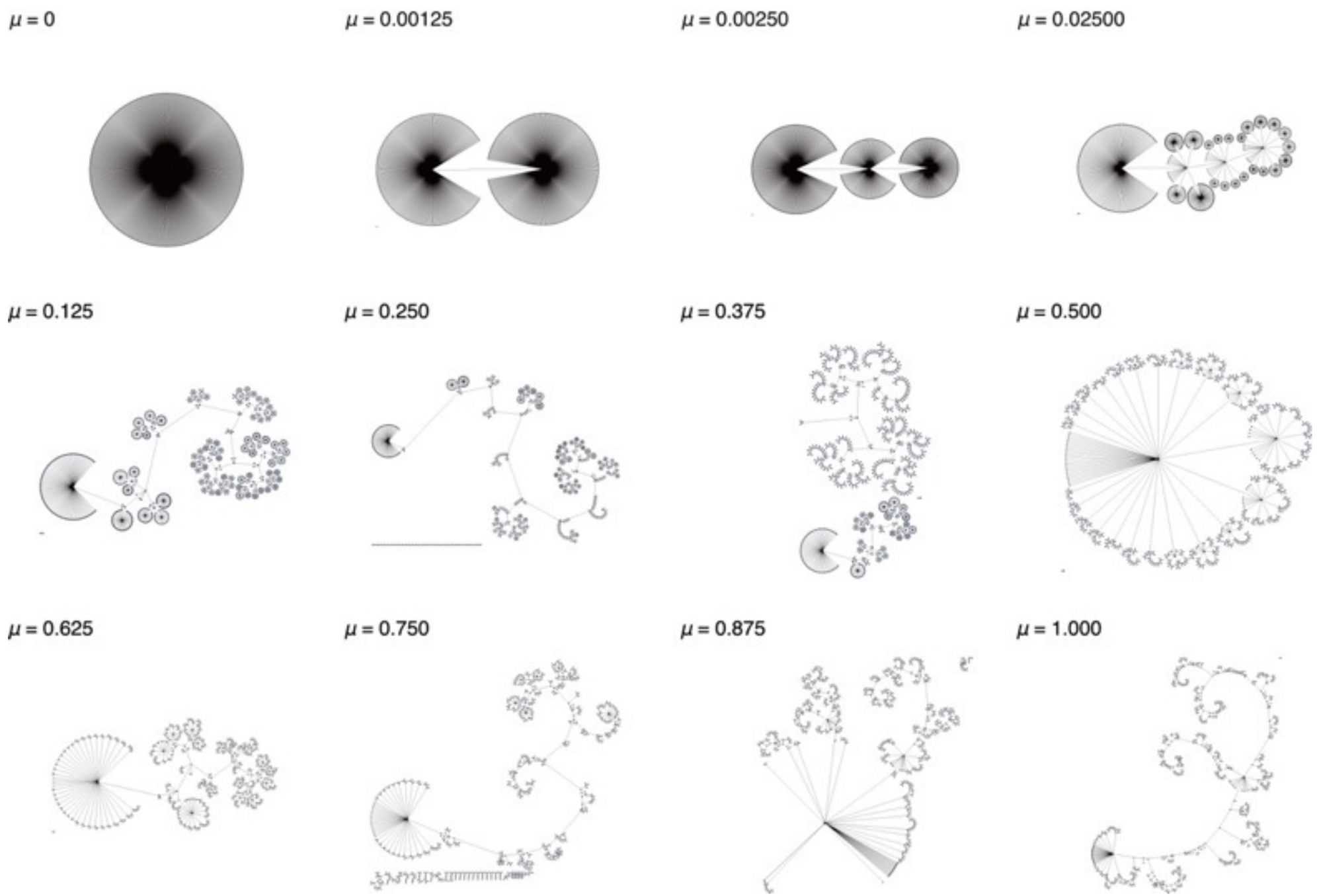
$$x_{n+1} = 4\mu\, x_n\, (\, 1 - x_n\,)$$



The state-transition networks for the logistic map

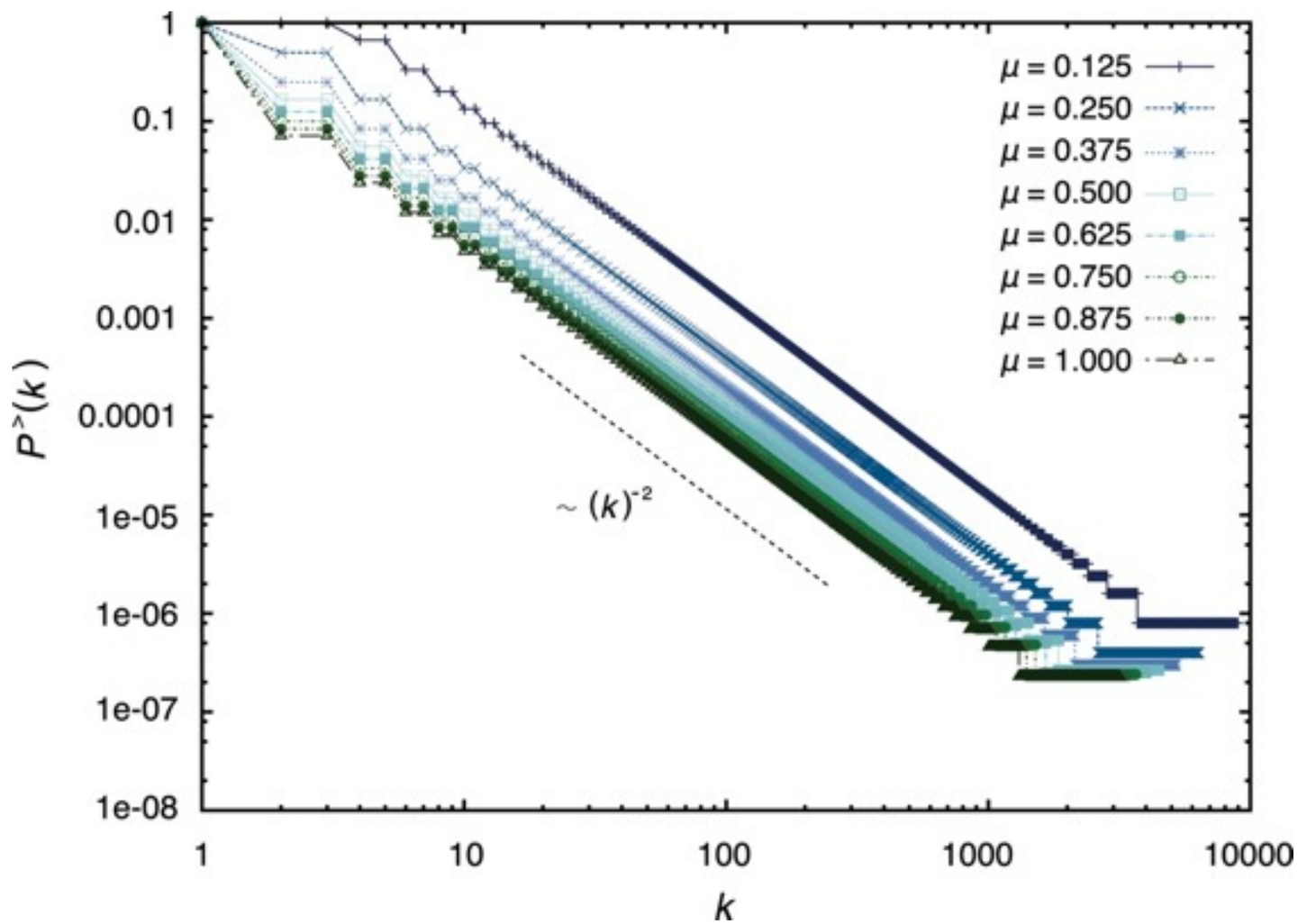
Control Parameter: ranging from $\mu = 0$ to $\mu = 1$ $\quad x_{n+1} = 4\mu\, x_n\, (1 - x_n)$

Round-Up into the decimal place, $d = 3$



The cumulative in-degree distributions of state-transition networks for the logistic map

from μ = 0.125 to 1.000 by 0.125 in the case d = 7



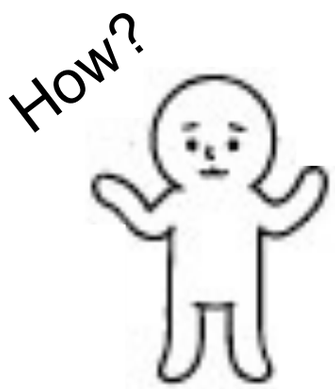
The dashed line has slope -2.
The networks are scale-free networks with the degree exponent $\gamma = 1$, regardless of the value of the parameter μ.

Does the *scale-free* property depend on the control parameter μ ?

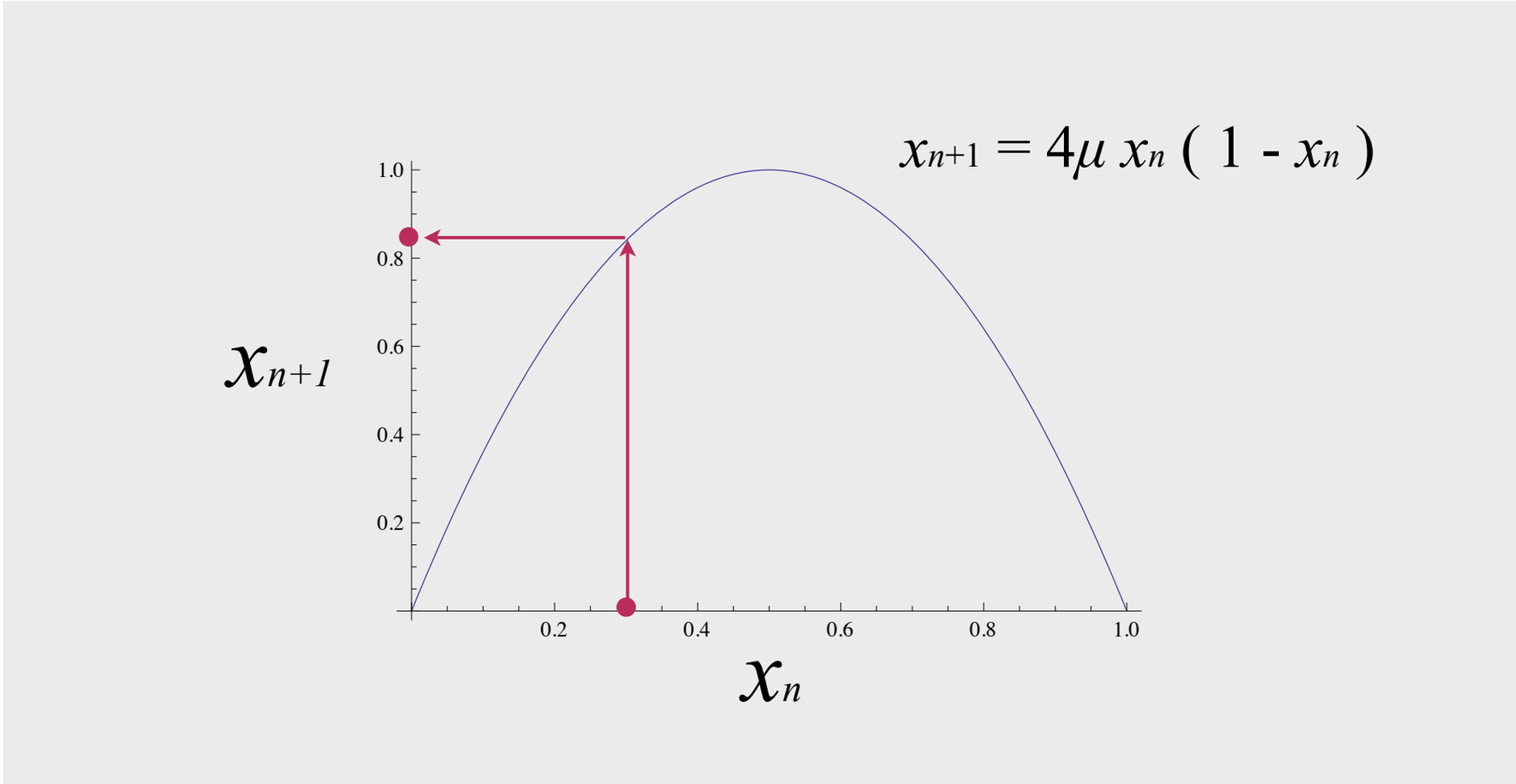
NO.

