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The Switched Injection-Locked Oscillator (SILO) Concept

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Abstract—A Switched Injection-Locked Oscillator (SILO) is one which exploits parameter and topology switching to facilitate one or more of the following features: (i) fast startup and quenching of an oscillation at a well-defined frequency and (ii) frequency division by a programmable integer. This paper describes an example LC oscillator circuit and associated switching and injection-locking mechanisms which accomplish these results.

1. Introduction

There is an increasing range of applications for simple oscillators whose frequency of oscillation is well-controlled and that can be started and stopped quickly. Injection-locking [1, 2] can be used to set the frequency precisely. Parameter and topology switching can be used to facilitate fast startup and quenching. The two techniques can be combined in a Switched Injection-Locked Oscillator (SILO) [3] which features both precise frequency control and fast start and stop.

2. Circuit under consideration

For the purposes of illustration, the circuit we consider in this work is the CMOS LC oscillator with tail injection shown in Fig. 1 [4, 5]. We expect that the ideas we illustrate can be applied to a wide range of oscillators which exhibit equivalent qualitative behavior.

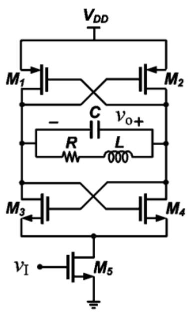

Figure 1: CMOS LC oscillator with tail injection.

This circuit comprises five transistors and a resonant tank circuit. The tank contains a capacitor C, an inductor L, and a linear resistor R. The five transistors M_1–M_5 form a negative resistor whose driving-point (DP) characteristic can be approximated reasonably well by a cubic nonlinear function $f(\cdot,\cdot)$, defined by

$$f(V_N, V_i) = -a(V_i)\, V_N \left(1 - \left(\frac{V_N}{V_{DD}}\right)^2\right), \quad (1)$$

where V_N is the voltage across the nonlinear resistor and V_i is an externally-applied input signal that controls the slope a of the DP characteristic at the origin.

The simplified circuit diagram is shown in Fig. 2.

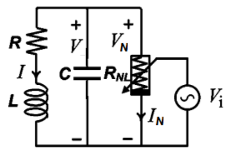

Figure 2: Simplified circuit diagram of CMOS LC oscillator with tail injection.

This circuit is described by two differential equations:

$$C\frac{dV}{dt} = -f(V, V_i) - I \quad (2)$$

$$L\frac{dI}{dt} = V - RI \quad (3)$$

where $f(\cdot,\cdot)$ is as defined in (1).

3. Normalized State Equations

By applying the linear transformation of the state variables and time

$$X = \frac{V}{V_{DD}}, \quad Y = \frac{RI}{V_{DD}}, \quad \tau = \frac{t}{\sqrt{LC}}, \quad (4)$$

and defining parameters

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}, \quad G(V_i) = R\, a(V_i), \quad \omega = \omega_i\sqrt{LC}, \quad (5)$$

the state equations can be reduced to a pair of ordinary differential equations as follows [4, 5]:

$$\frac{dX}{d\tau} = Q\left(G(V_i)X(1 - X^2) - Y\right) \quad (6)$$

$$\frac{dY}{d\tau} = \frac{1}{Q}(X - Y). \tag{7}$$

4. Bifurcation Diagram

The qualitative behavior of the circuit is summarized in the bifurcation diagram shown in Fig. 3.

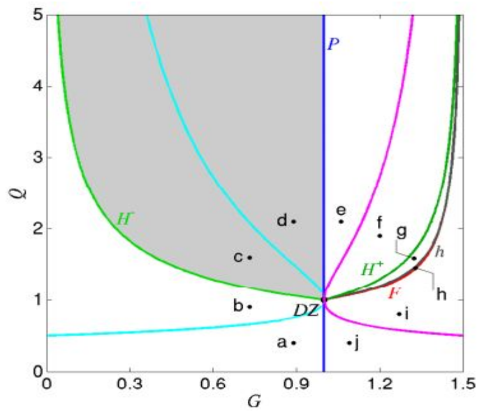

Figure 3: Two-parameter bifurcation diagram of the normalized circuit model.

