Network Analysis for Understanding Dynamics
- Wikipedia, Music & Chaos -

TAKASHI IBA
井庭 崇
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Associate Professor
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twitter in Japanese: takashiiba
twitter in English: taka_iba
Takashi Iba  井庭 崇

Studying ...

1997 -
Neural Computing (Optimization & Learning)
The Science of Complexity (Social and Economic Systems)
Research Methodology (Multi-Agent Modeling & Simulation)
Software Engineering (Model-Driven Development, UML)

2004 -
Sociology (Autopoietic Systems Theory)
Network Analysis
Pattern Language (A method to describe tacit knowledge)
Creativity

Teaching at SFC, Keio University

Social Systems Theory
Pattern Language
Simulation Design
Science of Complex Systems


Network Analysis for Understanding Dynamics
- Wikipedia, Music & Chaos -

時間発展のネットワーク分析

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Network Analysis for Understanding Dynamics

want to capture the process / dynamics of phenomena as a whole.

introducing network analysis in a new way.

to build a directed network by connecting between nodes, based on the sequential order of occurrence.

a new viewpoint “dynamics as network”, which is a distinct from “dynamics of network” and “dynamics on network.”
Dynamics
Dynamics
When Network Scientists talk about dynamics...

**Dynamics of Networks**
the evolution of network structure in time

**Dynamics on Networks**
the information spread or interaction on networks
Dynamics of Networks

Structural Change of Alliance Networks of Nations

the evolution of network structure in time


古川園智樹, 鈴木祐太, 井庭 崇, 「国家間同盟ネットワークの歴史的変化」, 情報処理学会 第58回数理モデル化と問題解決研究会, Vol.2006, No.29, 2006年3月, pp.93-96
Dynamics of Networks
Alliance Networks of Nations

the evolution of network structure in time
When Network Scientists talk about dynamics...

**Dynamics of Networks**
the evolution of network structure in time

**Dynamics on Networks**
the information spread or interaction on networks
Dynamics on Networks

Iterated Games on Alliance Network of Nations, as an example

the information spread or interaction on networks


古川智樹, 高田 佑介, 井庭 崇, 「ネットワーク上におけるジレンマゲーム」, 情報処理学会ネットワーク生態学研究会第2回サマースクール, 2006
When Network Scientists talk about dynamics...

Dynamics of Networks
the evolution of network structure in time

Dynamics on Networks
the information spread or interaction on networks
Dynamics of Networks

Dynamics on Networks

Dynamics as Networks
Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Dynamics as Networks
Sequential Collaboration Network of Wikipedians

the dynamics of system/phenomena are represented as networks
Dynamics as Networks

Sequential Collaboration Network of Wikipedians

The Sequential Collaboration Network of an Article on Wikipedia (English) "Star Wars Episode IV: A New Hope"

the dynamics of system/phenomena are represented as networks
Dynamics as Networks

Sequential Collaboration Network of Wikipedians

The Sequential Collaboration Network
of an Article on Wikipedia (English)
"Australia"

the dynamics of system/phenomena are represented as networks
Dynamics as Networks

Sequential Collaboration Network of Wikipedians

The Sequential Collaboration Network of an Article on Wikipedia (English) "Damien (South Park)"

The dynamics of system/phenomena are represented as networks
Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Dynamics as Networks
the dynamics of system/phenomena are represented as networks

- Chord Networks of Music
- Chord-Transition Networks of Music
- Collaboration Networks of Wikipedia
- Collaboration Networks of Linux
- Co-Purchase Networks of Books, CDs, DVDs
- State Networks of Chaotic Dynamical Systems
Chord Networks of Music

Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Sequential Chord Network of Music
Sequential Chord Network of Music
(Carpenters)

I NEED TO BE IN LOVE
WE’VE ONLY JUST BEGUN
TOP OF THE WORLD
RAINY DAYS AND MONDAY

GOODBYE TO LOVE
SING
ONLY YESTERDAY
I WON’T LAST A DAY WITHOUT YOU
SOLITAIRE

PLEASE MR. POSTMAN

JAMBALAYA (ON THE BAYOU)

HURTING EACH OTHER

THERE’S A KIND OF HUSH (ALL OVER THE WORLD)

FOR ALL WE KNOW

BLESS THE BEASTS AND CHILDREN

(HEY LONG TO BE) CLOSE TO YOU

YESTERDAY ONCE MORE

THOSE GOOD OLD DREAMS

MAKE BELIEVE IT’S YOUR FIRST TIME
Carpenters

major 19 songs

I NEED TO BE IN LOVE
WE’VE ONLY JUST BEGUN
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THOSE GOOD OLD DREAMS
MAKE BELIEVE IT’S YOUR FIRST TIME
Sequential Chord Network of Music [Integrated]

Carpenters
major 19 songs

weight distribution

in-degree distribution

out-degree distribution
Chord-Transition Networks of Music

Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Sequential Chord-Transition Network of Music
Sequential Chord-Transition Network of Music

I NEED TO BE IN LOVE
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THOSE GOOD OLD DREAMS
MAKE BELIEVE IT’S YOUR FIRST TIME
Sequential Chord-Transition Network of Music [Integrated]

Carpenters
major 19 songs

in-degree distribution

out-degree distribution

weight distribution
Papers & Presentations

- 広瀬 隼也, 井庭 崇, 「コードを基にした楽曲ネットワークの可視化と分析」, 情報処理学会 第5回ネットワーク生態学シンポジウム, 沖縄, 2009年3月


Collaborators

Junya Hirose
Former Graduate Student
Collaboration Networks of Wikipedia

Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

• The characteristics of collaboration patterns of all articles in a certain language.