The *scale-free* property is independent of the control parameter μ.

How the scale-free network is emerged?



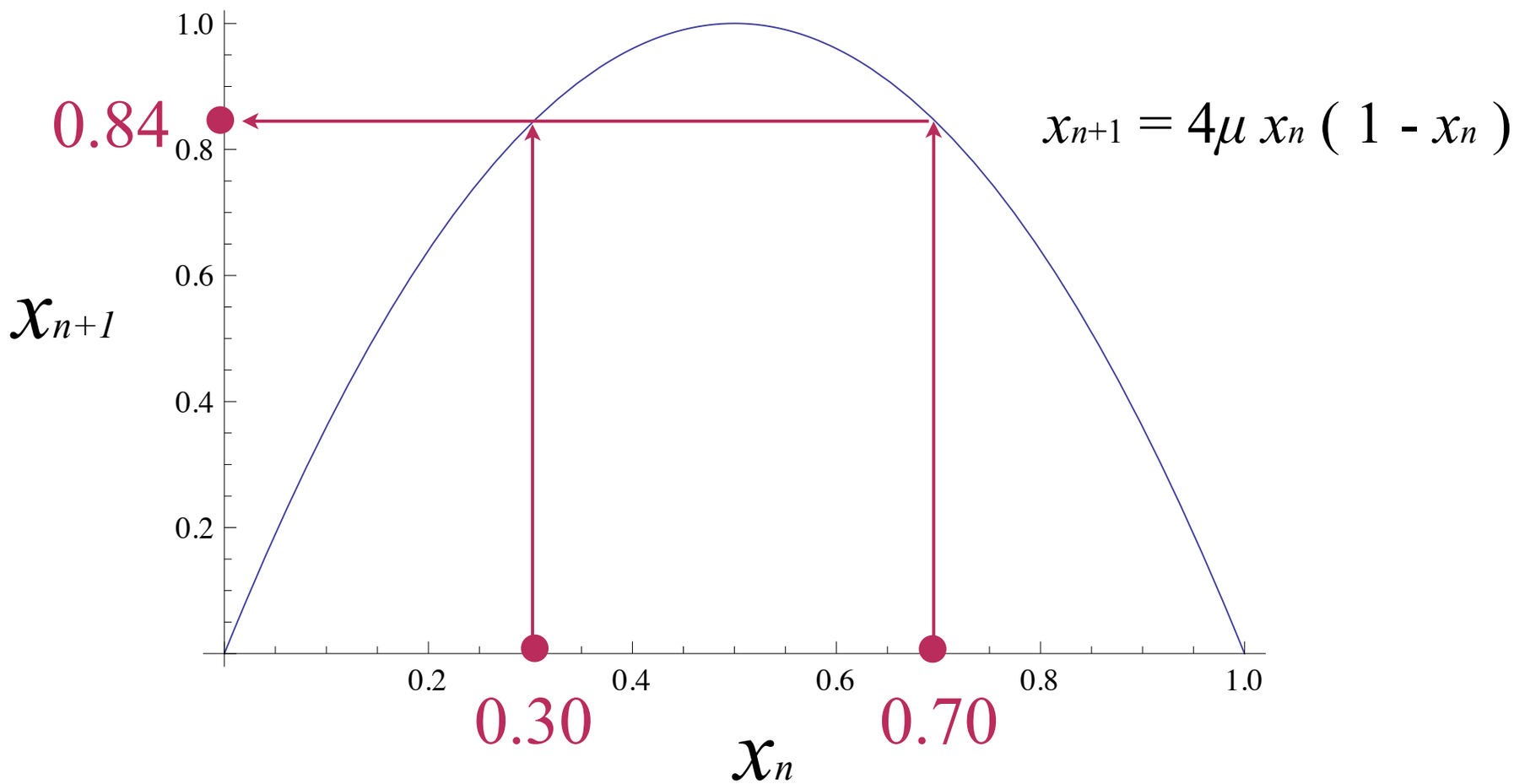
How the scale-free network is emerged?



How the scale-free network is emerged?

If x is a real number, namely in the *mathematically ideal* condition,

the state 0.84 must have only 2 previous values: 0.30 and 0.70.

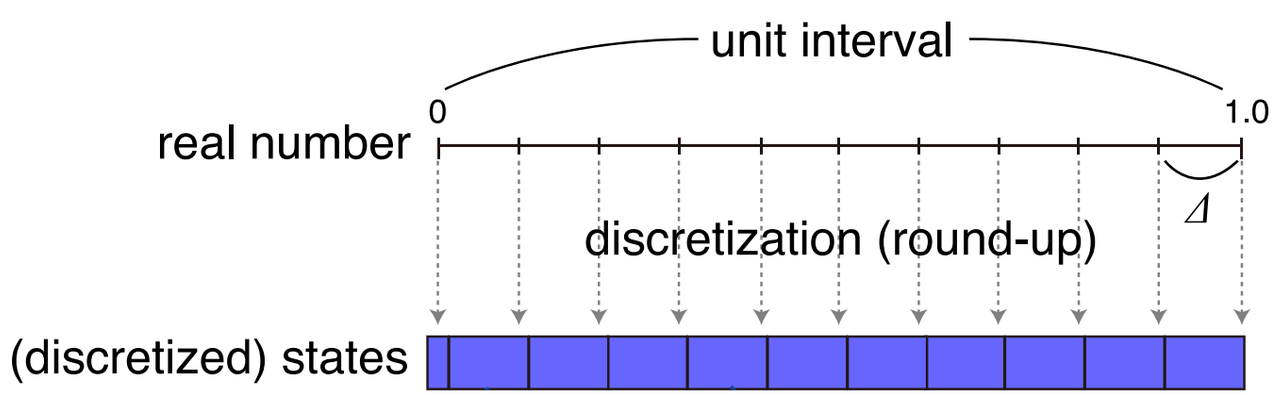


In the condition,
the state-transition network cannot be a scale-free network ...

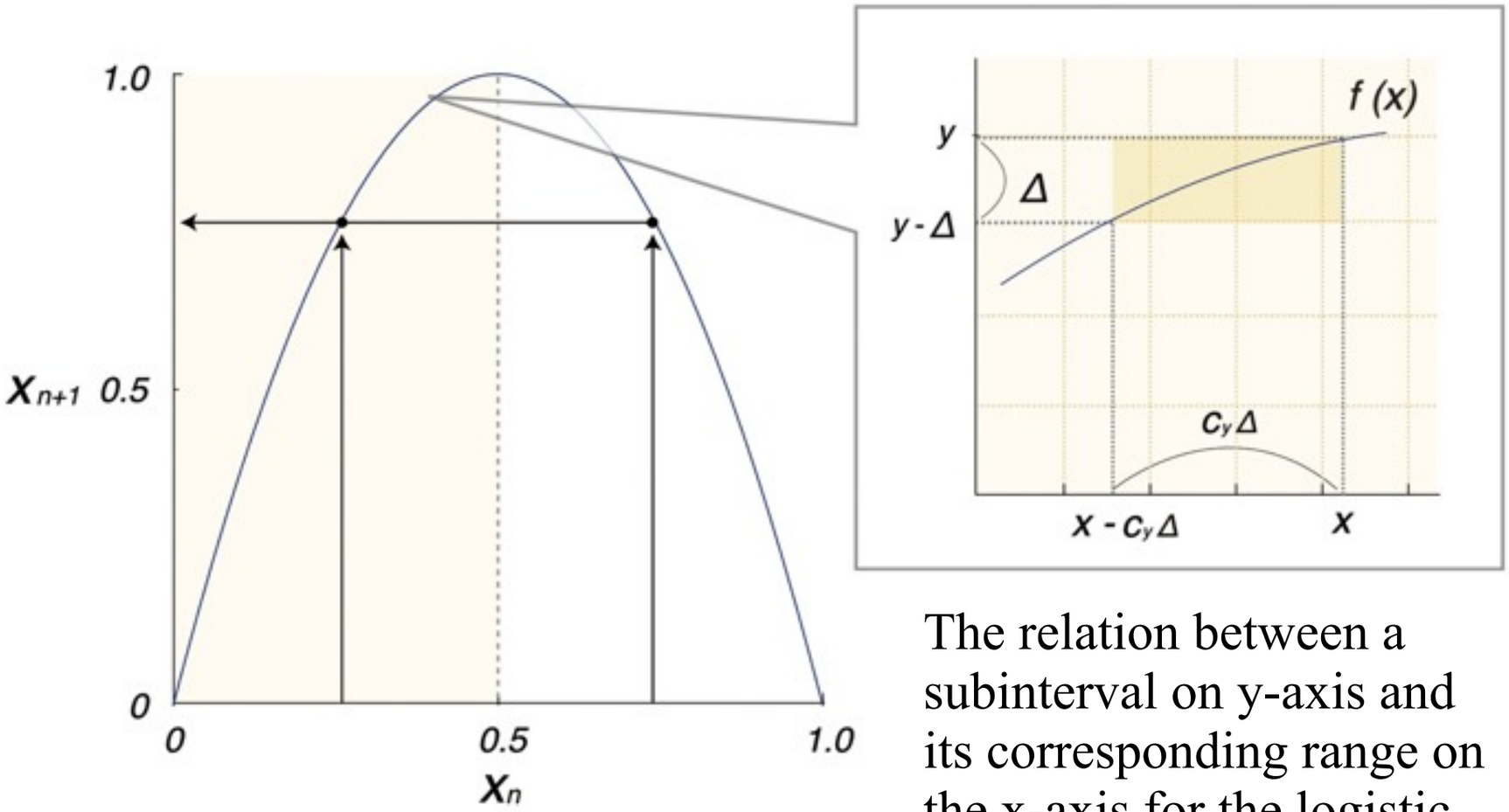
How the scale-free network is emerged?

How the scale-free network is emerged?

The trick is discretization.



How the scale-free network is emerged?



The relation between a subinterval on y-axis and its corresponding range on the x-axis for the logistic map function.

ex. $\mu = 1$, $d = 3$

x_{n+1}	in-degree
1.000	31
0.999	14
0.998	10
0.997	8
0.996	8
0.995	6
0.994	6
0.993	6
0.992	6

Mathematical Derivation

1. Number of the states whose in-degree is higher than zero

Suppose that f is the unimodal map of the interval $[0,1]$ that is concave downward, symmetrical with respect to the critical point, and $f(0) = f(1) = 0$ (see Fig. 7). While there are two solutions for the inverse function f^{-1}, we shall indicate a solution on the left hand of parabola as f_L^{-1}. Notice that, due to the symmetrical nature, in-degree k is always an even number but the critical point.

