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Multiscale resolution of networks of granular media network evolution — a network of networks

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Abstract—The experimental technique of Digital Image Correlation is used to measure the displacement of clusters of glass beads in an assembly across strain intervals throughout a plane strain compression test. For each strain interval a complex network is constructed whose connectivity captures similarity of kinematical response of the observed clusters. We propose a method of obtaining a network of networks which collates the temporal activity of these kinematical networks through loading. This is done by considering convex combinations of network adjacency matrices with coefficients determined using a distance kernel function. The use of a distance kernel allows the introduction of a network interaction score which enables the deformation history to be partitioned into temporal sequences of different kinematical behaviour. A network of networks in each of these temporal ranges helps identify areas within the material responsible for onset of failure, the localized failure zone and shear band activity.

1. Introduction

Granular materials (e.g., soil, rock, powders etc.) are ubiquitous in Nature. Their behaviour in response to stimulus such as shear and compression is not fully understood despite such knowledge being crucial to improved understanding of processes involving these stimuli. Recently the behaviour of granular media has been studied using complex networks of the fabric [1] and kinematics [2]. These works considered collections of networks individually but it has become apparent in other arenas that consideration of network of networks is important (e.g., [3, 4]). Here we propose a method of combining evolving networks as a superposition whose coefficients allow for a multiscale resolution examination of network interaction. We demonstrate the usefulness of this network of networks with respect to collections of kinematic network representations of a deforming experimental assembly of glass beads which exhibits localized failure through shear banding.

2. Glass beads experiment

We subject a low porosity specimen (approx. 140 by 40 by 80 mm) of glass beads to plane strain compression. The specimen sits on a translatable base which is kinematically free to slide only in the in-plane direction, thus allowing for unconstrained translation along a shear band when such a structure emerges. Axial deformation rates are small, about 0.05 mm/min. Every 0.025% global axial strain, or every 45 sec throughout shear, digital images are collected, as well as readings of macroscopic axial and out-of-plane forces and axial and in plane displacements.

The digital image correlation (DIC) technique used here requires the subject material to possess local material colour variations at the scale of interest. To achieve the required variation in our granular sample, we mixed together materials of different colour. The glass bead material is a bi-disperse mixture of 1.0-mm opaque orange (40% by mass) and 1.5-mm (60% by mass) opaque purple, green and yellow beads. The median grain size is roughly 1.25 mm. Typical macroscopic stress-strain data are given in Fig. 1(a).

DIC is used to compute in-plane displacements across the out-of-plane specimen surface. DIC mathematically tracks pixel gray level value patterns manifested within small subsets of pixels, here comprising about three to four grains across (e.g., [5, 6]). Incremental DIC analyses are performed every 0.15% axial strain, representing about 4% gross shear strain across a shear band. Figure 1(b) shows the observed displacement field at the final observed strain increment. A single persistent shear band develops in the sample, the measured thickness of which is around 7 mean grain diameters.

3. Network representation of DIC data

The kinematic data provided by DIC is resolved to a fixed number of observation sites on a mesh (i.e., a grid) which itself is fixed throughout the deformation. At each site and across a specified strain interval the displacement of an identifiable cluster of grains is recorded. To convert this information to a network requires the specification of what constituents a network node and also a method to connect nodes with network links. In [2] we proposed two ways — the k-net and S-map — of defining links given network nodes represent the individual observation sites. Here, we use the k-net formalism to help present our net-

