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Bursting oscillations in a memristor-based dynamic model

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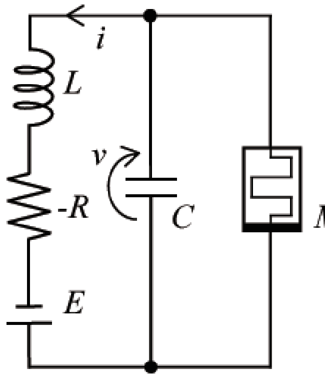
Abstract—Memristive resistor (memristor) has attracted many researchers' attentions. In this study, we report that there exist various bursting oscillations in a memristor-based simple dynamic model. The successive bursts are triggered by a gradually increasing memory resistance (memristance) in the model. An average interburst interval of the bursting oscillations is measured. In addition, the other oscillatory phenomena around the boundary existing regions of the bursting oscillations are shown.

1. Introduction

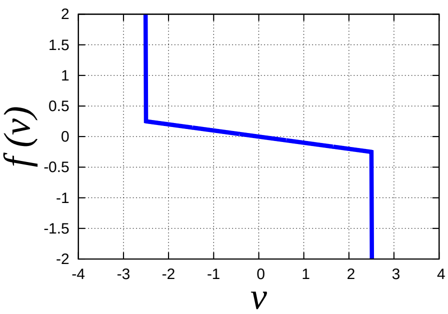
Memristive resistor is an electronic component with resistance that changes over time as a function of existing state and the voltages applied across them and/or currents driven through them. Such element (named as "memristor") was first predicted by L.O. Chua in 1971 [1], and the implementation was reported by a team of Hewlett-Packard company in 2008 [2]. Diverse applications for the circuit element are expected in a wide variety of fields [3–11]. One of the examples is for neuromorphic learning systems, because the memristor is regarded as a prospective candidate for developing nonvolatile analog synaptic devices in a small size [4, 8, 9].

In our previous study [10], we studied a simple memristor-based circuit model which is in the modified form of the model [11], and reported memristive behaviors with periodic injections of a pulse voltage. The model consists of a few circuit elements (one linear resistor, one inductor, one capacitor, one voltage source, and one voltage-controlled memristor). In this study, we will study a dynamic model by replacing the linear resistance in the circuit by a negative one, and by applying a DC backward voltage source instead of the periodic pulse voltages. We are interested in what happens if the above two parameter values (the negative resistance and the DC backward voltage) are changed in this dynamics.

In this paper, we investigate the memristor-based dynamic model numerically, and report that there exist interesting nonlinear oscillations in the dynamics. In particular, we will show that various bursting oscillations appear continuously in a certain parameter regime. If we apply a DC backward bias voltage to the dynamics, the memristance gradually increases with time, and eventually triggers the burst oscillations of the other state variables. An occurrence rate of the bursts can be varied by changing the negative resistance value and/or the value of the DC voltage.



(a) Schematic drawing.



(b) Memristor function $f(v)$ ($\alpha = 0.1$ and $\beta = 200$).

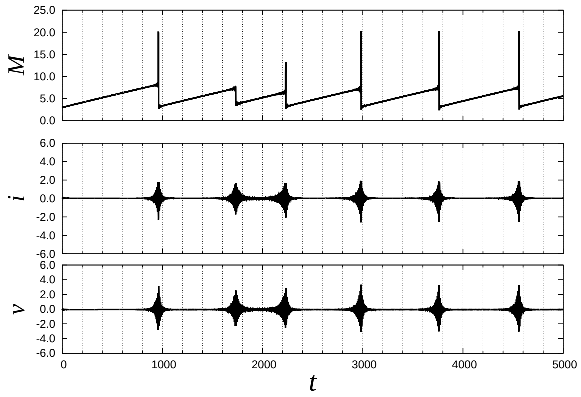
Figure 1: The circuit model.