• The commonality and differences of collaboration patterns among Wikipedias written in various languages.
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles
**Method: Sequential collaboration network**

Building a *sequential* collaboration network, connecting a relation from editor A to editor B, if editor B follows on work done by editor A.
Sequential Collaboration Network of Article
“Collaborative Innovation Networks” in English Wikipedia

The number of Nodes = 51
Average path length = 6.399

The number of Nodes = 594
Average path length = 6.577
Sequential Collaboration Network of Article “Switzerland” in English Wikipedia

The number of Nodes = 3998
Average path length = 5.468
Sequential Collaboration Network of Article “Fondue” in English Wikipedia

The number of Nodes = 457
Average path length = 10.485
■ Method: Sequential collaboration network

Building a *sequential* collaboration network, connecting a relation from editor A to editor B, if editor B follows on work done by editor A.
Our Previous Study: Featured Articles in English Wikipedia

The order of each sequential collaboration network
(The number of editors in each article)

Linear graph
2,545 articles [Jun 27 2009]

Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles
Analysis 1: Comparison of 12 different languages

Target Languages

Rank 1: English
Rank 2: German
Rank 3: French
Rank 4: Polish
Rank 5: Italian
Rank 6: Japanese
Rank 7: Spanish
Rank 8: Dutch
Rank 9: Portuguese
Rank 10: Russian
...
Rank 15: Finnish
...
Rank 20: Turkish

Analyzing ALL articles as of January 1\textsuperscript{st}, 2011 in each language.

The ranking based on the data as of January 6\textsuperscript{th}, 2011.
The order of each sequential collaboration network
(The number of editors in each article)
The order of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph
Rank 1
3,490,325 articles

The order of each sequential collaboration network
(The number of editors in each article)

Double logarithmic graph

The average path length of each sequential collaboration network
The order of each sequential collaboration network
(The number of editors in each article)

Double logarithmic graph
The order of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph

The average path length of each sequential collaboration network

Rank 3
1,039,251 articles
The order of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph

Rank 4
752,734 articles
The order of each sequential collaboration network
(The number of editors in each article)

Double logarithmic graph
Japanese

Rank 6
718,974 articles

The order of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph
The order of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph

Rank 7
676,866 articles
Dutch

Rank 8
656,079 articles

The order of each sequential collaboration network
(The number of editors in each article)

Double logarithmic graph

The average path length of each sequential collaboration network
The order of each sequential collaboration network
(The number of editors in each article)

Double logarithmic graph

The average path length of each sequential collaboration network

Rank 9
638,747 articles
Rank 10
627,139 articles

The average path length of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph
The order of each sequential collaboration network (The number of editors in each article)

Double logarithmic graph

The average path length of each sequential collaboration network

Rank 15
255,712 articles
The order of each sequential collaboration network
(The number of editors in each article)

Rank 20
152,262 articles
Result of Analysis 1: Comparison of 12 different languages

- Scatter plot of all articles exhibits a tilted triangle in all languages.

- The height of triangle gets shorter as the number of articles decreases.
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles
Analysis 2: Distribution of account and IP users
The order of each sequential collaboration network (The number of editors in each article)

Scatter plot of articles in English Wikipedia

Double logarithmic graph
Scatter plot of articles with number of IP users / number of total editors

The order of each sequential collaboration network
(The number of editors in each article)

The average path length of each sequential collaboration network

Double logarithmic graph
Scatter plot of articles with number of IP users / number of total editors

\[
P_{IP} = 0.0
\]
\[
P_{IP} = 0.1
\]
\[
P_{IP} = 0.2
\]
\[
P_{IP} = 0.3
\]
\[
P_{IP} = 0.4
\]
\[
P_{IP} = 0.5
\]
\[
P_{IP} = 0.6
\]
\[
P_{IP} = 0.7
\]
\[
P_{IP} = 0.8
\]
Scatter plot of articles with number of IP users / number of total editors

The order of each sequential collaboration network (The number of editors in each article)
Result of Analysis 2: Distribution of account and IP users

• Top and right area of the “triangle” in scatter plot consist of articles which ratios of users is high.

• As a result, both the average path length and order of network can be large in these areas.

$P_{IP} = 0.0$  
$P_{IP} = 0.6$
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles
Analysis 3: Distribution of Featured Articles

3,372 featured articles / 3,732,033 articles
In English Wikipedia
The order of each sequential collaboration network (The number of editors in each article)

Scatter plot of all articles in English Wikipedia

Double logarithmic graph
Scatter plot of featured articles on the all articles in English Wikipedia

The average path length of each sequential collaboration network

(Double logarithmic graph)

The order of each sequential collaboration network (The number of editors in each article)
Scatter plot of featured articles on the all articles in English Wikipedia

Double logarithmic graph

The order of each sequential collaboration network
(The number of editors in each article)
Result of Analysis 3: Distribution of Featured Articles

• Features articles are located at a certain area in the scatter plot.