In order to obtain the number of nodes who have incoming link, $N_{k>0}$, we shall describe a relation between y and x with using the size of subinterval Δ and the coefficient c_y (see Fig. 7). We then have $y = f(x)$ and $y - \Delta = f(x - c_y\Delta)$, and, with the inverse function, $x = f_L^{-1}(y)$ and $x - c_y\Delta = f_L^{-1}(y - \Delta)$. Cancelling y from these equations gives the equation:

$$c_y = \frac{x - f_L^{-1}(f(x) - \Delta)}{\Delta}.$$

On the other hand, cancelling x gives the equation:

$$c_y = \frac{f_L^{-1}(y) - f_L^{-1}(y - \Delta)}{\Delta}.$$

Accordingly, since $k_y = 2c_y$ by definition, in-degree of the state y is calculated as:

$$k_y = \frac{2}{\Delta}\left(f_L^{-1} - f_L^{-1}(y - \Delta)\right).$$

Let the coordinates of the point where $k = 4$, namely $c_y = 2$, be (X, Y), and therefore it must satisfy that $Y = f(X)$ and $c_Y = \left(f_L^{-1} - f_L^{-1}(Y - \Delta)\right)/\Delta = 2$. We now consider the number of states for two ranges: $c_y \geq c_Y$ and $0 < c_y < c_Y$. First, the number of states in the range $c_y \geq c_Y$ is calculated as:

$$N_{c_y \geq c_Y} = \begin{cases} \dfrac{\mu - Y}{\Delta} + 1 & if \;\; Y \geq 0 \\ \dfrac{\mu}{\Delta} + 1 & if \;\; Y < 0. \end{cases}$$

Second, the number of states in the range $0 < c_y < c_Y$ is calculated as:

$$N_{0 < c_y < c_Y} = \begin{cases} \dfrac{X}{\Delta} + 1 & if \;\; X \geq 0 \\ 0 & if \;\; X < 0. \end{cases}$$

Note that, since the map function is concave downward, $f(0) = f(1) = 0$, and focusing the left hand of the parabola, there are only two combinations of the conditions: $X \geq 0 \;\wedge\; Y \geq 0$ and $X < 0 \;\wedge\; Y < 0$. Therefore, the number of states in the range $0 < c$, namely the number of the states whose in-degree is higher than zero, $N_{k>0}$, is given as follows:

$$N_{k>0} = \begin{cases} \dfrac{\mu + X - Y}{\Delta} + 2 & if \;\; X \geq 0 \;\wedge\; Y \geq 0 \\ \dfrac{\mu}{\Delta} + 1 & if \;\; X < 0 \;\wedge\; Y < 0. \end{cases}$$

2. Cumulative in-degree distribution

As shown in some literatures [30, 31], cumulative degree distribution can be calculated from the rank / frequency plots as:

$$P^{>}(k) = \frac{r}{N},$$

where r is the rank and N is a total number of nodes. Based on the observation that the in-degree becomes higher as y is getting to be higher, the rank of state y is expressed as:

$$r_y = \frac{\mu - y}{\Delta} + 1.$$

Replacing r and N of the cumulative distribution function $P^{>}(k)$, we obtain:

$$P^{>}(k_y) = \begin{cases} \dfrac{\mu + \Delta - y}{\mu + 2\Delta + X - Y} & if \;\; X \geq 0 \;\wedge\; Y \geq 0 \\ \dfrac{\mu + \Delta - y}{\mu + \Delta} & if \;\; X < 0 \;\wedge\; Y < 0. \end{cases}$$

3. Cumulative in-degree distribution in the case of the logistic map

In the case of the logistic map $f(x) = 4\mu x(1 - x)$, a solution of the inverse function is given by $f_L^{-1}(y) = \left(1 - \sqrt{1 - y/\mu}\right)/2$, and the coordinates of the point where $k = 4$, (X, Y), is given by:

$$(X, Y) = \left(\frac{1}{2} - \frac{|-1 + 16\mu\Delta|}{16\mu},\;\; \frac{\Delta}{2} - \frac{1}{64\mu} + \mu - 4\mu\Delta^2\right)$$

Solving the equation of k_y for y, we obtain:

$$y = \mu - \frac{1}{4\mu k^2} - \frac{\Delta}{4}\left(-2 + \mu\Delta k^2\right).$$

Since $0 < \Delta \leq 1/10$ is always satisfied in the condition of this paper, the condition $X \geq 0 \;\wedge\; Y \geq 0$ means $1/(8 + 16\Delta) \leq \mu \leq 1$; and the condition $X < 0 \;\wedge\; Y < 0$ means $0 < \mu < 1/(8 + 16\Delta)$. Thus, the number of states whose in-degree is higher than zero is given by:

$$N_{c_y > 0} = \begin{cases} \dfrac{1 - 4|-1 + 16\mu\Delta| + 32\mu(1 + \Delta + 8\mu\Delta^2)}{64\mu\Delta} & if \;\; \dfrac{1}{8 + 16\Delta} \leq \mu \leq 1 \\ \dfrac{\mu}{\Delta} & if \;\; 0 < \mu < \dfrac{1}{8 + 16\Delta}. \end{cases}$$

Note that the maximum in-degree is observed at the highest point of the parabola $y^* = \mu$, namely at the critical point $x^* = 1/2$, and the maximum in-degree is estimated by:

$$k^{max} = \frac{1}{\sqrt{\mu\Delta}}.$$

Consequently, in the case $1/(8 + 16\Delta) \leq \mu \leq 1$, the cumulative in-degree distribution $P^{>}(k_y)$ is given by:

$$P^{>}(k) = \frac{16}{1 - 4|-1 + 16\mu\Delta| + 32\mu(1 + 3\Delta + 8\mu\Delta^2)} \times \left(\frac{1}{k} + \mu\Delta k\right)^2.$$

In the case $0 < \mu < 1/(8 + 16\Delta)$, the distribution is given by:

$$P^{>}(k) = \begin{cases} \dfrac{1}{4\mu(\mu + \Delta)}\left(\dfrac{1}{k} + \mu\Delta k\right)^2 & if \;\; \dfrac{1 - \sqrt{\Delta/\mu + 1}}{\Delta} < k \\ 1 & if \;\; 0 < k \leq \dfrac{1 - \sqrt{\Delta/\mu + 1}}{\Delta}. \end{cases}$$

Thus, it is proved that the cumulative in-degree distribution of the DST network for the logistic map follows a law given by:

$$P^{>}(k) \propto \left(\frac{1}{k} + \mu\Delta k\right)^2,$$

where in-degree k is in the range $0 < k \leq k^{max}$ and $k^{max} = 1/\sqrt{\mu\Delta}$. Recall that $0 < \mu\Delta < 1$, therefore the second term makes a large effect on the distribution, only when k is quite close to k^{max}. It means that the "hub" states will have higher in-degree than the typical scale-free network whose in-degree distribution follows a strict power law. Fig. 8 shows the fitness between the results of numerical computation and mathematical predictions.

In order to understand the distribution when the size of subintervals, Δ, is enough small, we take the limit of Δ into 0. In the case $1/(8 + 16\Delta) \leq \mu \leq 1$,

$$\lim_{\Delta \to 0} P^{>}(k) = \frac{16}{-5 + 32\mu + 128\mu^2} k^{-2},$$

and in the case $0 < \mu < 1/(8 + 16\Delta)$,

$$\lim_{\Delta \to 0} P^{>}(k) = \begin{cases} \dfrac{1}{4\mu^2} k^{-2} & if \;\; \dfrac{1}{2\mu} < k \\ 1 & if \;\; 0 < k \leq \dfrac{1}{2\mu}. \end{cases}$$

Thus, it is clear that the cumulative in-degree distribution of the DST network for the logistic map follows a strict power law when Δ is enough small as follows:

$$P^{>}(k) \propto k^{-2}.$$

In conclusion, the DST networks for the logistic map become "perfect" scale-free networks whose degree distribution strictly follows a power law with the exponent $\gamma = 1$, as the size of subintervals is getting smaller.

Consequently, in the case $1/(8+16\Delta) \leq \mu \leq 1$, the cumulative in-degree distribution $P^{>}(k_y)$ is given by:

$$P^{>}(k) = \frac{16}{1-4|-1+16\mu\Delta|+32\mu(1+3\Delta+8\mu\Delta^2)} \times \left(\frac{1}{k}+\mu\Delta k\right)^2.$$

In the case $0 < \mu < 1/(8+16\Delta)$, the distribution is given by:

$$P^{>}(k) = \begin{cases} \frac{1}{4\mu(\mu+\Delta)}\left(\frac{1}{k}+\mu\Delta k\right)^2 & if \ \frac{1-\sqrt{\Delta/\mu+1}}{\Delta} < k \\ 1 & if \ 0 < k \leq \frac{1-\sqrt{\Delta/\mu+1}}{\Delta}. \end{cases}$$

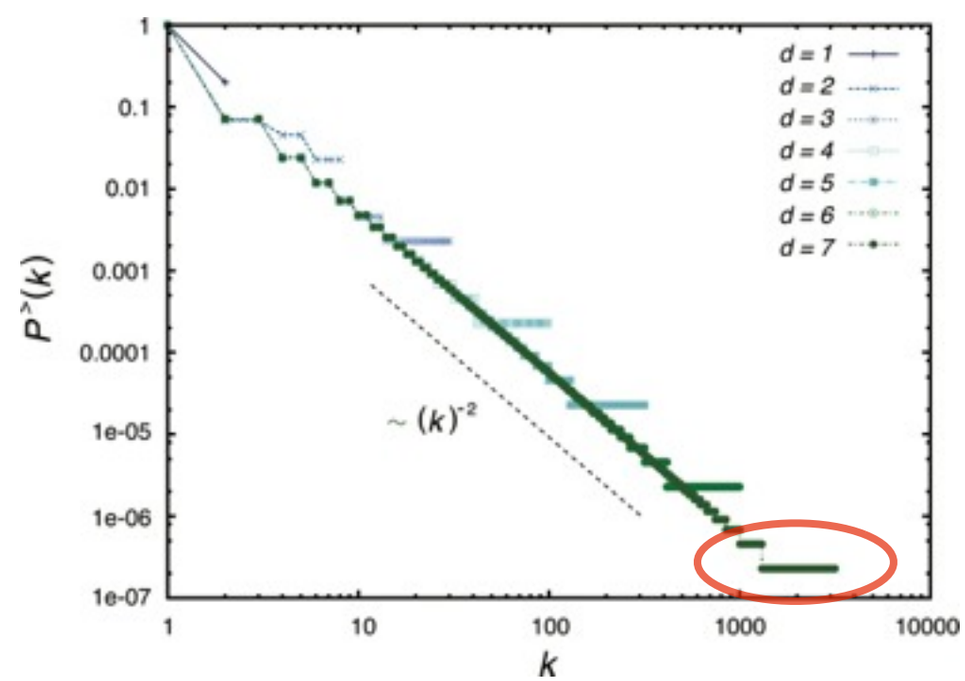
Summarizing

The cumulative in-degree distribution follows a law given by:

$$P^{>}(k) \propto \left(\frac{1}{k}+\mu\Delta k\right)^2$$

The second term makes a large effect on the distribution, only when k is quite close to k^{max}, because $0 < \mu\Delta < 1$.