2. Circuit model

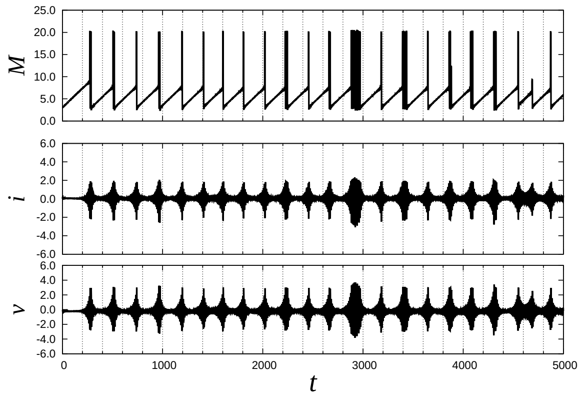
Figure 1(a) presents a schematic diagram of the circuit model in which a capacitor C and a series of some elements (an inductor L, a negative resistor $-R$, and a DC backward voltage source E) are connected in parallel to a voltage-controlled memristor. Referring to [11], we adopt the simple memristor model which takes into account the activation change of its state. The memristance M changes between two limiting values M_1 and M_2 ($0 < M_1 < M_2$). Then, the time-varying state of M can be written as follows:

$$\frac{dM}{dt} = f(v)[\theta(v)\theta(M - M_1) + \theta(-v)\theta(M_2 - M)], \quad (1)$$

where v is the voltage applied across the memristor, $f(v)$ represents the voltage-controlled memristor characteristic



(a) $E = 0.05$.



(b) $E = 0.2$.

Figure 2: Time series plots of the state variables for $E = 0.05$ and $E = 0.2$ ($R = 0.4$).

(memristor function), and $\theta(\cdot)$ is a step function. The memristor function $f(v)$ is written as in the form: $f(v) = -\beta v + 0.5(\beta - \alpha)(|v + V_T| - |v - V_T|)$, where α and β are positive constants, and V_T is a threshold voltage.

From the Kirchhoff's law, the circuit equation can be represented as follows:

$$\frac{dv}{dt} = \frac{\left(-i - \dfrac{v}{M}\right)}{C}, \qquad \frac{di}{dt} = \frac{v + Ri + E}{L}. \tag{2}$$

In the following results, we adopt the same parameter values in [11]: $M_1 = 3.0\Omega, M_2 = 20.0\Omega, \beta = 200, \alpha = 0.1, V_T = 2.5$V, $L = 2$ H and $C = 1$F, and employ the values of $-R$ [Ω] and E [V] as control parameters. We solve the Eqs.(1) and (2) by using Runge-Kutta method with a step size $h = 0.01$[1]. The initial condition (M_0, v_0, i_0) is set to $M_0 = 3.0$ and $v_0 = i_0 = 0.0$, throughout this study.

[1]The order of complexity of the system is 3 referring to the Eq.(18) in [1]. From the viewpoint of the memristor constitutive relation [12], the state variables (v, i, φ(the flux)) may be appropriate. In this study, we assume $M(t)$ in Eq.(1) instead of the memductance $W(\varphi)$ to describe the voltage–current characteristic of the memristor.

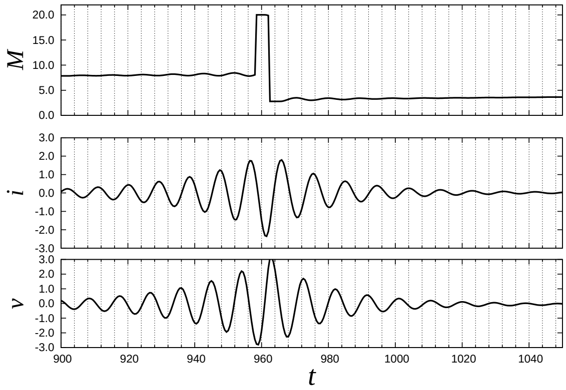


Figure 3: Magnified drawing of the first bursting oscillation of Fig.2 (a).

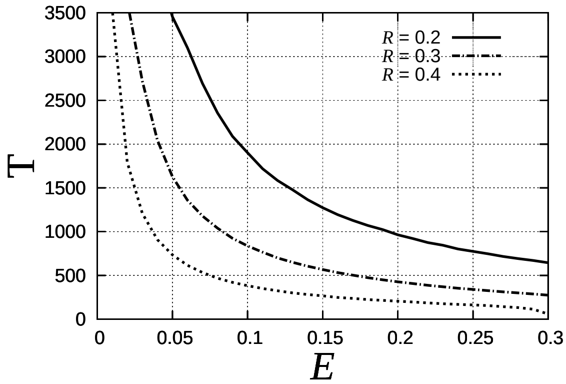


Figure 4: Average interburst interval T in terms of E for three values of R.