• It implies that there would be characteristic patterns of collaboration producing good results.
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

- Method: Sequential collaboration network
- Analysis 1: Comparison of 12 different languages
- Analysis 2: Distribution of account and IP users
- Analysis 3: Distribution of Featured Articles
Editorial Collaboration Networks of Wikipedia Articles in Various Languages

• Scatter plot of all articles commonly exhibits a tilted triangle in all languages, but the height of triangle gets shorter as the number of articles decreases.

• Top and right area of the “triangle” in scatter plot consist of articles which the ratios of IP users are high.

• Features articles are located at a certain area in the scatter plot.
Collaborators

Satoshi Itoh
A Former Graduate Student, Iba Lab.

Daiki Muramatsu
Student, Iba Lab.

Bui Hong Ha
Former Student, Iba Lab.

Natsumi Yotsumoto
Former Student, Iba Lab.

Ko Matsuzuka
Student, Iba Lab.

Peter Gloor
Research Scientist, MIT CCI

Keiichi Nemoto
Visiting Scholar, MIT CCI
Fuji Xerox
Papers & Presentations

- 伊藤 諭志, 伊藤 貴一, 熊坂 賢次, 井庭 崇, 「マスコラボレーションにおけるコンテンツ形成プロセスの分析」, 人工知能学会第20回セマンティックウェブとオントロジー研究会, 2009
- 四元 菜つみ, 井庭 崇, 「Wikipediaにおけるコラボレーションネットワークの成長」, 情報処理学会第7回ネットワーク生態学シンポジウム, 2011
Collaboration Networks of Linux

Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Linux-Activists Mailing List & comp.os.minix Newsgroup

Takashi Iba (2008)
Linux-Activists Mailing List (Nov. 1991)
Linux-Activists Mailing List (Nov. 1 – 14, 1991)
1991/8

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| Linus Benedict Torvalds |
| Jyrki Kuoppala |
| James da Silva |
| Peter Holzer |
| Adam David |
| Alan Barclay |

“What would you like to see most in minix?”

8/25 (20:57)

8/26 (11:16)

8/28 (12:56)

8/25 (23:44)

8/26 (15:39)

8/26 (16:20)

8/27 (0:32)

8/27 (14:34)
comp.os.minix Newsgroup (Dec. 1991)

1991/12

Linus
Benedict
Torvalds

Jyrki
Kuoppala

James da
Silva

Peter
Holzer

Adam
David

Alan
Barclay

Michael
Haardt

Ari
Lemmke

Paul
Allen

Ricardo
Santos

C. G.
Albone

Oleg
Moroz

Miquel van
Smorenburg

Drew
Eckhardt

"Status of LINUX?"
12/19 (12:46)

12/19 (23:35)

12/20 (7:02)
comp.os.minix Newsgroup (Jan. 1992)
comp.os.minix Newsgroup (Aug. 1991)
comp.os.minix Newsgroup (Oct. 1991)
comp.os.minix Newsgroup (Jan. 1992)
Co-Purchase Networks of Books, CDs, DVDs

Dynamics as Networks

the dynamics of system/phenomena are represented as networks
Rakuten Books: A famous Japanese Online Store

http://books.rakuten.co.jp/

We've got POS data of random-sampled 30,000 customers (2005-2006)
- Customer ID (masked)
- When purchasing
- Which book (CD, DVD) purchasing

* sample customers of books, CDs, and DVDs are different one another.
Building Co-Purchase Network from data
Co-Purchase Network of Books

Number of nodes = 68,701
Number of edges = 113,940

* Target Customers = 30,000
* All Links are Visualized.
Co-Purchase Network of CD

Number of nodes = 14,038
Number of edges = 22,727

* Target Customers = 30,000
* All Links are Visualized.
Co-Purchase Network of DVD

Number of nodes = 10,875
Number of edges = 24,535

* Target Customers = 30,000
* All Links are Visualized.
In-Degree Distribution follow Power Law

Sequential Connection

Book

CD

DVD

$\gamma=2.56$

$\gamma=2.33$

$\gamma=2.09$
Out-Degree Distribution follow Power Law

Sequential Connection

Book: $\gamma=2.57$

CD: $\gamma=2.43$

DVD: $\gamma=1.99$
Weight Distribution follow Power Law

Sequential Connection

Books

CDs

DVDs
Mapping Genre in Co-Purchase Network of DVDs

Node Coloring

low $k$ high

Sequential Connection

Target Customers = 30,000
Weight > 1
Papers & Presentations

- 井庭 崇, 北山 雄樹, 伊藤 諭志, 西田 壽介, 吉田 真理子,「オンラインストアにおける商品の共購買ネットワークの分析」, 情報処理学会 第5回ネットワーク生態学シンポジウム, 2009

Collaborators

- Satoshi Itoh
  Former Graduate Student, Iba Lab.
- Yuki Kitayama
  Former Graduate Student, Iba Lab.
- Mariko Yoshida
  Former Student, Iba Lab.
- Ryosuke Nishida
  Former Graduate Student, Iba Lab.
- Masaya Mori
  Rakuten Institute of Technology
State Networks of Chaotic Dynamical Systems

Dynamics as Networks

the dynamics of system/phenomena are represented as networks
The networks of state transitions in several discretized chaotic dynamical systems are *scale-free networks*.