It means that the "hub" states have higher in-degree than the typical scale-free network whose in-degree distribution follows a strict power law.



Consequently, in the case $1/(8+16\Delta) \le \mu \le 1$, the cumulative in-degree distribution $P^{>}(k_y)$ is given by:

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Summarizing

The cumulative in-degree distribution follows a law given by:

$$P^{>}(k) \propto \left(\frac{1}{k}+\mu\Delta k\right)^2$$

The second term makes a large effect on the distribution, only when k is quite close to k^{max}, because $0 < \mu\Delta < 1$.

It means that the "hub" states have higher in-degree than the typical scale-free network whose in-degree distribution follows a strict power law.

taking the limit

In the case $1/(8+16\Delta) \le \mu \le 1$,

$$\lim_{\Delta\to 0} P^{>}(k) = \frac{16}{-5+32\mu+128\mu^2}k^{-2},$$

and in the case $0 < \mu < 1/(8+16\Delta)$,

$$\lim_{\Delta\to 0} P^{>}(k) = \begin{cases} \frac{1}{4\mu^2}k^{-2} & if \ \frac{1}{2\mu} < k \\ 1 & if \ 0 < k \le \frac{1}{2\mu}. \end{cases}$$

Summarizing

In the case Δ is extremely small (d is extremely large),
The cumulative in-degree distribution follows a power law:

$$P^{>}(k) \propto k^{-2}$$

Consequently, in the case $1/(8+16\Delta) \leq \mu \leq 1$, the cumulative in-degree distribution $P^{>}(k_y)$ is given by:

$$P^{>}(k) = \frac{16}{1-4|-1+16\mu\Delta| + 32\mu(1+3\Delta+8\mu\Delta^2)} \times \left(\frac{1}{k}+\mu\Delta k\right)^2.$$

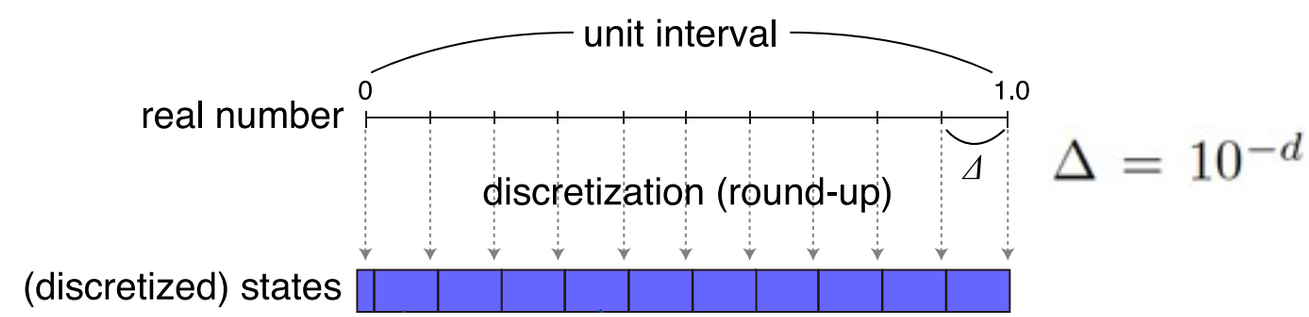
In the case $0 < \mu < 1/(8+16\Delta)$, the distribution is given by:

$$P^{>}(k) = \begin{cases} \frac{1}{4\mu(\mu+\Delta)}\left(\frac{1}{k}+\mu\Delta k\right)^2 & if \ \frac{1-\sqrt{\Delta/\mu+1}}{\Delta} < k \\ 1 & if \ 0 < k \leq \frac{1-\sqrt{\Delta/\mu+1}}{\Delta}. \end{cases}$$

Summarizing

The cumulative in-degree distribution follows a law given by:

$$P^{>}(k) \propto \left(\frac{1}{k}+\mu\Delta k\right)^2$$



$$\Delta = 10^{-d}$$

taking the limit

In the case $1/(8+16\Delta) \leq \mu \leq 1$,

$$\lim_{\Delta\to 0} P^{>}(k) = \frac{16}{-5+32\mu+128\mu^2}k^{-2},$$

and in the case $0 < \mu < 1/(8+16\Delta)$,

$$\lim_{\Delta\to 0} P^{>}(k) = \begin{cases} \frac{1}{4\mu^2}k^{-2} & if \ \frac{1}{2\mu} < k \\ 1 & if \ 0 < k \leq \frac{1}{2\mu}. \end{cases}$$

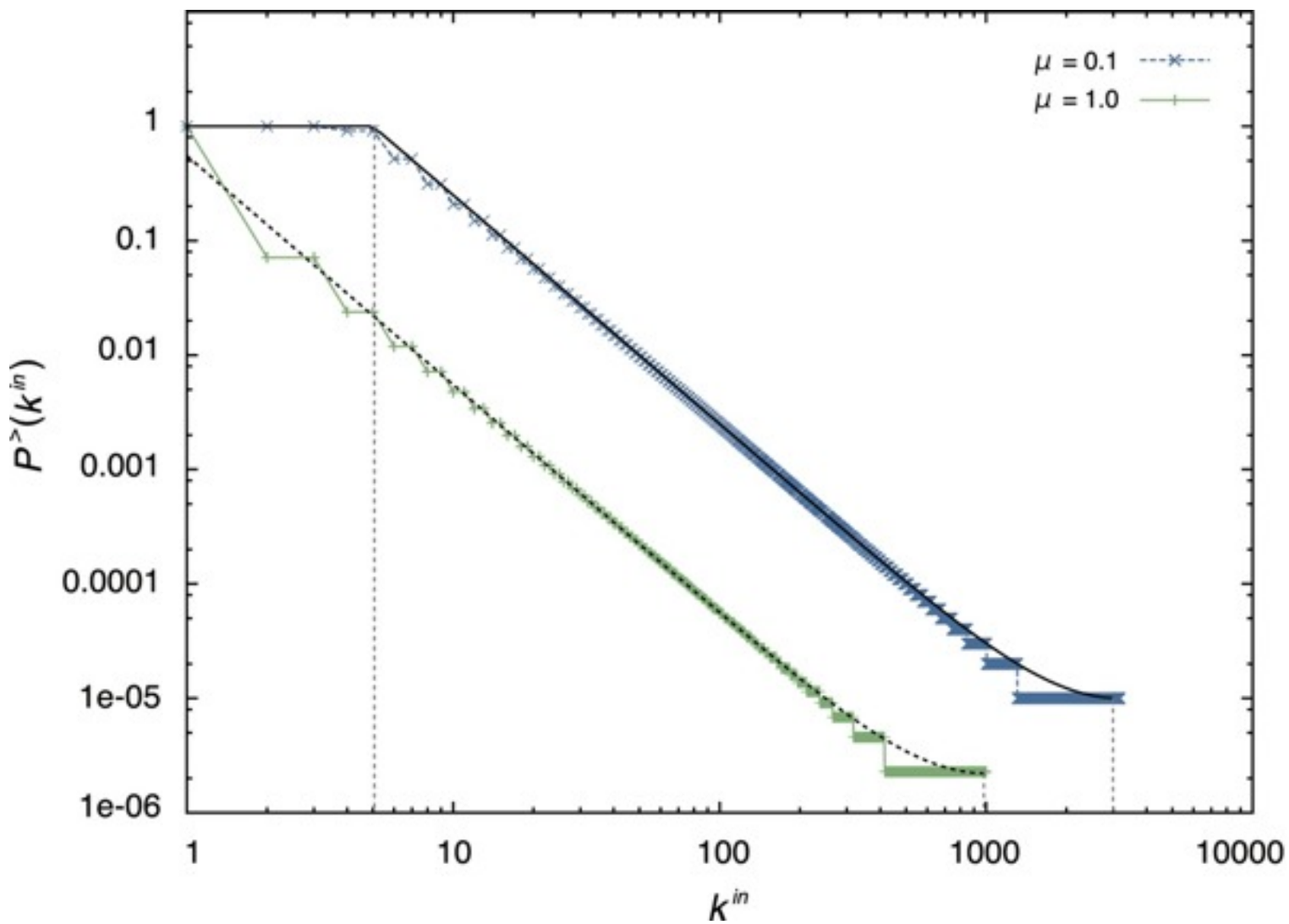
Summarizing

In the case *Δ* is extremely small (*d* is extremely large), The cumulative in-degree distribution follows a power law:

$$P^{>}(k) \propto k^{-2}$$

Comparison between the results of numerical computations and mathematical predictions

with $\mu = 0.1$ and $\mu = 1.0$, in the case $d = 6$



Are there any other maps whose
state-transition networks are scale-free networks?

Yes!

One-dimensional maps

Sine map

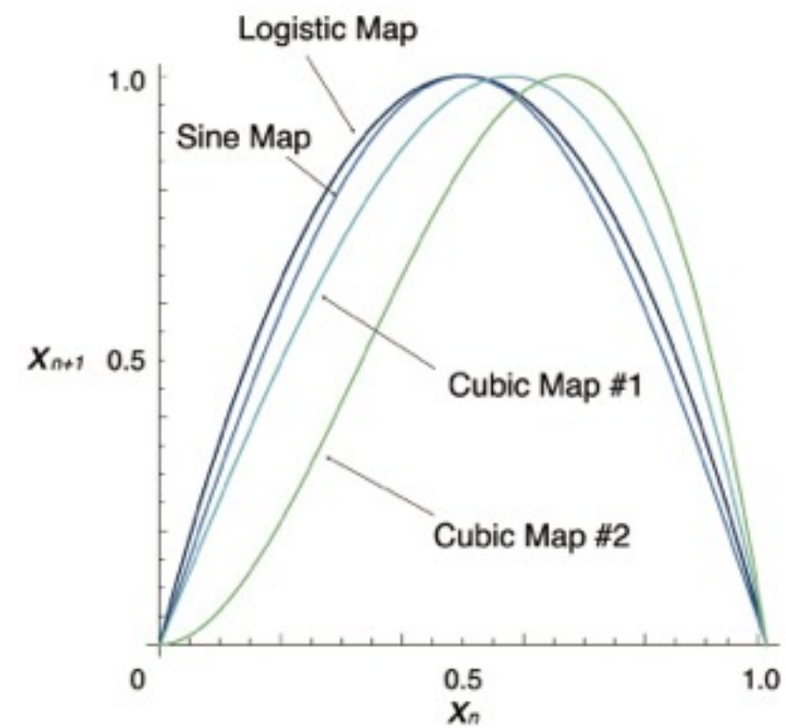
$$f(x) = \mu sin(\pi x)$$

Cubic map #1

$$f(x) = 3\sqrt{3}\mu x(1 - x^2)/2$$

Cubic map #2

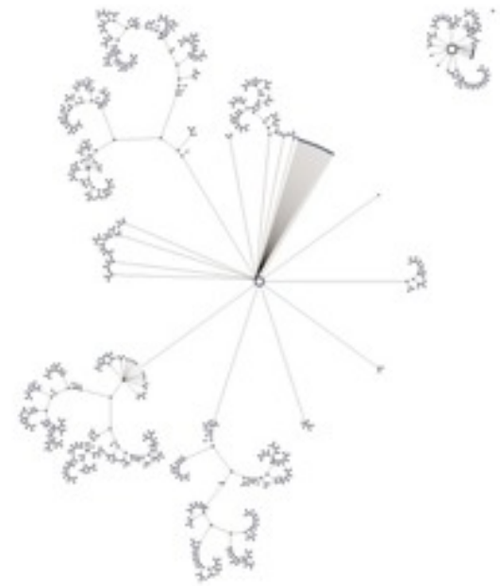
$$f(x) = 27\mu x^2(1 - x)/4$$



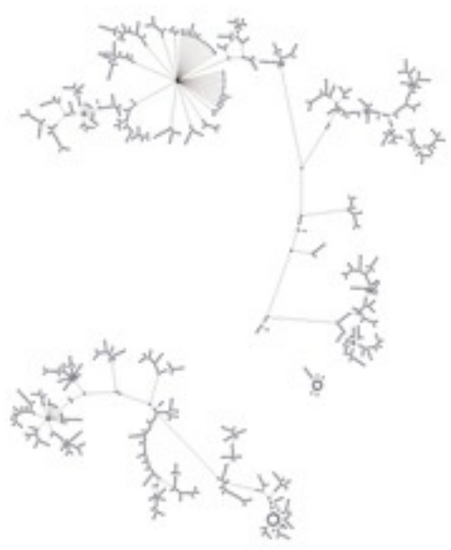
The state-transition networks for other one-dimensional maps