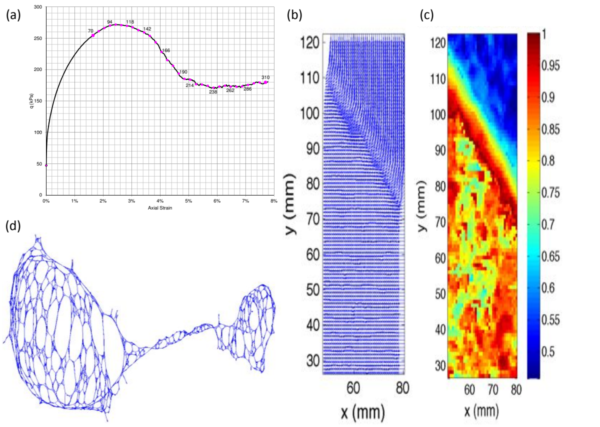

Figure 1: (a) The stress-strain response of the glass beads to plane strain compression. (b) The displacement field at the final observed strain interval clearly identifies the fully formed shear band zone. (c) The (relative) node closeness centrality of the network mapped back to the observation sites. (d) Abstract representation of the k-net at the final observed strain interval.

work of networks method. Specifically, network nodes represent the individual observation sites and network links are drawn if the corresponding displacement information is close. The term close is relative and in a k-net each node is connected to a minimum of k other nodes based on the k closest matches of observed displacement based on the Euclidean norm. The value of k is selected to be the minimum value such that the resulting k-net *first* forms a single component network (i.e., each node is reachable to every other node by a finite shortest network path length).

In [2] we constructed k-nets for a plane strain compression test of mixed concrete at each observed strain interval of the deformation and demonstrated that appropriate properties of the resulting networks identified, or highlighted, important kinematical and rheological changes occurring within the sample. For example, in Fig. 1 we show an abstract representation of the k-net constructed at the final recorded strain interval of the glass bead test and the network property of closeness centrality mapped back to the observation sites of the physical sample. We see at this post-failure strain interval the dominant and persistent shear band is fully formed with observation sites within the band corresponding to the "neck" of the abstract representation of the k-net and these sites/nodes have high relative values of closeness centrality.

Our goal here is to introduce a way of collating the information in all of these network representations, or a suitably chosen temporal subset, so that essential features of the physical system can be probed through a multi-resolution analysis. The method is quite general and points the way towards combining information in collections of networks of a system each constructed with respect to different properties or aspects of the system.

4. Network of networks

For a given observed property of interest a network can be constructed based on the relationship between measurements of properties throughout deformation (see, Fig. 1). These networks can be studied in isolation to provide useful interpretations and understanding of the response behaviour of a granular material to a deformation program at multiple scales — the scale determined by the property of interest. Here, we turn our attention to the problem of usefully collating the information obtained from different networks. In particular, we consider how one can construct a single network from a number of particle property networks whose own properties can further reveal insights to grain rearrangements. We suggest two methods of performing a superposition of networks to produce a network of networks with the second method providing a means for examining granular materials at multiple resolutions.

4.1. Convex combination

Suppose we have constructed networks N_i , $i = 1, \ldots, n$ based on n different properties of interest — or using the same property at n different times through a deformation program — where N_i represents the adjacency matrix of each particle property network. We can combine the information embedded in each network by considering the network of networks given by

$$N = \sum_{i=1}^{n} \alpha_i N_i \qquad (1)$$

where without loss of generality we consider convex combinations so that for $\alpha_i \geq 0$ we have

$$\sum_{i=1}^{n} \alpha_i = 1. \qquad (2)$$

Clearly, $\alpha_i = 1$ and $\alpha_j = 0 \; \forall j \neq i$ recovers each individual particle property network. A second choice of weighting in the superposition is $\alpha_i = 1/n \; \forall i$. This weights all property networks equally resulting in a weight matrix N. We can identify areas of high and low connectivity over the duration of the experiment using weighted network properties such as *node strength* or *average node strength*.

4.2. Kernel estimation

A second method providing a multi-resolution analysis is to combine the particle property networks in a manner inspired by kernel density estimation of network community structures [7]. We suggest placing a kernel on each network and describe the interaction between networks by modelling the strength of interaction by a decaying function based on the distance between the networks. This sets up a multiple interaction system, or network of networks, and the accumulated interaction levels of the networks reflects the intrinsic structures of the networks and the material they represent. Each kernel function has a bandwidth and tuning of this parameter allows a multiscale or multi-resolution aspect of the analysis.