It is found in Logistic map, Sine map, Cubic map, General symmetric map, Gaussian map, Sine-circle map.
Highly complex behavior is generated, although it’s governed by a very simple rule.
Fundamentals: Logistic Map

\[ x_{n+1} = 4\mu \, x_n \left( 1 - x_n \right) \]

a simple population growth model (non-overlapping generations)

\[ x_n \quad \text{... population (capacity)} \quad 0 \leq x_n \leq 1 \quad \text{(variable)} \]

\[ \mu \quad \text{... a rate of growth} \quad 0 \leq \mu \leq 1 \quad \text{(constant)} \]

\[ x_0 = \text{an initial value} \]

\[ n = 0 \quad x_1 = 4\mu \, x_0 \left( 1 - x_0 \right) \]

\[ n = 1 \quad x_2 = 4\mu \, x_1 \left( 1 - x_1 \right) \]

\[ n = 2 \quad x_3 = 4\mu \, x_2 \left( 1 - x_2 \right) \]


The behavior depends on the value of control parameter $\mu$.

$$x_{n+1} = 4\mu x_n (1 - x_n)$$
Fundamentals: Logistic Map

\[ x_{n+1} = 4\mu \, x_n \, (1 - x_n) \]

This diagram shows long-term values of \( x \).
This diagram shows long-term values of $x_n$.
I want a map! for taking an overview of the whole.

map
1. a. A representation, usually on a plane surface, of a region of the earth or heavens.
   
   b. Something that suggests such a representation, as in clarity of representation.

2. Mathematics: The correspondence of elements in one set to elements in the same set or another set.

- The American Heritage Dictionary of the English Language
I want a map of the map! for taking an overview of the whole behavior of the iterated map.

Logistic Map

\[ x_{n+1} = 4 x_n (1 - x_n) \]

map

1. a. A representation, usually on a plane surface, of a region of the earth or heavens.
   b. Something that suggests such a representation, as in clarity of representation.
2. Mathematics: The correspondence of elements in one set to elements in the same set or another set.

- The American Heritage Dictionary of the English Language
Existing methods are NOT for drawing a map for taking an overview of the whole.

They merely visualize the trajectory of an instance of the system’s behavior, giving an initial value.
Visualizing State Transitions of a System for taking an overview of the whole.

$$x_{n+1} = 4\mu x_n (1 - x_n)$$

state: the value of $x$
map

$x_{n+1} = 4\mu x_n (1 - x_n)$
$x$ is treated as \textit{discrete}
which implies the finite number of states

abstraction

target world

$x_{n+1} = 4\mu x_n (1 - x_n)$
$x$ is a real number
which has no limit to smallness (detail)
$x_{n+1} = 4\mu x_n (1 - x_n)$

$x$ is treated as **discrete** which implies the finite number of states
Discretizing into the finite number of states

To subdivide a unit interval of variables into a finite number of subintervals

\[ f(x) = 4\mu x(1 - x) \]

\[ x_{n+1} = f(x_n) \]
Discretizing into the finite number of states

To subdivide a unit interval of variables into a finite number of subintervals

\[ f(x) = 4\mu x(1 - x) \]

The discretization is carried out by rounding the value \( x \) to \( d \) decimal places.

Therefore, \( \Delta = 10^{-d} \)
Obtaining the set of state transitions

To apply the map $f$ into all states in order to obtain all state transitions.

The discretization is carried out by rounding the value $x$ to $d$ decimal places. Therefore, $\Delta = 10^{-d}$

(ex.) round-up

$d = 1$: 0.14      0.2
$d = 2$: 0.146     0.15

the set of state
Obtaining the set of state transitions

To apply the map $f$ into all states in order to obtain all state transitions.

$x_{n+1} = h(f(x_n))$

where $h()$ is the rounding function, and $f(x) = 4\mu x(1 - x)$
Building state-transition networks

To connect each state into its successive state

State-transition network is a “map” of the whole behavior of the system, so one can take an overview from bird’s-eye view.
State-transition networks

The order of the network $N = \frac{1}{\Delta} + 1$.

Each connected component must have only one loop or cycle: fixed point or periodic cycle.

The whole behavior is often mapped into more than one connected components, which represent basins of attraction.

The in-degree of each node may exhibit various values.

The out-degree of every node must be always equal to 1, since it is a deterministic system.
State-transition networks as a “dried-up river”

node: geographical point in the river
link: a connection from a point to another

The direction of a flow in the river is fixed.

The network is *dried-up* river, where there are no water flows on the riverbed.

There are many confluences of two or more tributaries.
There are no branches of the flow.
Numerical simulation represents an instance of flow on the state-transition networks.

Determining a starting point and discharging water, you will see that the water flow downstream on the river network. That is a happening you witness when conducting a numerical simulation of system’s evolution in time.