Logistic map
$\mu = 1.0$, $d = 3$

Sine map
$\mu = 1.0$, $d = 3$

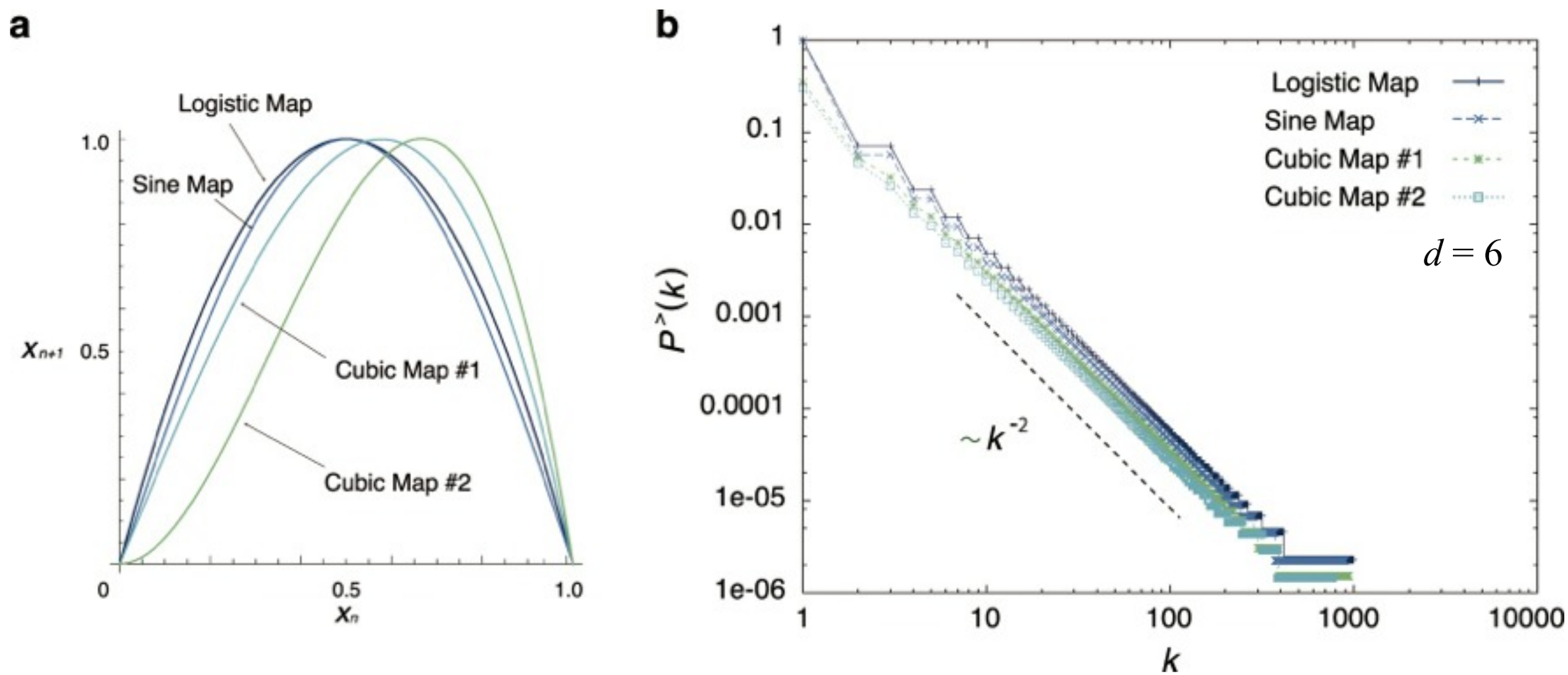


Cubic map #1
$\mu = 1.0$, $d = 3$



Cubic map #2
$\mu = 1.0$, $d = 3$

The cumulative in-degree distributions of state-transition networks for other one-dimensional maps



The dashed line has slope -2

The networks are scale-free networks with the degree exponent $\gamma = 1$.

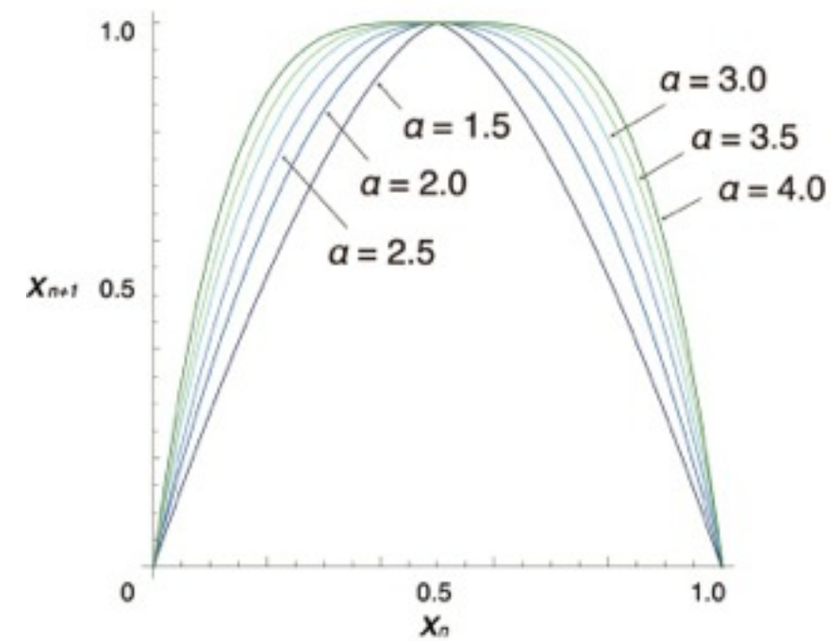
One-dimensional maps

General Symmetric map

$$f(x) = \mu\left(1 - |2x - 1|^{\alpha}\right)$$

The parameter α is controlling the flatness around the top.