For two networks i and j we define G_{ij} to be the Hamming distance between the networks [8]. Since each network has the same identifiable node structure this distance can reflect the differences in the link structure between the two networks. To model the interaction between networks we follow [7] and select a kernel function of the form

$$K(x/h) = \exp(-x^2/2h^2) \quad (3)$$

The parameter h is the *bandwidth* of the kernel and determines the spread, or scale of the analysis. We can also gauge the level of importance a network achieves for a given bandwidth by considering the network accumulated impact score (essentially the node AI score of [7]) defined by

$$\beta_i = \sum_{j=1}^{n} K\left(\frac{G_{ij}}{h}\right) \quad (4)$$

For a given bandwidth, or resolution, we also obtain a weighted network whose properties can be studied to reveal information about the material. That is, consider the weights α in (1) to be of the form

$$\alpha_i = \beta_i / \sum_j \beta_j. \quad (5)$$

5. Networks of kinematical networks

The most straightforward way of combining a number of networks is through the convex combination expressed in (1) and (2) with all weights equal. If we are combining n adjacency matrices this results in a weighted adjacency matrix and one of the simplest network property to resolve features is node strength or its average which is a generalization of node degree of unweighted networks to weighted networks. In Fig. 2(a) we show the node-strength from the network of network convex sum and in Fig. 2(b) we present the average node strength (i.e., the node strength divided by the node degree). We remark that the node strength of this network of networks when mapped back to the location of the observation sites is relatively uninformative similar to the node degree of individual k-networks in [2] for a similar loading test of mixed concrete sand. In contrast the average node strength of this network of networks appears capable of resolving all of the important kinematical activity throughout the entire loading program. We see that the prominent shear band region is clearly delineated, however, observational sites of higher relative average node strength have also been resolved.

In our second proposed method of obtaining a convex combination network of networks using a kernel function we must first calculate the distance between each network. Recall, we select the distance metric of Hamming distance which counts the discrepancy in network connections (i.e., links) between each network. This pairwise Hamming distance matrix is passed through the kernel function (3) for a given bandwidth. We have found a useful way of selecting a suitable bandwidth is to choose h to be a multiple of the average value of the Hamming distance, say $\bar{G}$, between networks. In Fig. 3(a)-(b) we show the effects of tuning the bandwidth at $0.5\bar{G}$ and $1.0\bar{G}$ respectively.

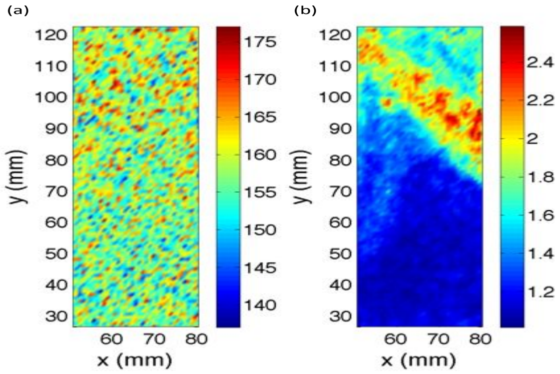

Figure 2: (a) The node strength of the convex sum of networks with equal weights. (b) The average node strength of the convex sum of networks with equal weights.