Numerical simulation represents an instance of flow on the state-transition networks.
Let’s draw a map of the logistic map!

for taking an overview of the whole

\[ x_{n+1} = 4\mu x_n (1 - x_n) \]
The state-transition networks for the logistic map

Control Parameter: $\mu = 1$ 
Therefore, $x_{n+1} = h(4x_n(1-x_n))$

Round-Up into the decimal place, $d = 1$ 
Therefore, 11 states

<table>
<thead>
<tr>
<th>$x_n$</th>
<th>$f$</th>
<th>$h$</th>
<th>$x_{n+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.36</td>
<td>0.40</td>
<td>0.4</td>
</tr>
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<td>0.2</td>
<td>0.64</td>
<td>0.70</td>
<td>0.7</td>
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<td>0.3</td>
<td>0.84</td>
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<tr>
<td>0.4</td>
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<td>1.00</td>
<td>1.0</td>
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<td>0.5</td>
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<td>1.00</td>
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</tr>
<tr>
<td>1.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.0</td>
</tr>
</tbody>
</table>
The state-transition networks for the logistic map

Control Parameter: $\mu = 1$

Round-Up into the decimal place, $d = 2$

Therefore, $x_{n+1} = 4 \times x_n \times (1 - x_n)$

Therefore, 101 states

<table>
<thead>
<tr>
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<th>$h$</th>
<th>$x_{n+1}$</th>
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</table>
The state-transition networks for the logistic map

Control Parameter: $\mu = 1$

Therefore, $x_{n+1} = 4x_n (1 - x_n)$

Round-Up into the decimal place, $d = 3$

Therefore, 1001 states

\[
\begin{array}{cccc}
x_n & f & h & x_{n+1} \\
0.000 & 0.00000 & 0.000 & 0.000 \\
0.001 & 0.003996 & 0.004 & 0.004 \\
0.002 & 0.007984 & 0.008 & 0.008 \\
0.003 & 0.011964 & 0.012 & 0.012 \\
0.004 & 0.015936 & 0.016 & 0.016 \\
0.005 & 0.019900 & 0.020 & 0.020 \\
0.006 & 0.023856 & 0.024 & 0.024 \\
0.007 & 0.027804 & 0.028 & 0.028 \\
0.008 & 0.031744 & 0.032 & 0.032 \\
0.009 & 0.035676 & 0.036 & 0.036 \\
0.010 & 0.039600 & 0.040 & 0.040 \\
0.011 & 0.043516 & 0.044 & 0.044 \\
0.012 & 0.047424 & 0.048 & 0.048 \\
0.013 & 0.051324 & 0.052 & 0.052 \\
0.014 & 0.055216 & 0.056 & 0.056 \\
0.015 & 0.059100 & 0.060 & 0.060 \\
0.016 & 0.062976 & 0.063 & 0.063 \\
\end{array}
\]
The state-transition networks for the logistic map

Control Parameter: $\mu = 1$

Round-Up into the decimal place, $d = 4$

Therefore, $x_{n+1} = 4 \cdot x_n \cdot (1 - x_n)$

Therefore, 10001 states

<table>
<thead>
<tr>
<th>$x_n$</th>
<th>$f$</th>
<th>$h$</th>
<th>$x_{n+1}$</th>
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<td>0.0016</td>
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</table>

...
The cumulative in-degree distributions of state-transition networks for the logistic map with $\mu = 1$ in the case from $d = 1$ to $d = 7$

The dashed line has slope $-2$.

The networks are scale-free networks with the degree exponent $\gamma = 1$, regardless of the value of $d$. 
The state-transition networks for the logistic map

Control Parameter: $\mu = 1$ Therefore, $x_{n+1} = 4x_n(1 - x_n)$

Round-Up into the decimal place, ranging from $d = 1$ to $d = 4$

$d = 1$  
$d = 2$  

$d = 3$  
$d = 4$

scale-free networks!
Does the *scale-free* property depend on the control parameter $\mu$?
Does the *scale-free* property depend on the control parameter $\mu$?

\[ x_{n+1} = 4\mu x_n (1 - x_n) \]
The state-transition networks for the logistic map

Control Parameter: ranging from $\mu = 0$ to $\mu = 1$ \hspace{1cm} $x_{n+1} = 4\mu x_n (1 - x_n)$

Round-Up into the decimal place, $d = 3$
The cumulative in-degree distributions of state-transition networks for the logistic map from $\mu = 0.125$ to 1.000 by 0.125 in the case $d = 7$.

The dashed line has slope -2.
The networks are scale-free networks with the degree exponent $\gamma = 1$, regardless of the value of the parameter $\mu$. 
Does the *scale-free* property depend on the control parameter $\mu$?

NO.

The *scale-free* property is independent of the control parameter $\mu$. 
How the scale-free network is emerged?
How the scale-free network is emerged?

\[ x_{n+1} = 4 \mu x_n (1 - x_n) \]
How the scale-free network is emerged?

If $x$ is a real number, namely in the *mathematically ideal* condition, the state 0.84 must have only 2 previous values: 0.30 and 0.70.

In the condition, the state-transition network cannot be a scale-free network ...
How the scale-free network is emerged?
How the scale-free network is emerged?

The trick is **discretization**.
How the scale-free network is emerged?

The relation between a subinterval on y-axis and its corresponding range on the x-axis for the logistic map function.

<table>
<thead>
<tr>
<th>$x_{n+1}$</th>
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<td>0.992</td>
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Mathematical Derivation

1. Number of the states whose in-degree is higher than zero

Suppose that $f$ is the unimodal function of the interval $[0, 1]$ that is concave downward, symmetrical with respect to the critical point, and $f(0) = f(1) = 0$ (see Fig. 7). While there are two solutions for the inverse function $f^{-1}$, we shall indicate a solution on the left hand of the parabola as $f_{y}^{-1}$. Notice that, due to the symmetrical nature of the in-degree $k$ is always an even number but the critical point.

