THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

IEICE Technical Report
PEM2015 FCB-3 (2015-11)

Permittivity measurements using a microwave resonator with a sample insertion mechanism

Yuto KATO† Masahiro HORIBE†

† National Metrology Institute of Japan, National Institute of Advanced Industrial Science and Technology

1-1-1 Umezono, Tsukuba, Ibaraki, 305-8568 Japan
E-mail: † {y-katou, masahiro-horibe}@aist.go.jp

Abstract Recently a new resonant method has been proposed. This method uses a resonator with a sample insertion mechanism. By adjusting the sample volume in a cavity, the magnitudes of changes in resonant properties can be controlled. By using this method, the permittivity can be perturbatively measured and evaluated even for high-loss or high-permittivity samples. In this paper, we characterized a cyclic olefin copolymer (COC) at 6.5 – 9.5 GHz frequency region by measuring TE_{10n} modes as a sample is inserted into the rectangular cavity, and quantitatively confirmed that the new method not only improved the accuracy but also reduced the uncertainty. This method is also expected to use for characterizing biomedical materials.

Keywords Permittivity measurement, Resonator, Uncertainty, Nonuniform field

1. Introduction

Broadband measurements of the complex permittivity are required for a wide range of applications ranging from evaluation of electronic material properties to qualification of agricultural products. Resonant methods are suitable for characterization of low-loss materials in a high-frequency region. In resonant methods, the relative permittivity is determined from the shift of the resonant frequency of a resonator with and without a sample. The loss tangent can be derived from the difference of the Q-factor with and without a sample in a resonator. Though they are the most accurate measurement method for low-loss materials, they cannot apply for non-perturbative samples such as high-loss or high-permittivity materials.

Recently a new resonant method has been proposed [1]. This method uses a resonator with a sample insertion mechanism. By adjusting the sample volume in a cavity, the magnitudes of changes in resonant properties can be controlled. Therefore, even for high-loss or high-permittivity samples, the permittivity can be perturbatively measured and evaluated. As a sample is inserted into a cavity, the resonant frequency and the Q-factor are decreased. The shifts of the resonant frequency and the inverse of the Q-factor are linearly related to both of the sample effective filling factor and the insertion length. The real part of the permittivity is calculated from the slope of the linear relationship for the resonant frequency, and the imaginary part can be derived from the slope for the inverse of the Q-factor. This technique has the potential to characterize high-loss samples including biomedical materials by resonator methods.

The purpose of this work is to test this new method. We designed and built a resonator with a sample insertion mechanism to confirm the validity of the modified perturbation analysis. Our resonator system is expected to use for characterizing high-loss or high-permittivity samples in the future. To this end, we performed preliminary experiments of the resonator by measuring a low-loss material as a first step of this study.

2. Method

This new method involves the modification of the treatment of the perturbation equation. In the perturbation analysis, we define three parameters,

$$x \equiv \frac{V_s'}{V_c}, \qquad y_r \equiv \frac{f_c - f_{cs}}{f_c}, \qquad y_i \equiv \frac{1}{Q_{cs}} - \frac{1}{Q_c} \qquad (1)$$

where V_c is the volume of the cavity, $V_s' = ht'w'$ is the effective volume of the sample, h is the insertion length of the sample, f_c, Q_c are the resonant frequency and the Q-factor of the cavity without the sample, and f_{cs}, Q_{cs} are the resonant frequency and the Q-factor of the cavity with the sample. The effective sample thickness t' and width w' are written by

$$t' = \frac{t}{2} + \frac{d}{2\pi}\sin\left(\frac{\pi t}{d}\right), w' = \frac{w}{2} + \frac{l}{2\pi n}\sin\left(\frac{n\pi w}{l}\right) \qquad (2)$$

where w and t are the width and thickness of the sample, and l and d are the dimensions of the cavity [1]. In (1), x is the relative volume fraction of the sample proportional to the insertion length h, y_r is the relative frequency shift, and y_i is the sample damping ratio. According to the perturbation theory, the relationships between x and y_r, y_i are given by

$$y_r = (\epsilon_r^s - 1)2x - b_r \quad (3)$$

$$y_i = (\epsilon_i^s)4x - b_i \quad (4)$$

where $\epsilon^s = \epsilon_r^s - j\epsilon_i^s$ is the complex permittivity of the sample, and $b = b_r - jb_i$ is the result of integration over the nonuniform fields inside the sample. In the conventional method [2], the effect of the non-uniform field is not taken into account, so b is assumed to be zero. If b is zero, the permittivity can be derived from full insertion and empty measurements by using $\epsilon_r^s = y_r/2x + 1$ and $\epsilon_i^s = y_i/4x$. On the other hand, in the new method [1], the non-uniform fields are considered, but there is one assumption; b is assumed to be constant when the filling factor is moderate. This means that there are linear regions between the insertion length and the changes in resonant properties. The permittivity can be derived from slopes of these linear relationships.