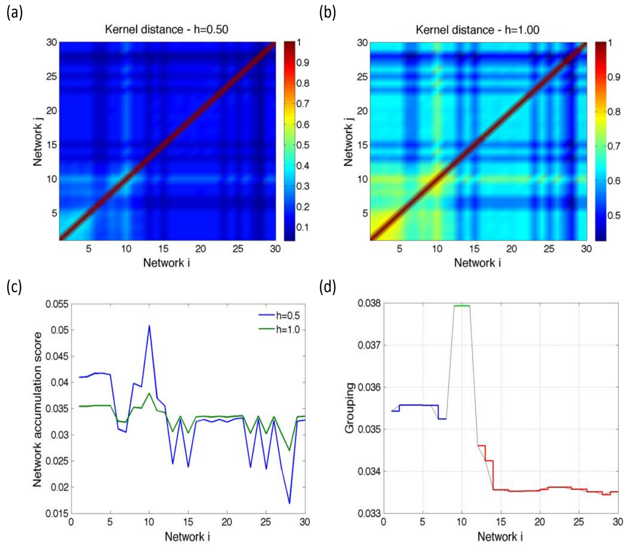

Figure 3: The Hamming distance matrix transferred through the Gaussian kernel function at bandwidths (a) $0.5\bar{G}$, (b) $1.0\bar{G}$. (c) The normalized network accumulation score at two bandwidths. (d) A simple-minded grouping of the network accumulation scores at bandwidth $\bar{G}$ identifies three temporal groupings.

We summarize the information in each of these kernel distributions by calculating the normalized version (5) of the network accumulation (interaction) score (4). This normalized score for each of the selected bandwidths is shown in Fig. 3(c). For lower bandwidths we converge to the equal weight case considered earlier and for higher bandwidths in this collection of networks we found qualitatively the same features as the trace shown for $\bar{G}$ over a wide range. If we sum the networks based on these accumulation scores and then examine the resulting network of networks with respect to average node strength we find essentially the same results as shown in Fig. 2(b). The strength of the kernel method of combining networks, however, is apparent when we use it to identify subsets of networks to combine in a fashion akin to the use of kernel functions to identify network community structures [7]. If we perform a simple temporal parsing of the network accumulation score whereby we reset the value of a score to be the value of the maximum value of its neighbour then after one iteration we can identify a crude grouping of the networks into three temporal regions (see, Fig. 3(d)). If we combine the networks in each of these three regions using the basic equal weight convex combination then we achieve a multi-resolution perspective of the important kinematical activity through loading. The average network node strength of each of these three networks of networks is displayed in Fig. 4. We note that the behaviour observed in the network of Fig. 2(b) is dominated by the activity of the fully formed shear band seen in Fig. 4(c). The behaviour of the sample at the *onset* of failure and *during* failure is captured by the network of networks shown in Fig. 4(a) and 4(b) respectively. Figs. 4(a)–(b) also reveal the activity in this failure zone is made up of (possibly) two competing criss-crossed bands before the final localized zone from left to right wins.

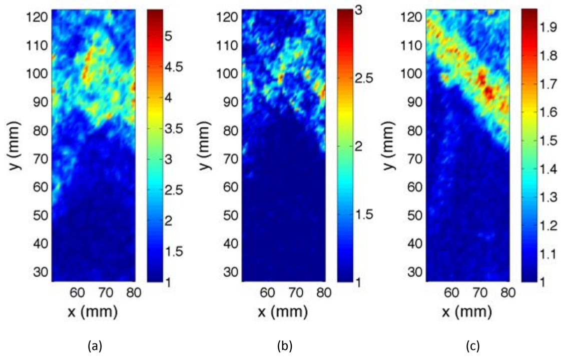

Figure 4: Average node strength of network of networks over the strain intervals (a) 1st to 8th (b) 8th to 11th and (c) 12th to the 30th and final interval.

6. Conclusion

We have proposed a method of obtaining a network of networks by considering convex combinations of adjacency matrices. The introduction of a kernel function to determine the weights of the convex combinations allows the flexibility of bandwidth tuning. This provides a way of obtaining a temporal grouping of the networks to highlight different aspects of network, or system, evolution. For the particular test case presented here we found three clear distinct regions in the deformation history. These corresponded to compression prior to peak shear stress, shear banding at peak failure and the deformation during steady-state behaviour. The development of an improved temporal partitioning method, an improved prescription for bandwidth tuning and identifying the correspondence between network of network properties to specific observed failure mechanisms in this and other granular tests is the subject of ongoing research.

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