In order to obtain the number of nodes which have incoming links, $N_{\text{in}}>0$, we shall describe a relation between $y$ and $x$ with using the size of subinterval $\Delta$ and the coefficient $c_{y}$ (see Fig. 7). We then have $y = f(x)$ and $y - \Delta = f(x - c_{y}\Delta)$, and, with the inverse function, $x = f_{y}^{-1}(y)$ and $x - c_{y}\Delta = f_{y}^{-1}(y - \Delta)$. Cancelling $y$ from these equations gives the equation:

$$c_{y} = \frac{x - f_{y}^{-1}(f(x) - \Delta)}{\Delta}$$

On the other hand, cancelling $x$ gives the equation:

$$c_{y} = \frac{f_{y}^{-1}(y) - f_{y}^{-1}(y - \Delta)}{\Delta}$$

Accordingly, since $k_{y} = 2c_{y}$ by definition, in-degree of the state $y$ is calculated as:

$$k_{y} = \frac{2}{\Delta} \left(f_{y}^{-1}(f_{y}^{-1}(y) - \Delta)\right)$$

Let the coordinates of the point where $k = 4$, namely $c_{y} = 2$, be $(X, Y)$, and therefore it must satisfy that $Y = f(X)$ and $c_{y} = \frac{Y}{\Delta}$, and the coordinates of the point where $k = 4$, $(X, Y)$, is given by:

$$(X, Y) = \left(\frac{1}{2}, \frac{2}{2} - \frac{1}{2} + 4\Delta, \Delta\right)$$

Solving the equation for $k_{y}$, we obtain:

$$y = \frac{\mu}{4\Delta} + \frac{1}{2\Delta} - 2\mu \Delta$$

Since $0 < \Delta < 1/16$ is always satisfied in the condition of this paper, the condition $X \geq 0 \land Y \geq 0$ means $1/(8 + 16\Delta) \leq \mu \leq 1$, and the condition $X < 0 \land Y < 0$ means $0 < \mu < 1/(8 + 16\Delta)$. Thus the number of states whose in-degree is higher than zero is given by:

$$N_{\text{in}} > 0 = \begin{cases} \frac{\mu + X + Y}{\Delta} + 2 & \text{if } X \geq 0 \land Y \geq 0 \\ \frac{\mu}{\Delta} + 1 & \text{if } X < 0 \land Y < 0. \end{cases}$$

Note that, since the map function is concave downward, $f(0) = f(1) = 0$, and focusing the left hand of the parabola, there are only two combinations of the conditions: $X \geq 0 \land Y \geq 0$ and $X < 0 \land Y < 0$. Therefore, the number of the states in the range $0 < c_{y} < c_{y}$ is calculated as:

$$N_{0 < c_{y} < c_{y}} = \begin{cases} \frac{X}{\Delta} + 1 & \text{if } X \geq 0 \\ 0 & \text{if } X < 0. \end{cases}$$

Consequently, in the case $1/(8 + 16\Delta) \leq \mu \leq 1$, the cumulative in-degree distribution $P^{\geq}(k_{y})$ is given by:

$$P^{\geq}(k_{y}) = \frac{16}{1 - 4\mu + 1 + 16\mu\Delta + 32\mu(1 + 3\Delta + 5\mu\Delta^{2})} \times \frac{1}{(1 + k + \mu\Delta)^{2}}$$

In the case $0 < \mu < 1/(8 + 16\Delta)$, the distribution is given by:

$$P^{\geq}(k_{y}) = \begin{cases} \frac{1}{4\mu(1 + k + \Delta)} & \text{if } 1 - \sqrt{\Delta + 1}/\mu + 1 < k \\ 1 & \text{if } 0 < k \leq 1 - \sqrt{\Delta + 1}/\mu + 1. \end{cases}$$

Thus, it is proved that the cumulative in-degree distribution of the DST network for the logistic map follows a law given by:

$$P^{\geq}(k_{y}) \propto \frac{1}{(1 + k + \mu\Delta)^{2}}$$

where in-degree $k$ is in the range $0 < k \leq k_{\text{max}}$ and $k_{\text{max}} = 1/\gamma \Delta$. Recall that $0 < \Delta \leq 1$ is always satisfied in the second term makes a large effect on the distribution, only when $k_{y}$ is quite close to $k_{\text{max}}$. It means that the “hub” states will have higher in-degree than the typical scale-free network whose in-degree distribution follows a strict power law. Fig. 8 shows the fitness between the results of numerical computation and mathematical predictions.