In order to verify the new method, we characterized a low-loss material by using a waveguide resonator with a sample insertion mechanism (Fig. 1). The waveguide size is WR-90 used for an X-band frequency region. The resonator length is 32.84 mm determined from the empty measurement of the cavity. We measured TE_{10n} mode resonances. The Q-factor without a sample is about 5000 – 7000 which becomes larger as a mode number is increased. The sample insertion hole is located at the center of the resonator, so we used odd number modes (3 – 15), because the amplitude of the electric field takes the maximum at the center for these modes, but TE_{101} mode was not used, because it is obscured by the cutoff frequency. The resolution of the sample insertion mechanism is 10 µm. The inserted sample is parallel to the electric field in the resonator. We used a cyclic olefin copolymer (COC) as a sample under test. This is a low-loss material used as a dielectric material in a coaxial cable. The width (w) and thickness (t) of the sample were 1.03 ± 0.01 mm and 1.02 ± 0.01 mm, respectively. According to the data sheet from the manufacturer, the relative permittivity and the loss tangent of the measured sample at 1 GHz are 2.3 and 1×10^{-4}, respectively.

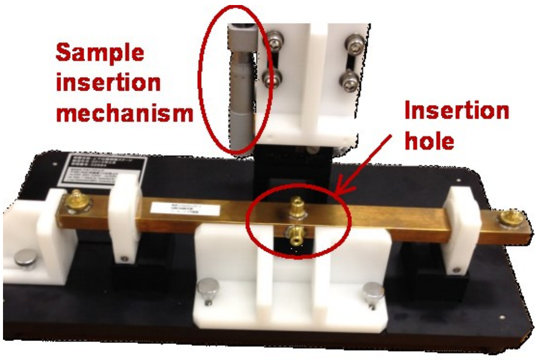

Fig. 1 Photo of the WR-90 rectangular waveguide cavity with the control mechanism for the sample.

3. Measurement result

Figure 2(a) shows the resonant traces of S_{21} at $\mathrm{TE}_{10,15}$ mode resonance. As the sample is inserted into the cavity, the resonant properties are continuously shifted. In Fig. 2(b), we plot the resonant frequency f_{cs} and the inverse of Q-factor $1/Q_{cs}$ against the insertion length. We can see the linear regions between the resonant properties and the insertion length, but there are also transition regions, or non-linear regions, when the sample is almost full-inserted or almost outside of the cavity.

The permittivity at $\mathrm{TE}_{10,15}$ mode resonance is calculated from the conventional and new methods, as summarized in Table 1. In the conventional method, which uses the difference between full-insertion and empty measurements, non-uniform fields or the transition regions are not considered. In the new method, which uses the slopes of linear regions, the transition regions are excluded for a linear fitting. Since we confirmed the transition region, we expect the permittivity from the new method is more reliable.

Table 1 Permittivity of COC at $\mathrm{TE}_{10,15}$ resonance.

Analysis	Transition region	ϵ_r^s	ϵ_i^s
New	Considered	2.42	9.5×10^{-4}
Conv.	NOT considered	2.22	7.0×10^{-4}

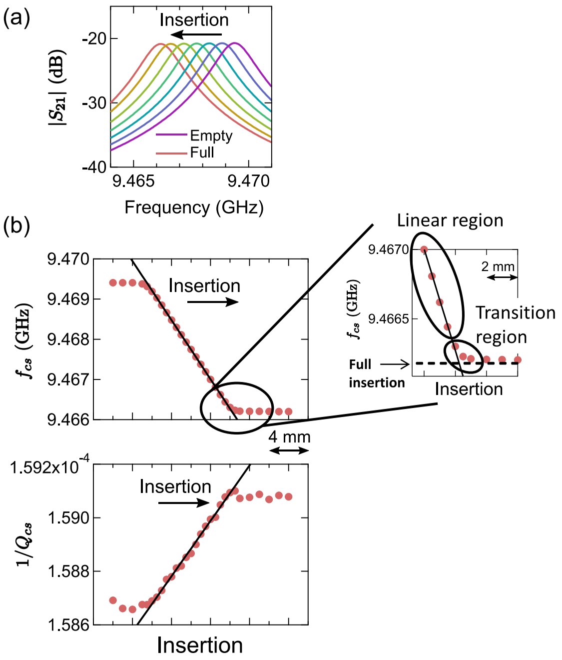

Fig. 2 (a) Resonant traces of S_{21} at $\mathrm{TE}_{10,15}$ mode for different insertions. (b) Resonant frequency and inverse of Q-factor plotted against the insertion length.