In the gray-shaded area in Fig. 3 (bounded by curves H^- and P labeled as c and d), the circuit admits a unique stable limit cycle; this region is the traditional "design area" for the unforced sinusoidal oscillator. H^- denotes a supercritical Hopf bifurcation [6] and corresponds to the classical condition for oscillation, the so-called "startup condition." P is the boundary representing the transition from one equilibrium point (in regions a, b, c and d) to three equilibria (in regions e, f, g, h, i and j); it corresponds to a pitchfork bifurcation.

The qualitative behavior of the normalized circuit is shown in Fig. 4 [6]. In regions a and b, the circuit has a single stable equilibrium point; it is a stable node in region a and a stable focus in b. In regions c and d, the circuit has a single unstable equilibrium point which is surrounded by a stable limit cycle; it is an unstable focus in region c and an unstable node in region d.

5. G switching

We first consider the effect of switching the parameter G. Our objective is to start and stop the oscillator by changing the stability of the operating point. When the circuit is in region c, it oscillates with a unique limit cycle, while the oscillation is quenched in regions a and b.

We wish to start and stop the oscillator quickly and, in addition, have it run at a well-defined frequency; we will use injection-locking to accomplish the latter. We use G-switching to change the stability of the operating point. This is accomplished by switching the bias voltage applied

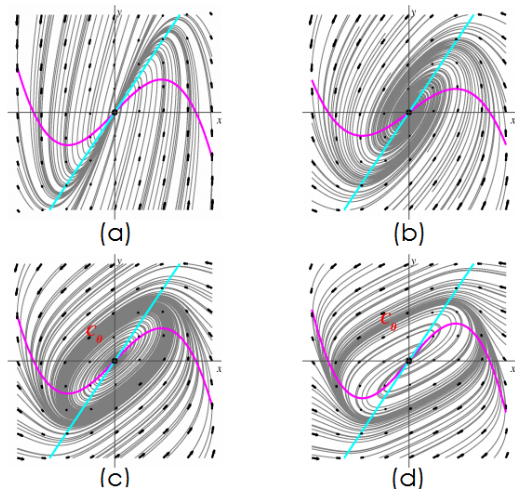

Figure 4: Flows in regions a, b, c, and d. (a) Stable node; (b) stable focus; (c) unstable focus; (d) unstable node.

to the gate of transistor M_5 in Fig. 1. A small periodic signal v_i is added in series with the dc bias to injection-lock the oscillator.

5.1. Example 1

Consider the case where Q is fixed ($Q = 2.1$) and G is switched between 0.20 and 0.26 (corresponding to points 1 and 2 in Fig. 5).

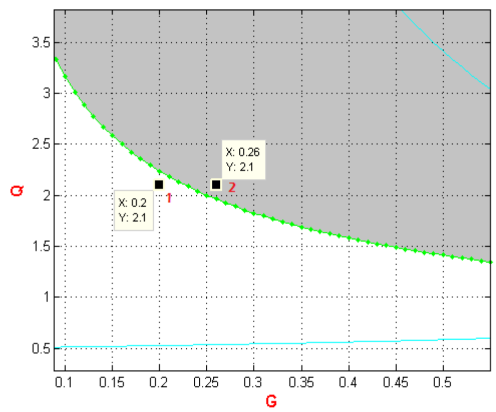

Figure 5: Zoom of two-parameter bifurcation diagram of the normalized circuit model.

The top two traces in Fig. 6 show the simulated input signal (switching between 3.0 V and 3.7 V with a period of 300.02 μs and a duty cycle of 66.67%; the superimposed injection signal is 1 V pk-pk at f_s = 520 kHz) and the transient response of this circuit.

Note that the oscillation decays rapidly when the circuit is moved into region b. The oscillation is relatively slow to start in region c because it must grow from zero each time. The oscillation would reach steady-state faster if it were started closer to the limit cycle. The lower trace in Fig. 6

shows what happens when the capacitor is pre-charged to a voltage that is close to the limit cycle. Note that the startup is significantly faster than the middle trace, and that the period of the oscillation in steady-state is once again determined by the injection signal. Thus, a combination of G-switching, injection-locking, and pre-charging can be used to make an oscillator start, oscillate at a prescribed injection frequency, and stop very quickly.