In order to understand the distribution when the size of subintervals, $\Delta$, is enough small, we take the limit of $\Delta$ into 0. In the case $1/(8 + 16\Delta) \leq \mu \leq 1$,

$$\lim_{\Delta \to 0} P^{\geq}(k_{y}) = \frac{16}{1 - 32\mu + 128\mu^{2}k^{-2}},$$

and in the case $0 < \mu < 1/(8 + 16\Delta)$,

$$\lim_{\Delta \to 0} P^{\geq}(k_{y}) = \begin{cases} \frac{1}{4\mu} & \text{if } 1 - \sqrt{\Delta + 1}/\mu + 1 < k \\ 1 & \text{if } 0 < k \leq \frac{1}{2\mu}. \end{cases}$$

Thus, it is clear that the cumulative in-degree distribution of the DST network for the logistic map follows a strict power law when $\Delta$ is enough small as follows:

$$P^{\geq}(k_{y}) \propto k^{-2}$$

In conclusion, the DST networks for the logistic map become “perfect” scale-free networks whose degree distribution strictly follows a power law with the exponent $\gamma = 1$, as the size of subintervals is getting smaller.
The cumulative in-degree distribution follows a law given by:

\[
P^>(k) \propto \left(\frac{1}{k} + \mu \Delta k\right)^2
\]

Consequently, in the case \(1/(8 + 16\Delta) \leq \mu \leq 1\), the cumulative in-degree distribution \(P^>(k)_d\) is given by:

\[
P^>(k) = \frac{16}{1 - 4|1 + 16\mu \Delta| + 32\mu(1 + 3\Delta + 8\mu \Delta^2)} \times \left(\frac{1}{k} + \mu \Delta k\right)^2.
\]

In the case \(0 < \mu < 1/(8 + 16\Delta)\), the distribution is given by:

\[
P^>(k) = \begin{cases} 
\frac{1}{4\mu(\mu + \Delta)} \left(\frac{1}{k} + \mu \Delta k\right)^2 & \text{if } \frac{1 - \sqrt{\Delta/\mu + 1}}{\Delta} < k \\
1 & \text{if } 0 < k \leq \frac{1 - \sqrt{\Delta/\mu + 1}}{\Delta}.
\end{cases}
\]

The second term makes a large effect on the distribution, only when \(k\) is quite close to \(k^\text{max}\) because \(0 < \mu \Delta < 1\).

It means that the “hub” states have higher in-degree than the typical scale-free network whose in-degree distribution follows a strict power law.
The cumulative in-degree distribution follows a law given by:

\[
P^>(k) \propto \left( \frac{1}{k} + \mu \Delta k \right)^2
\]

Consequently, in the case \(1/(8 + 16\Delta) \leq \mu \leq 1\), the cumulative in-degree distribution \(P^>(k_d)\) is given by:

\[
P^>(k) = \frac{16}{1 - 4\mu + 16\mu \Delta + 32\mu (1 + 3\Delta + 8\mu \Delta^2)} \times \left( \frac{1}{k} + \mu \Delta k \right)^2.
\]

In the case \(0 < \mu < 1/(8 + 16\Delta)\), the distribution is given by:

\[
P^>(k) = \begin{cases} 
\frac{1}{4\mu(\mu + \Delta)} \left( \frac{1}{k} + \mu \Delta k \right)^2 & \text{if } \frac{1 - \sqrt{\Delta/\mu + 1}}{\Delta} < k \\
1 & \text{if } 0 < k \leq \frac{1 - \sqrt{\Delta/\mu + 1}}{\Delta}.
\end{cases}
\]

Taking the limit

In the case \(1/(8 + 16\Delta) \leq \mu \leq 1\),

\[
\lim_{\Delta \to 0} P^>(k) = \frac{16}{-5 + 32\mu + 128\mu^2} k^{-2},
\]

and in the case \(0 < \mu < 1/(8 + 16\Delta)\),

\[
\lim_{\Delta \to 0} P^>(k) = \begin{cases} 
\frac{1}{4\mu^2} k^{-2} & \text{if } \frac{1}{2\mu} < k \\
1 & \text{if } 0 < k \leq \frac{1}{2\mu}.
\end{cases}
\]

The second term makes a large effect on the distribution, only when \(k\) is quite close to \(k^\text{max}\) because \(0 < \mu \Delta < 1\).

It means that the “hub” states have higher in-degree than the typical scale-free network whose in-degree distribution follows a strict power law.

In the case \(\Delta\) is extremely small (\(d\) is extremely large),

The cumulative in-degree distribution follows a power law:

\[
P^>(k) \propto k^{-2}
\]
Consequently, in the case $1/(8 + 16\Delta) \leq \mu \leq 1$, the cumulative in-degree distribution $P^>(k)$ is given by:

$$P^>(k) = \frac{16}{-5 + 32\mu + 128\mu^2} k^{-2},$$

and in the case $0 < \mu < 1/(8 + 16\Delta)$,

$$\lim_{\Delta \to 0} P^>(k) = \begin{cases} 
\frac{1}{4\mu^2} k^{-2} & \text{if } \frac{1}{2\mu} < k \\
1 & \text{if } 0 < k \leq \frac{1}{2\mu}.
\end{cases}$$

Taking the limit

In the case $0 < \mu < 1/(8 + 16\Delta)$, the distribution is given by:

$$P^>(k) = \begin{cases} 
\frac{1}{4\mu(\mu + \Delta)} \left( \frac{1}{k} + \mu\Delta k \right)^2 & \text{if } \frac{1 - \sqrt{\Delta/\mu + 1}}{\Delta} < k \\
1 & \text{if } 0 < k \leq \frac{1 - \sqrt{\Delta/\mu + 1}}{\Delta}.
\end{cases}$$

Summarizing

The cumulative in-degree distribution follows a law given by:

$$P^>(k) \propto \left( \frac{1}{k} + \mu\Delta k \right)^2$$

In the case $\Delta$ is extremely small ( $d$ is extremely large),

The cumulative in-degree distribution follows a power law:

$$P^>(k) \propto k^{-2}$$
Comparison between the results of numerical computations and mathematical predictions with $\mu = 0.1$ and $\mu = 1.0$, in the case $d = 6$
Are there any other maps whose state-transition networks are scale-free networks?