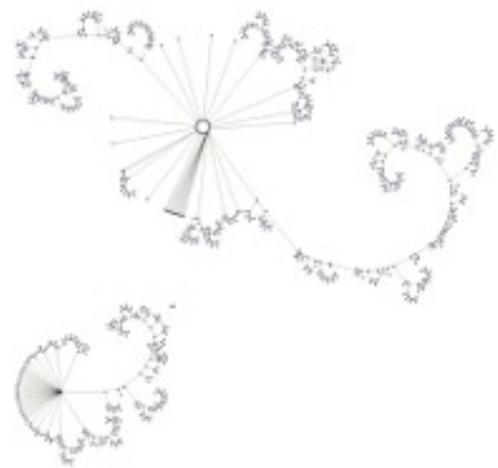
α = 2.0: the logistic map



The state-transition networks for the general symmetric map

α = 1.5

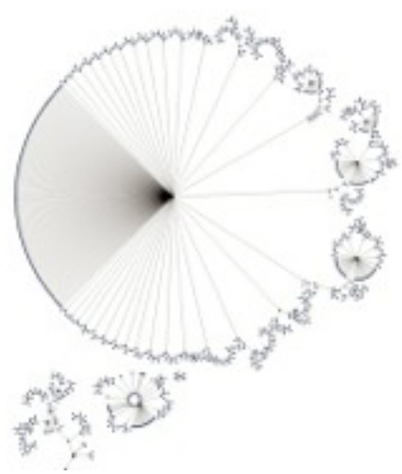
α = 2.0



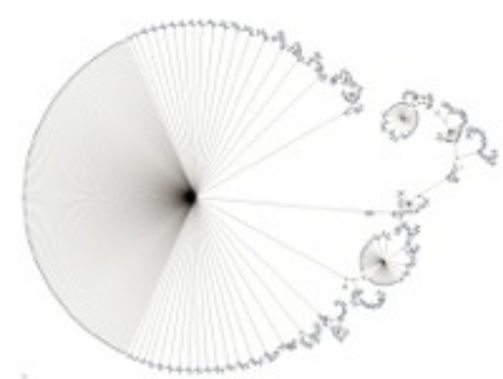
α = 2.5

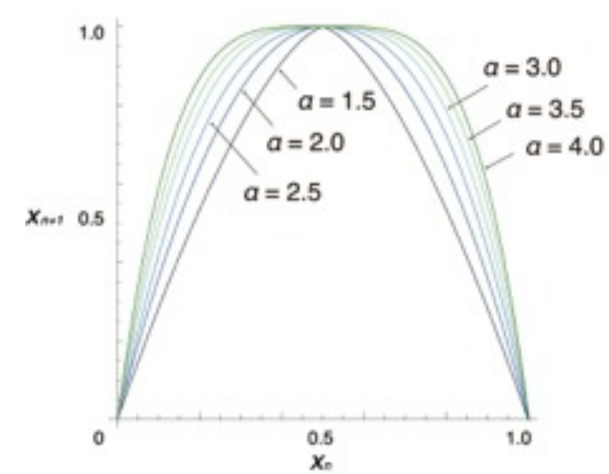
α = 3.0

α = 3.5



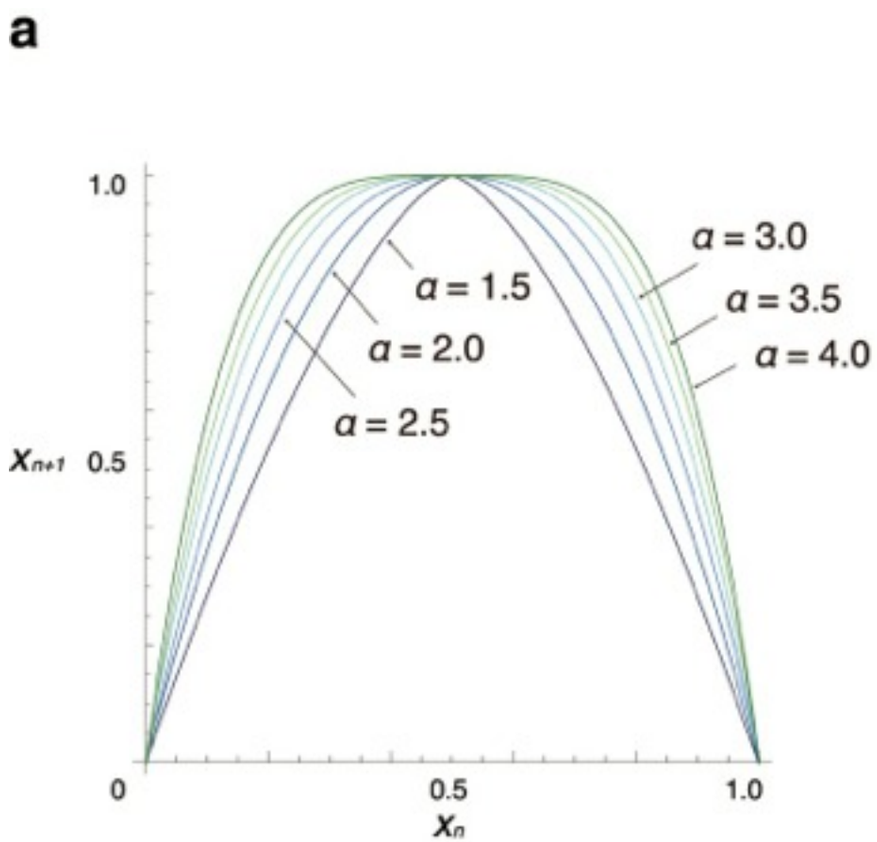
α = 4.0

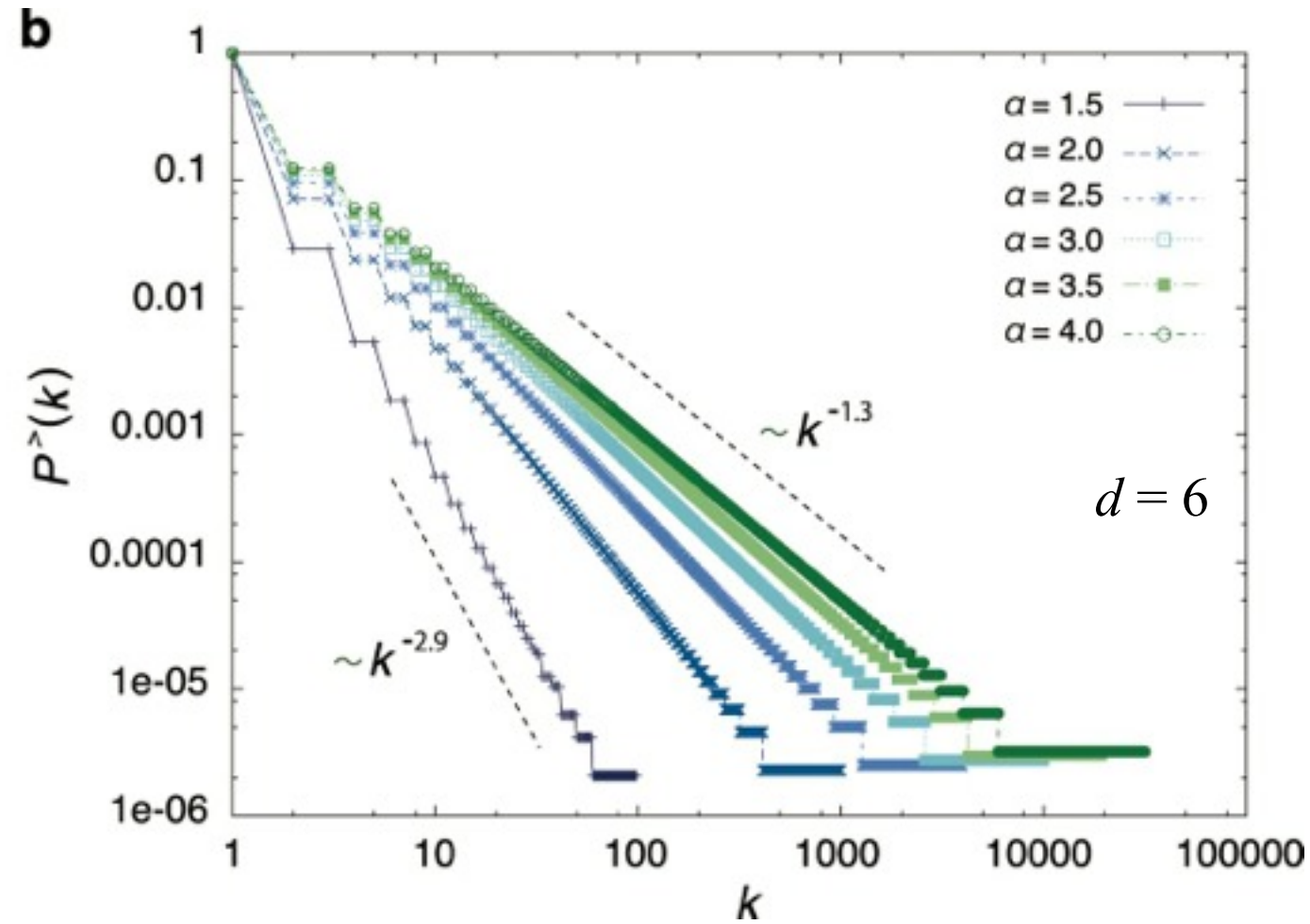


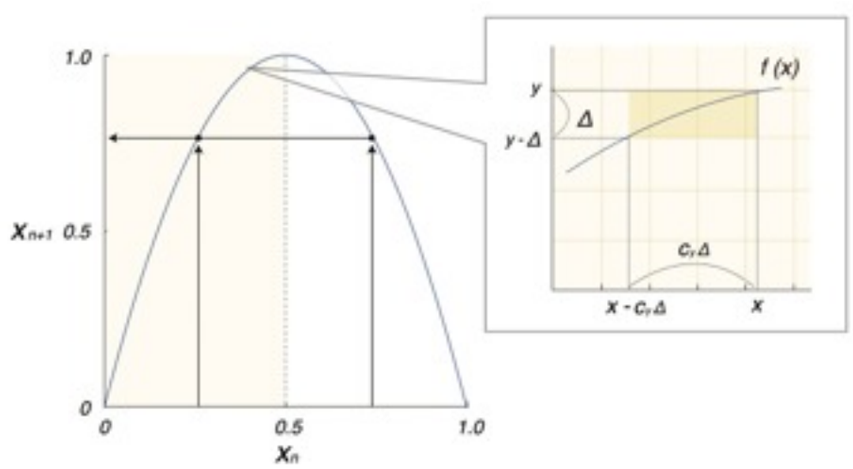


$d = 3$

The cumulative in-degree distributions of state-transition networks for the general symmetric map





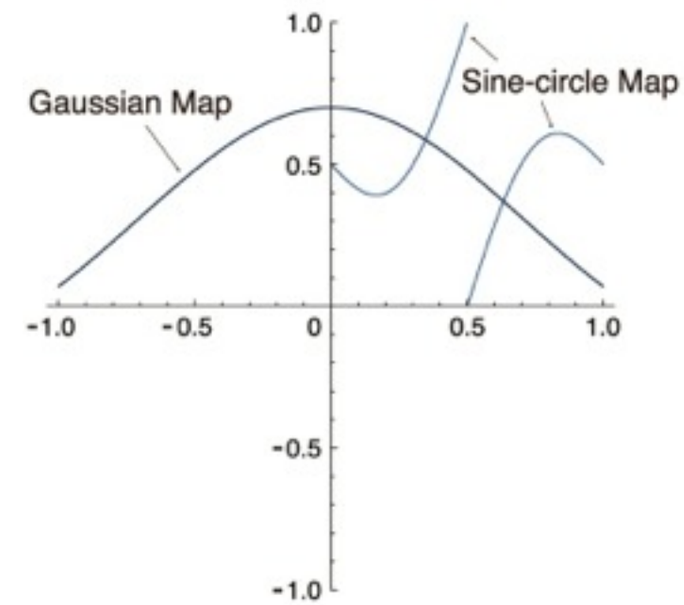


The parameter α influences the degree exponent γ, while the scale-free property is maintained.

Other types of maps

Gaussian map —— *exponential*

$$f(x) = b + e^{-ax^2}$$



Sine-circle map —— *discontinuous*

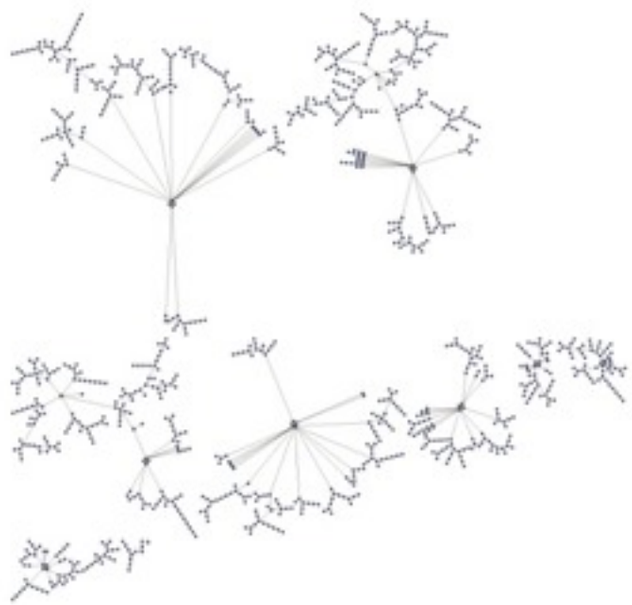
$$f(x) = x + b - (a/2\pi)x \quad (mod\ 1)$$

Delayed logistic map —— *two-dimensional*

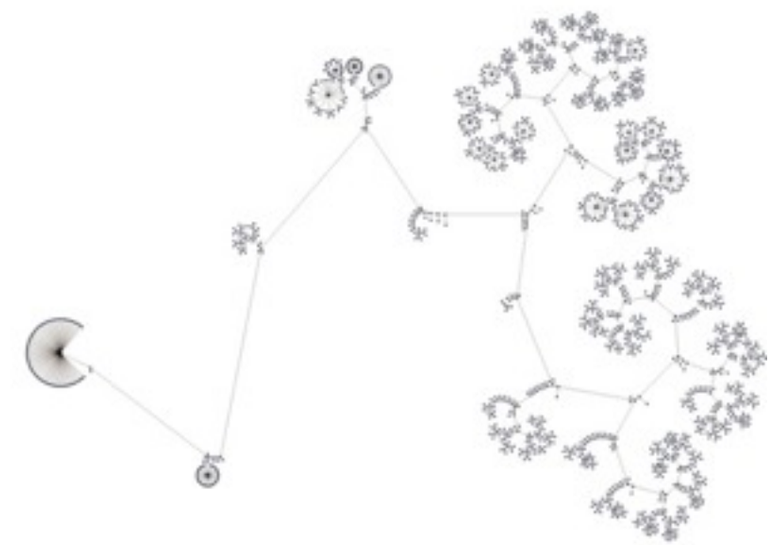
$$f(x, y) = \Big(ax(1-y), x\Big)$$

The state-transition networks for the other types of maps

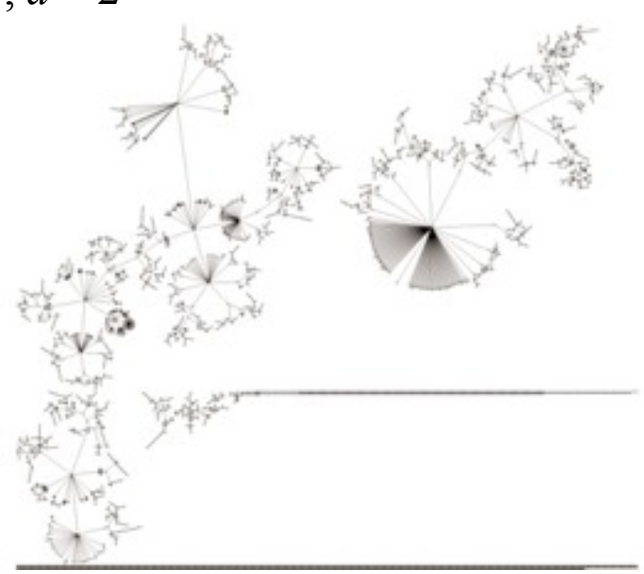
Sine-circle map
$a = 4.0$, $b = 0.5$. $d = 3$