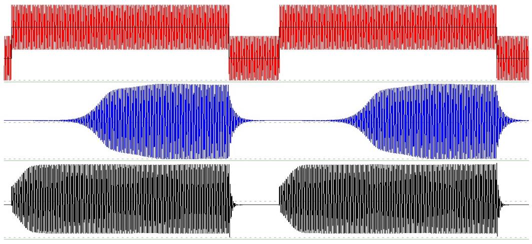

Figure 6: Transient simulation of G switching with injection-locking without and with pre-charging. The frequency in steady-state is $f_s = 520$ kHz. Top: injected signal $V_I(t)$; middle: output signal $V(t)$ without pre-charging; bottom: output with pre-charging showing faster startup.

6. ρ switching

In this section, we consider the injection-locked oscillator acting as a frequency divider [7]. The principle of operation is shown schematically in Fig. 7. The free-running

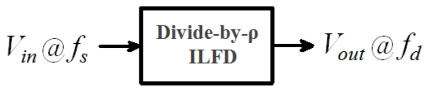

Figure 7: Injection-locked frequency divider.

oscillator oscillates with frequency f_0. When an input signal at frequency f_s is applied, the output of the driven oscillator has an average zero-crossing rate f_d. The ratio f_s/f_d is called the *rotation number* (denoted ρ). Under suitable conditions, ρ may be constant over a range of input frequencies, called the locking range. When operating in this mode, the circuit is called an injection-locked frequency divider (ILFD). In particular, when ρ takes on integer values, the ILFD is commonly called a "divide-by-ρ" circuit.

6.1. Example 2

The circuit shown in Fig. 1 has been studied extensively in the literature [4, 5]. In particular, Daneshgar *et al.* have determined the Arnold tongues (locking regions) corresponding to rotation numbers of 2, 4, and 6; these are summarized in Fig. 8. To interpret this diagram, consider a sinusoidal injection voltage v_i having an amplitude of 1.0V. The ILFD divides by 2 when the input frequency f_d

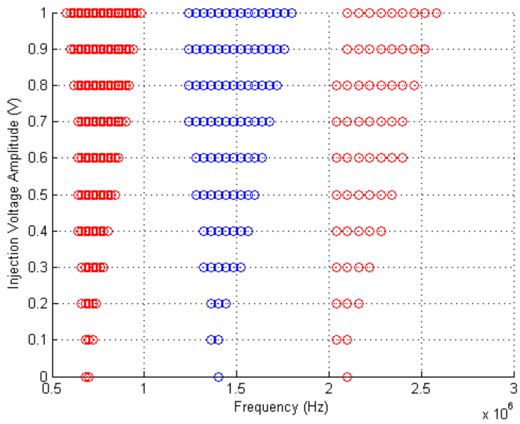

Figure 8: Bifurcation diagram showing SPICE simulations of Arnold tongues (locking regions) corresponding to (from left to right) ρ = 2, 4 and 6.

lies in the range of 550 kHz to 1.0 MHz, it divides by 4 when 1.25 MHz $\leq f_d \leq$ 1.75 MHz, and by 6 for 2.1 MHz $\leq f_d \leq$ 2.6 MHz.

Assume that we wish the ILFD to divide by 4 instead of 2 in the range 600 kHz $\leq f_d \leq$ 900 kHz. By changing the value of f_0, the circuit can be switched to the "divide-by-4" regime. In practice, it is difficult to change the value of L; it is easier to change the values of C, R and $a(\cdot)$. With an appropriate rescaling of these parameters, as shown schematically in Fig. 9, f_0 can be changed without affecting G or Q. Thus, the qualitative locking behavior of the ILFD is unchanged, as shown in Fig. 10.

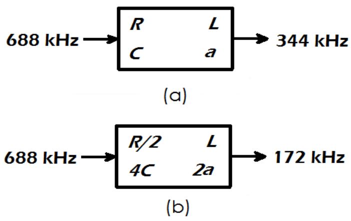

Figure 9: ρ-switched ILFD. (a) divide-by-2 parameter values; (b) divide-by-4 parameter values.