3. Bursting oscillations

When we apply the DC backward bias voltage $E = 0.1$ in Fig. 1(a), various bursting oscillations appear in terms of the value of $-R$[2]. Figures 2 (a) and (b) show typical examples of the bursting oscillations for $R = 0.05$ and 0.2, respectively. In the results, the value of M gradually increases with time, and the large amplitude of oscillations of v and i (v– and i–bursts, respectively) appear in association with increasing in M. To show what happened when $|v|$ exceeded the threshold value V_T more in detail, in Fig.3, we present magnified drawing of the bursting oscillations of Fig.2 (a). When the $|v|$ exceeds the threshold value V_T, the value of M suddenly increases or decreases which is

[2]To observe various bursting oscillations as shown in this section, we adopt the negative resistor and the backward DC voltage source in Fig.1(a).

shown at a certain time. This is because the memristor is characterized by the piecewise-linear nonlinearity as shown in Fig 1 (b) for $\alpha = 0.1$ and $\beta = 200$, and the M changes rapidly when $|v| > V_T$, and moves slower when $|v| < V_T$.

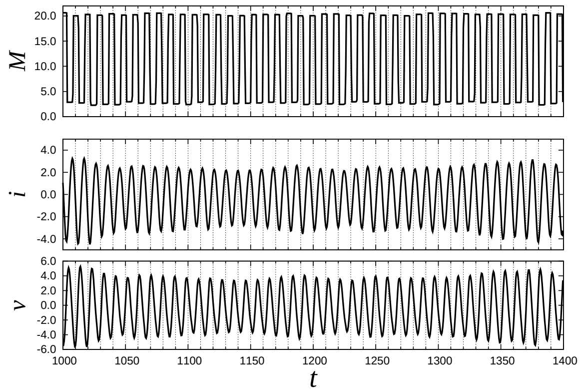
When we change the value of E, the rise velocity of the M is changed, and an occurrence rate of the v– and i–bursts can be varied as a consequence. For $E = 0.2$, compared with the result for $E = 0.05$, frequent occurrences of the bursts are observed in the same time domain. We measure an average interburst interval (= T) of such bursting oscillations for $0 \leqq t \leqq 10^6$. The T corresponds to a long-time average interval between two neighboring v–bursts. Figure 4 shows the T in terms of E for three values of R. From the figure, the T becomes smaller for larger value of E in any of three cases. For further large E, the bursting oscillatory phenomenon disappears. Instead, the other solution presented in Fig. 5 is observed. There exists fluctuation in the time series plots of the state variables as shown in Fig. 5(a), and the corresponding trajectory and the points on the Poincaré mapping in the v–i plane (we take mapped points when the flow of the v penetrates the hyperplane from + to −) is presented in Fig. 5(b)[3]. From the figures, the amplitude of oscillation fluctuates over time although the oscillation period is nearly periodic.

The bursting oscillations appear for $0.12 \leqq R \leqq 0.43$ as far as our numerical results are concerned. Around the upper boundary, there exist turbulent bursts. The turbulent bursts mean that a disordered large amplitude of oscillation appears irregularly. For $R = 0.44$ and 0.45, such turbulent bursts are observed as shown in Figs. 6(a) and (b), respectively. Figure 7 shows the magnified drawing of the oscillation in Fig.6 (b). On the other hand, around the lower boundary of the existing region of the bursting oscillations, the other kind of periodic oscillation appears. Figure 8 shows that there exists the nonlinear periodic oscillation as a steady-state after transient.

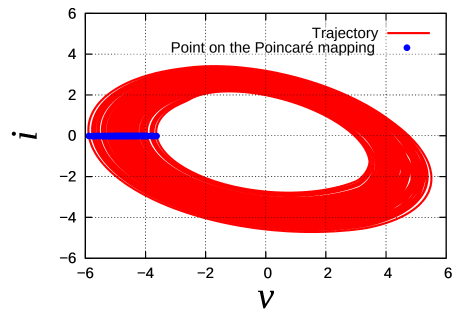
4. Conclusions

We investigated the memristor-based dynamic model in Fig. 1 numerically, and reported various oscillatory phenomena. In particular, we observed that the successive bursts are triggered by the gradually increasing memristance in the circuit model with injection of the DC backward bias voltage E. We measured the average interburst interval of such bursting oscillations in terms of the E. It should be noted that the occurrence rate can be changed easily by modulating the values of E (and/or the $-R$). In addition, we investigated the oscillatory phenomena around the boundary existing regions of the bursting oscillations.

[3]We draw the both results for $1000 \leqq t \leqq 1400$, so the transient is removed.