Yes!
One-dimensional maps

Sine map

\[ f(x) = \mu \sin(\pi x) \]

Cubic map #1

\[ f(x) = 3\sqrt{3} \mu x (1 - x^2)/2 \]

Cubic map #2

\[ f(x) = 27 \mu x^2 (1 - x)/4 \]
The state-transition networks for other one-dimensional maps

Logistic map
$\mu = 1.0, d = 3$

Sine map
$\mu = 1.0, d = 3$

Cubic map #1
$\mu = 1.0, d = 3$

Cubic map #2
$\mu = 1.0, d = 3$
The cumulative in-degree distributions of state-transition networks for other one-dimensional maps

The dashed line has slope -2
The networks are scale-free networks with the degree exponent $\gamma = 1$. 
One-dimensional maps

General Symmetric map

\[ f(x) = \mu \left(1 - |2x - 1|^\alpha\right) \]

The parameter \( \alpha \) is controlling the flatness around the top.

\( \alpha = 2.0 \): the logistic map
The state-transition networks for the general symmetric map

\[ \alpha = 1.5 \quad \alpha = 2.0 \quad \alpha = 2.5 \]

\[ \alpha = 3.0 \quad \alpha = 3.5 \quad \alpha = 4.0 \]

\[ d = 3 \]
The cumulative in-degree distributions of state-transition networks for the general symmetric map

The parameter $\alpha$ influences the degree exponent $\gamma$, while the scale-free property is maintained.
Other types of maps

Gaussian map —— exponential
\[ f(x) = b + e^{-ax^2} \]

Sine-circle map —— discontinuous
\[ f(x) = x + b - (a/2\pi)x \pmod{1} \]

Delayed logistic map —— two-dimensional
\[ f(x, y) = \left(ax(1 - y), x\right) \]
The state-transition networks for the other types of maps

Sine-circle map  
$a = 4.0$, $b = 0.5$, $d = 3$

Gaussian map  
$a = 1.0$, $b = -0.3$, $d = 3$

Delayed logistic map  
$a = 2.27$, $d = 2$
The cumulative in-degree distributions of state-transition networks for the other types of maps

The dashed line has slope -2
the networks are scale-free networks with the degree exponent $\gamma = 1$. 

$$d = 6 \quad d = 3$$
Chaotic maps that have scale-free state-transition networks

<table>
<thead>
<tr>
<th>Map</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logistic map</td>
<td>one-dimensional</td>
</tr>
<tr>
<td>Sine map</td>
<td>one-dimensional</td>
</tr>
<tr>
<td>Cubic map</td>
<td>one-dimensional</td>
</tr>
<tr>
<td>Gaussian map</td>
<td>one-dimensional, exponential</td>
</tr>
<tr>
<td>Sine-circle map</td>
<td>one-dimensional, discontinuous</td>
</tr>
<tr>
<td>Delayed logistic map</td>
<td>two-dimensional</td>
</tr>
</tbody>
</table>
Do all chaotic maps have scale-free state-transition networks?

No.
NOT scale-free state-transition networks of chaotic maps

Tent map
\( a = 2.0, d = 3 \)

Binary Shift map
\( d = 3 \)

Gingerbreadman map
\( d = 1 \)
NOT scale-free state-transition networks of chaotic maps

Cusp map
\[ a = 2.0, \quad d = 4 \]

Pincher map
\[ a = \cdot, \quad d = 4 \]

Henon map
\[ a = 1.4, \quad b = 0.3, \quad d = 2 \]

Lozi map
\[ a = 1.7, \quad b = 0.5, \quad d = 2 \]

Henon area-preserving map
\[ a = 0.24, \quad d = 2 \]

Standard (Chirikov) map
\[ a = 1.0, \quad d = 1 \]
Discretized-State-Transition (DST) Networks

- node (state)
- link (transition)
- connected components (basins of attraction)
- loop (fixed point)
- cycle (periodic cycle)

- building networks

- in-degree distribution
  - probability $P(k)$
  - log-log plot
  - scale-free networks

- functions:
  - Logistic map
  - Sine map
  - Cubic map
  - Sine-circle map
  - Gaussian map
  - Delayed logistic map
Papers & Presentations


Dynamics as Networks
the dynamics of system/phenomena are represented as networks

- Chord Networks of Music
- Chord-Transition Networks of Music
- Collaboration Networks of Wikipedia
- Collaboration Networks of Linux
- Co-Purchase Networks of Books, CDs, DVDs
- State Networks of Chaotic Dynamical Systems
want to capture the process / dynamics of phenomena as a whole.

introducing network analysis in a new way.

to build a directed network by connecting between nodes, based on the sequential order of occurrence.

a new viewpoint “dynamics as network”, which is a distinct from “dynamics of network” and “dynamics on network.”
Network Analysis for Understanding Dynamics
- Wikipedia, Music & Chaos -

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