Fig. 11 shows a circuit implementation of the idea described schematically in Fig. 9. Switches are used to change the values of the resistor, the capacitor, and the slope of the nonlinearity. Fig. 12 shows the output switching between 344 kHz and 172 kHz in response to the control signal, when a constant injected signal at 644 kHz is applied.

7. Conclusion

In this paper, we have exploited knowledge of the bifurcation diagram of a CMOS LC oscillator with tail injection

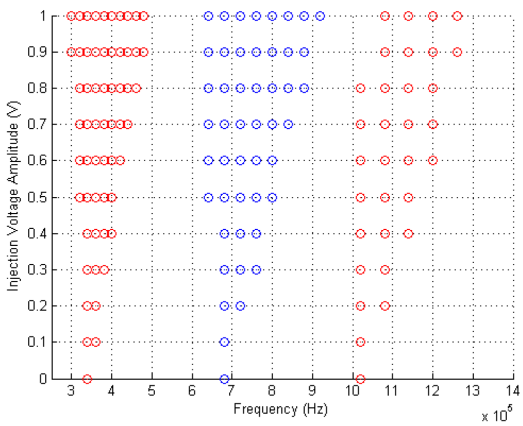

Figure 10: Bifurcation diagram showing SPICE simulations of Arnold tongues in the ρ-switched circuit corresponding to (from left to right) ρ = 2, 4 and 6. f_0 has been halved relative to Fig. 8 so that the ILFD divides by 4 instead of 2 in the range 600 kHz $\leq f_d \leq$ 900 kHz. .

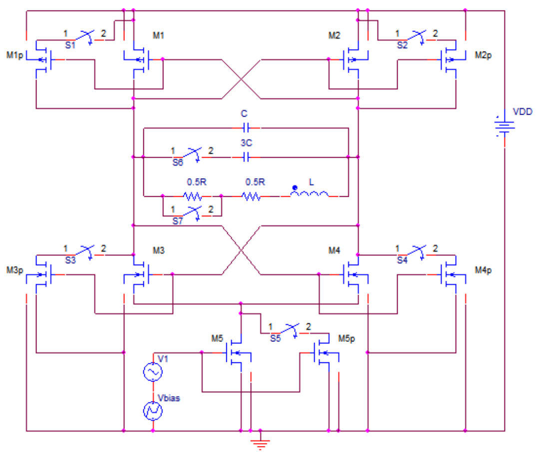

Figure 11: CMOS LC oscillator with tail injection and ρ-switching.

to show qualitatively how to start and stop the oscillator by changing the stability of the unique operating point. This is accomplished by switching the value of the normalized parameter G between two values using an injection signal V_I. Furthermore, the frequency of the steady-state oscillation can be injection-locked to a small superimposed injection signal applied to transistor M_5. We call this G-switching.

When a periodic signal is applied to the input V_I, the injection-locked oscillator behaves as a frequency divider. The ratio of the zero-crossing rates of the input and output signals is denoted by ρ. By changing the parameters of the circuit, the ILFD can be switched from one value of ρ to another, thereby implementing ρ-switching. This is equivalent to changing the division ratio in a programmable frequency divider.

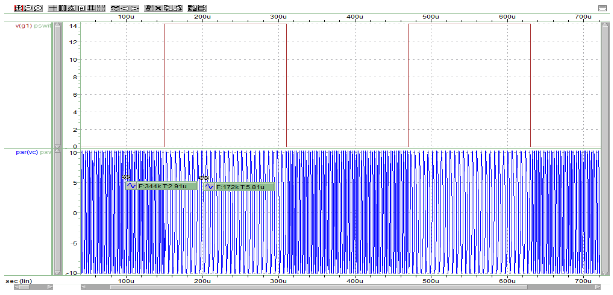

Figure 12: Transient simulation of ρ switching with a 688 kHz injection signal. Top: switch control signal; bottom: output showing injection-locked frequency switching between divide-by-4 (172 kHz) and divide-by-2 (344 kHz).

Acknowledgments

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