(a) Time series plot.



(b) Trajectory and the mapped points in the v–i plane.

Figure 5: Oscillatory phenomenon observed for $E = 0.35$ and $R = 0.4$.

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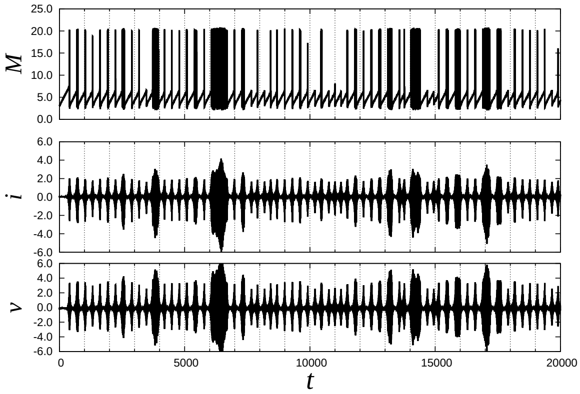
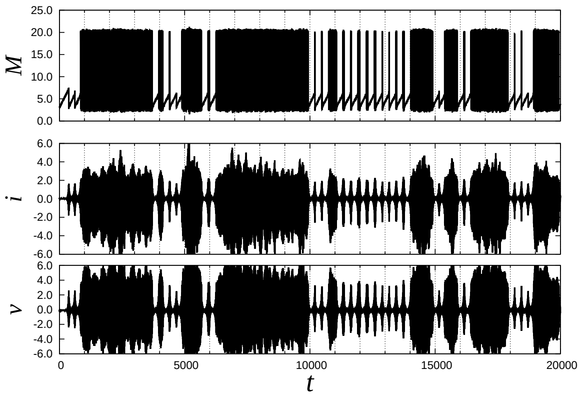


(a) $R = 0.44$.



(b) $R = 0.45$.

Figure 6: Turbulent bursts ($E = 0.1$).

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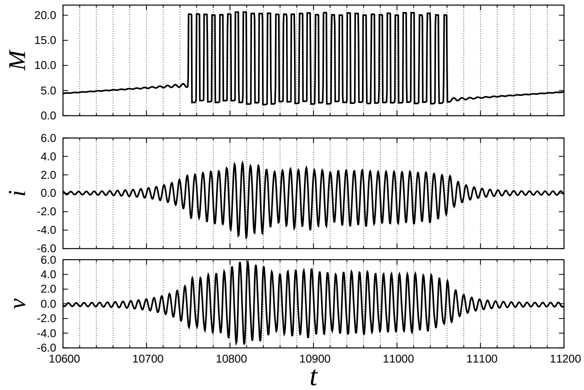
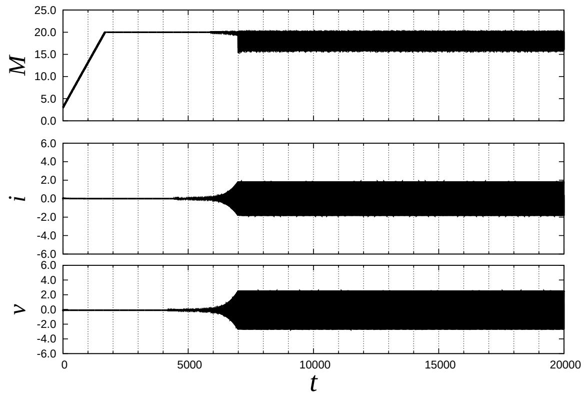
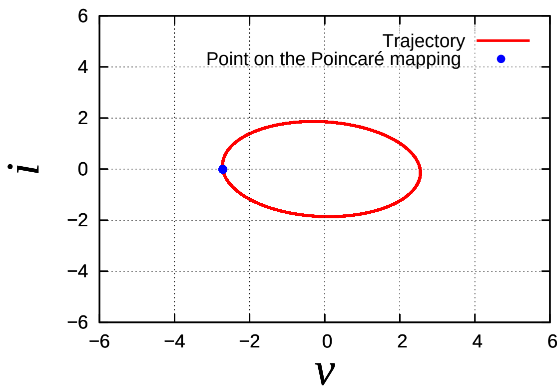


Figure 7: Magnified drawing of the oscillation of Fig.6 (b).



(a) Time series plot.



(b) Trajectory and the mapped points in the v–i plane.

Figure 8: Periodic oscillation as a steady-state after transient ($E = 0.1$ and $R = 0.11$).