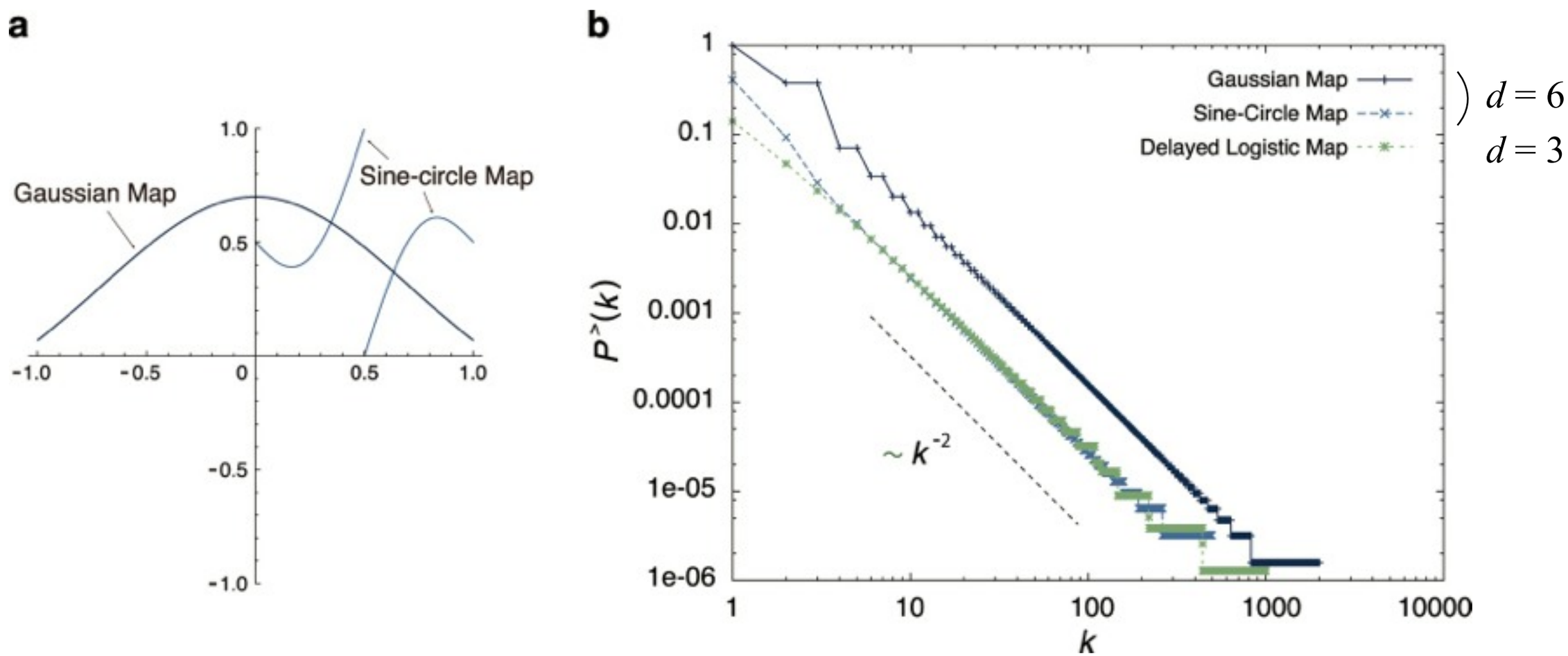
Gaussian map
$a = 1.0$, $b = -0.3$, $d = 3$



Delayed logistic map
$a = 2.27$, $d = 2$



The cumulative in-degree distributions of state-transition networks for the other types of maps



The dashed line has slope -2
the networks are scale-free networks with the degree exponent $\gamma = 1$.

Chaotic maps
that have scale-free state-transition networks

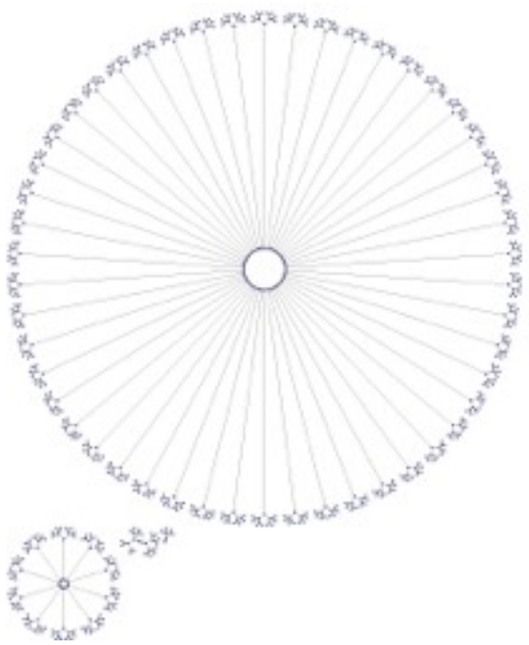
Logistic map	*one-dimensional*
Sine map	*one-dimensional*
Cubic map	*one-dimensional*
Gaussian map	*one-dimensional, exponential*
Sine-circle map	*one-dimensional, discontinuous*
Delayed logistic map	*two-dimensional*

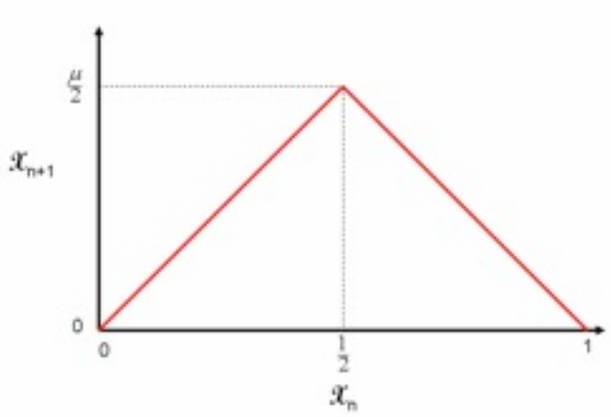
Do all chaotic maps have
scale-free state-transition networks?

No.

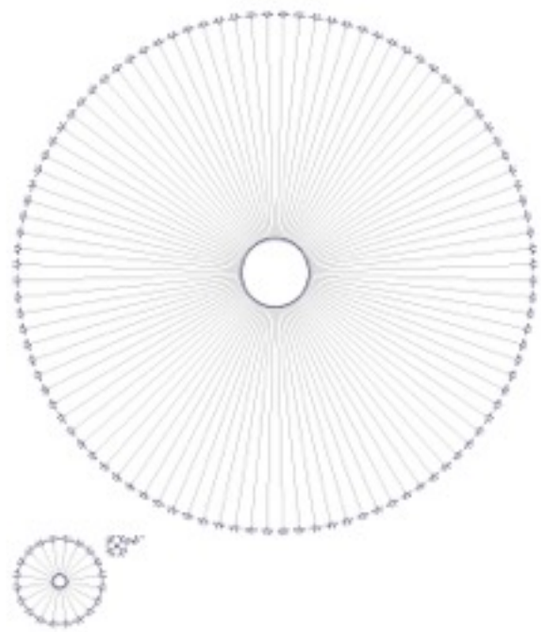
NOT scale-free state-transition networks of chaotic maps

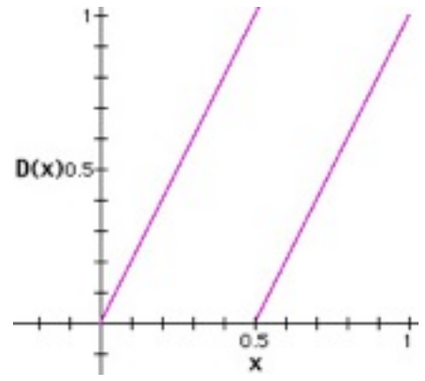
Tent map
$a = 2.0.\ d = 3$



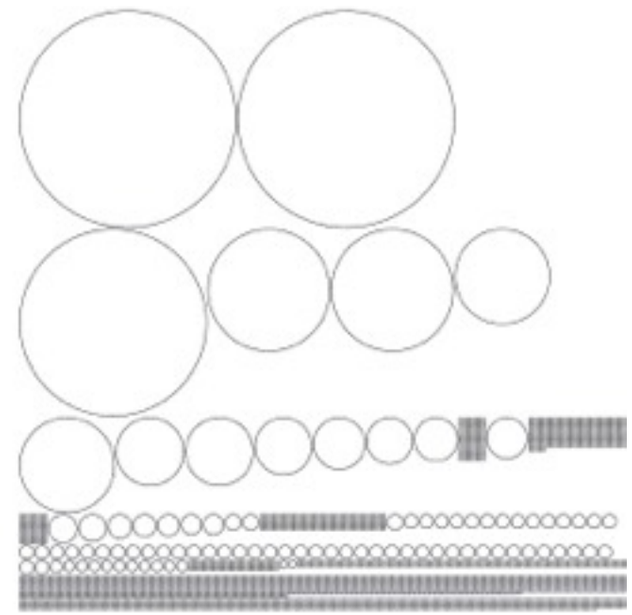


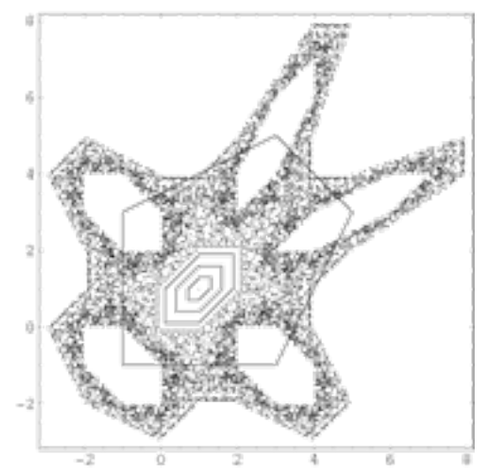
Binary Shift map
$d = 3$





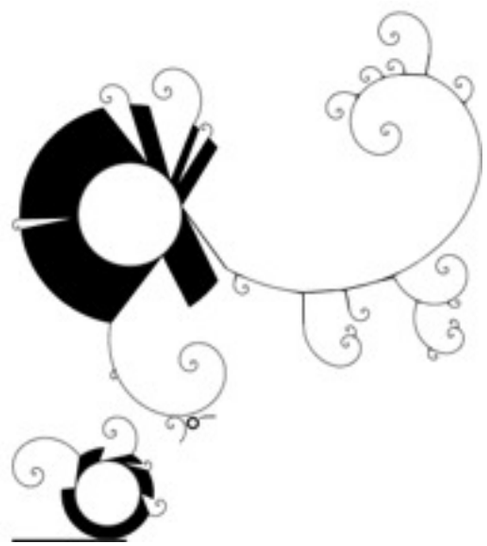
Gingerbreadman map
$d = 1$





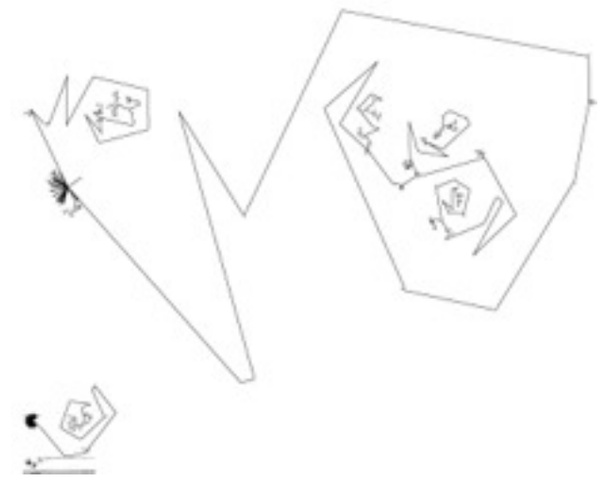
NOT scale-free state-transition networks of chaotic maps