Figure 3(a) and (b) show real and imaginary part of permittivities, respectively, measured for TE_{103} to $TE_{10,15}$ modes calculated from the conventional and new methods. In Fig. 3(a), we can see the difference between two perturbation analyses. We expect the new analysis is more reliable because it considers the transition region, but results are slight deviated from the data sheet value ($\epsilon_r^s = 2.3$ at 1 GHz). Although the reason is not clear, the effect of leakage fields through a insertion hole should be considered.

Figure 4(a) and (b) show the uncertainties of the real and imaginary part of permittivities, respectively, calculated from the conventional and new methods. Uncertainties are evaluated by considering the following contributions: uncertainties of resonant properties [3], those of dimensions of the sample and cavity, uncertainty of sample position, and the fitting uncertainty. Although $u(\epsilon_r^s)$ for two methods are nearly the same, $u(\epsilon_i^s)$ is considerably reduced by using the new method. This is attributed to a linear fitting process. In a linear fitting, several results for different insertions are analyzed, so the effect of the Q-factor uncertainty is moderated in the new method.

4. Conclusion

We built and tested a waveguide resonator with a sample insertion mechanism. We confirmed the resonant properties are linearly related to the insertion length when the filling fraction of a sample in a cavity is moderate. We also clarified the modification of analysis has an impact on the permittivity determination and reduces the uncertainty. Note that the linear relationship is, to the best of our knowledge, only an experimental fact, or an empirical fact, so we need more considerations to rigorously verify the new analysis with a linear fitting.

References

[1] N. D. Orloff, J. Obrzut, C. J. Long, T. Lam, P. Kabos, D. R. Novotny, J. C. Booth, and J. A. Liddle, “Dielectric Characterization by Microwave Cavity Perturbation Corrected for Nonuniform Fields,” *IEEE Trans. Microw. Theory Techn.*, vol. 62, no. 9, pp. 2149-2159, Sep., 2014.

[2] C. G. Montgometry, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits*, Stevenage, U. K.: IET, 1948.

[3] Y. Kato and M. Horibe, “Comparison of Calculation Techniques for Q-Factor Determination of Resonant Structures Based on Influence of VNA Measurement Uncertainty,” *IEICE Trans. Electron.*, vol. E97-C, no. 6, pp. 575-582, Jun., 2014.

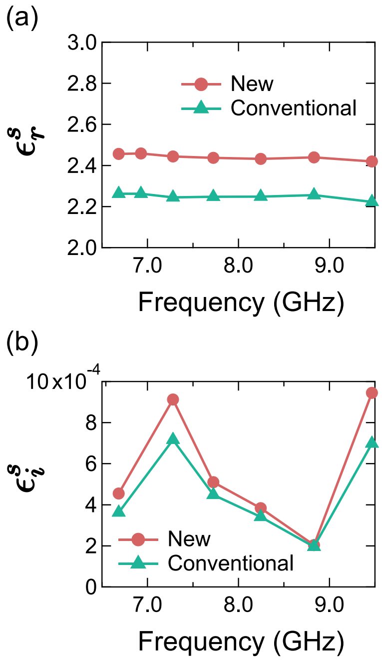

Fig. 3 (a) Real and (b) imaginary part of the permittivity of a COC sample at TE_{103} to $TE_{10,15}$ modes.

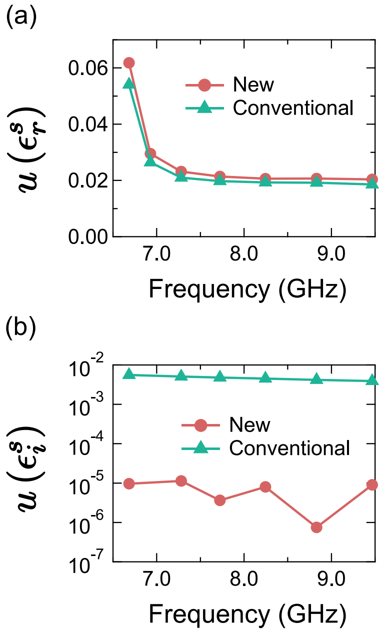

Fig. 4 Uncertainty of (a) Real and (b) imaginary part of the permittivity of a COC sample at TE_{103} to $TE_{10,15}$ modes.