Cusp map
$a = 2.0.\ d = 4$

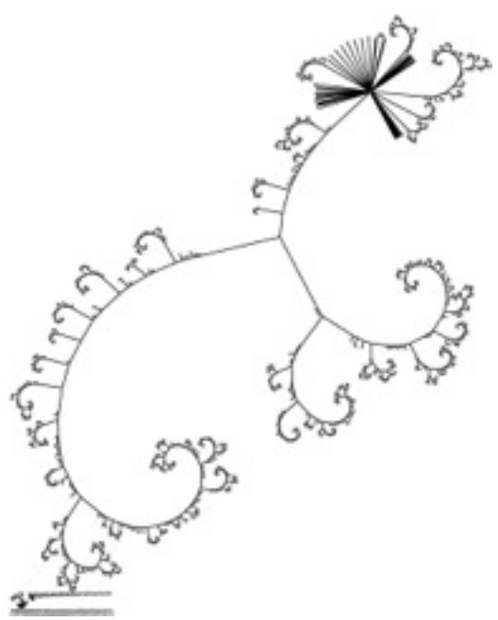


Pincher map
$a =.\ d = 4$

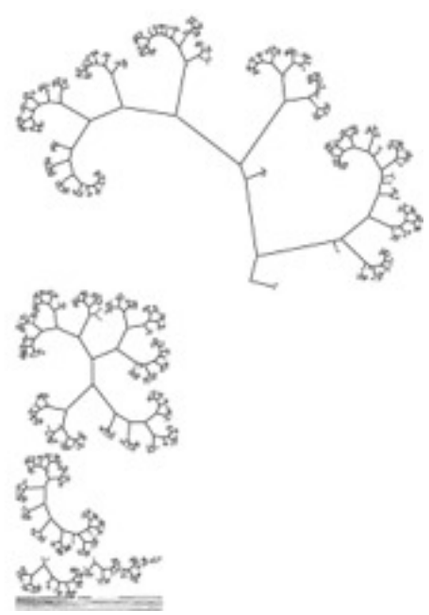
Henon map
$a = 1.4,\ b = 0.3,\ d = 2$



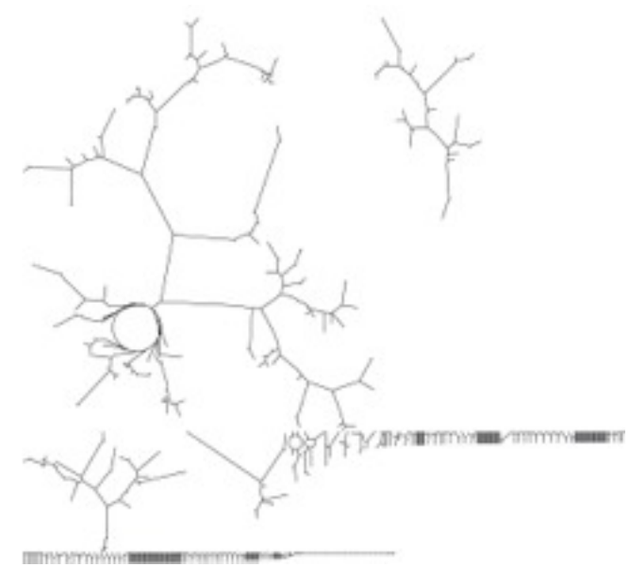
Lozi map
$a = 1.7,\ b = 0.5,\ d = 2$

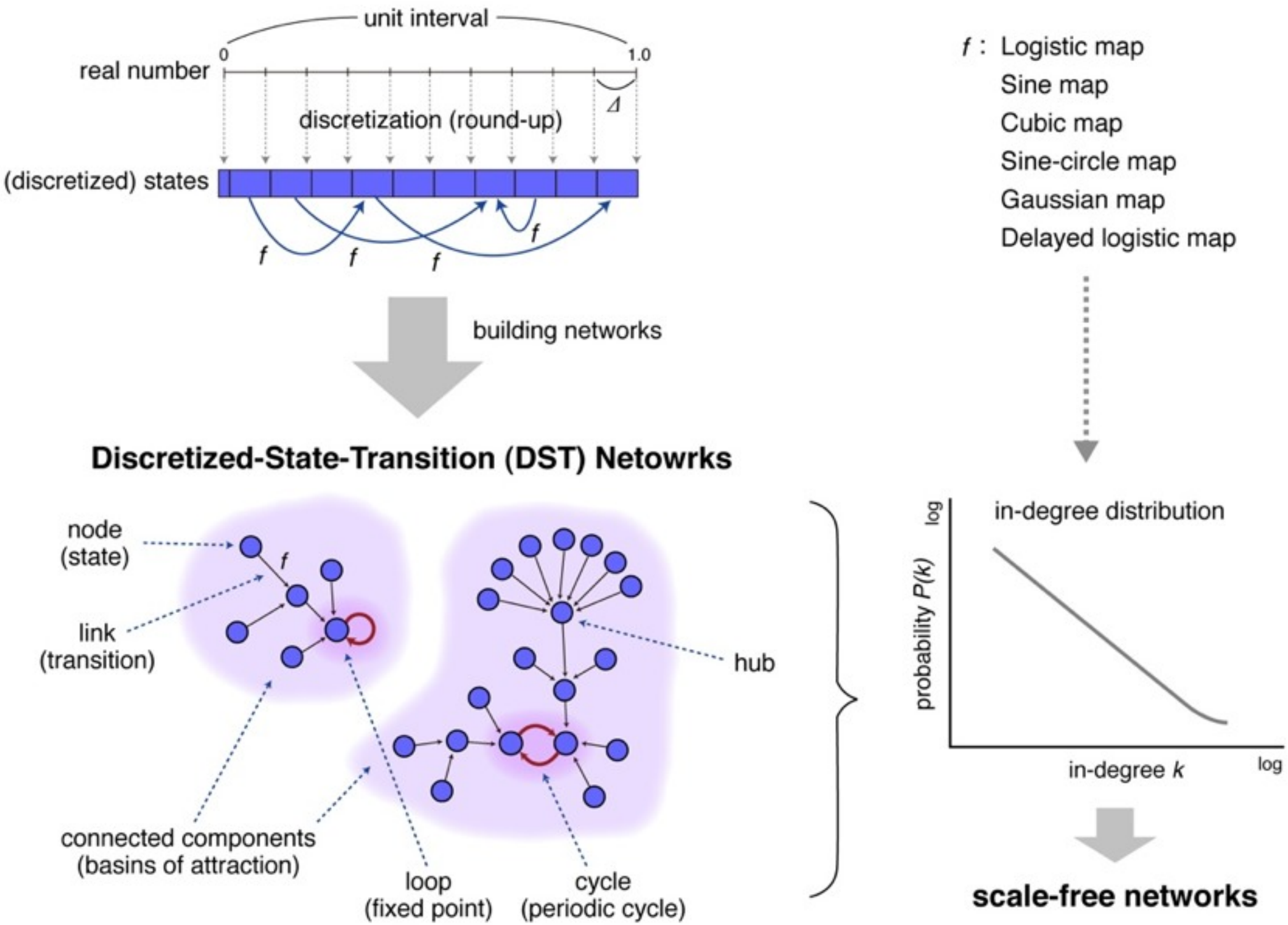


Henon area-preserving map
$a = 0.24.\ d = 2$



Standard (Chirikov) map
$a = 1.0.\ d = 1$



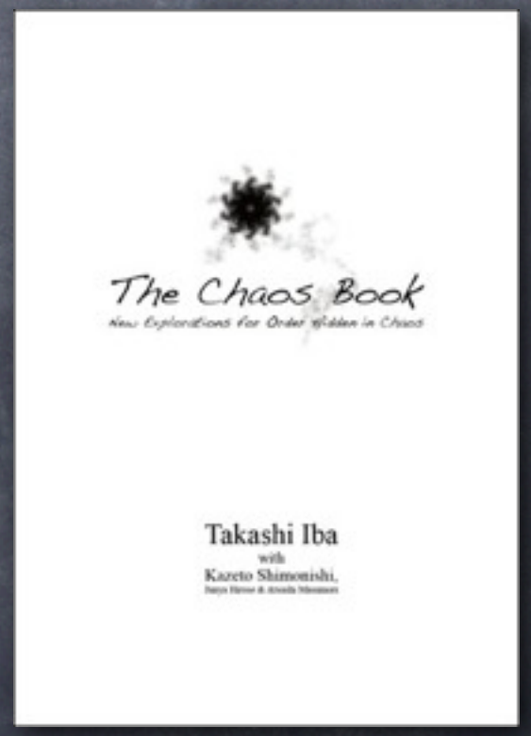

unit interval
real number
0
1.0
Δ
discretization (round-up)
(discretized) states
f
f
f
f
building networks
Discretized-State-Transition (DST) Netowrks
node
(state)
f
link
(transition)
hub
connected components
(basins of attraction)
loop
(fixed point)
cycle
(periodic cycle)
f : Logistic map
Sine map
Cubic map
Sine-circle map
Gaussian map
Delayed logistic map
in-degree distribution
log
probability P(k)
in-degree k
log
scale-free networks

Papers & Presentations

- T. Iba, "Hidden Order in Chaos: The Network-Analysis Approach To Dynamical Systems", Unifying Themes in Complex Systems Volume VIII: Proceedings of the Eighth International Conference on Complex Systems, Sayama, H., Minai, A. A., Braha, D. and Bar-Yam, Y. eds., NECSI Knowledge Press, Jun., 2011, pp.769-783

- T. Iba, "Scale-Free Networks Hidden in Chaotic Dynamical Systems", arXiv: 1007.4137v1, 2010

T. Iba with K. Shimonishi, J. Hirose, A. Masumori, "The Chaos Book: New Explorations for Order Hidden in Chaos", 2011



Dynamics as Networks

the dynamics of system/phenomena are represented as networks

- Chord Networks of Music
- Chord-Transition Networks of Music
- Collaboration Networks of Wikipedia
- Collaboration Networks of Linux
- Co-Purchase Networks of Books, CDs, DVDs
- State Networks of Chaotic Dynamical Systems

Network Analysis for Understanding Dynamics

- want to capture the process / dynamics of phenomena as a whole.
- introducing network analysis in a new way.
- to build a directed network by connecting between nodes, based on the sequential order of occurrence.
- a new viewpoint "dynamics as network", which is a distinct from "dynamics of network" and "dynamics on network."

Network Analysis for Understanding Dynamics
- Wikipedia, Music & Chaos -

時間発展のネットワーク分析

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