DOA Estimation of Direct Wave in Multipath Environments Using FFT-FOCUSS with Multiple Thinned Arrays

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1. Introduction

In the estimation of the direction of arrival (DOA) of radio waves using FOCUSS [1], the improvement effect of a method introducing FFT as a pre-processing step (FFT-FOCUSS) has been confirmed. In this paper, we attempt a performance analysis of FFT-FOCUSS in DOA estimation of the first arrival wave in a multipath environment using multiple thinned arrays.

2. DOA Estimation by Compressed Sensing with FFT

2.1. Compressed Sensing with FFT

As shown in Fig.1, we use a K-element uniform linear array with element spacing d for the DOA estimation. The component corresponding to the desired signal frequency is extracted from the frequency spectrum obtained by the FFT of the received signal at each element. Thus, we have the array vector $\boldsymbol{x}$ from all element components. With the first element of the array as the phase reference point, $\boldsymbol{x}$ is reconstructed using N angle bins as follows:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \quad \in \mathbb{C}^{K\times 1} \qquad (1)$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_N)] \quad \in \mathbb{C}^{K\times N} \qquad (2)$$

$$\boldsymbol{s} = [s_1, \ldots, s_N]^T \quad \in \mathbb{C}^{N\times 1} \qquad (3)$$

where $\boldsymbol{A}$ is an N-column angle-bin mode matrix and $\boldsymbol{s}$ is the corresponding N-dimensional signal waveform vector. As a thinned array, we consider an array with one element thinned out from the second to the $(K-1)$ th element.

2.2. FOCUSS

FOCUSS [1] finds $\boldsymbol{s}$ by minimizing $\|\boldsymbol{s}\|_p^p$ (l_p-norm to the pth power) under the condition $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s}$, and we obtain the vector $\hat{\boldsymbol{s}}$, which is an estimate of $\boldsymbol{s}$ by the iterative update as follows:

$$\boldsymbol{W}_m = \mathrm{diag}\left(|\hat{s}_{m-1}(1)|^{1-\frac{p}{2}}, \ldots, |\hat{s}_{m-1}(N)|^{1-\frac{p}{2}}\right) \qquad (4)$$

$$\boldsymbol{q}_m = (\boldsymbol{A}\boldsymbol{W}_m)^{-}\boldsymbol{x} \qquad (5)$$

$$\hat{\boldsymbol{s}}_m = \boldsymbol{W}_m\boldsymbol{q}_m \qquad (6)$$

Note that $(\cdot)^{-}$ in Eq.(5) denotes a pseudo-inverse, which is computed using a regularization parameter [1].

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3. Performance Analysis by Simulation

Under the conditions in Table 1, we carried out a computer simulation. The root means squared error (RMSE) for the first arrival wave (direct wave in a multipath environment) is used to evaluate the estimation accuracy. Here, we employ the three types of arrays, which are with all four elements (#1,2,3,4), with the third element thinned out (#1,2,4), and with the second element thinned out (#1,3,4), respectively. Fig.2 shows the cumulative distribution function (CDF) of RMSE for each array and RMSE of the median of DOA estimates from the three arrays. It is found that the median filtering achieves better performance than others.

4. Conclusion

It is shown that the median filtering of DOA estimates from the multiple thinned arrays improves the estimation accuracy of the first arrival wave in a multipath environment.

References

[1] I. F. Gorodnitsky and B. D. Rao, IEEE Trans. Signal Processing, Vol.45, No.3, pp.600-616, March 1997.

Table 1 Simulation conditions.

Carrier Frequency	2.45GHz	Number of snapshots	1024
Sampling frequency	1MHz	Search area	$-90°\sim90°$
Number of elements	4	Angle bin spacing	$1°$ ($N = 181$)
Element spacing	0.5λ	Norm value p	0.8
Number of waves	2	Regularization parameter of FOCUSS	10^{-3}
Baseband frequency (CW)	100kHz		
Input power (1st wave, 2nd wave)	(1.0, 0.5)	Convergence condition	$\frac{\|\hat{s}_m - \hat{s}_{m-1}\|}{\|\hat{s}_m\|} < 10^{-3}$
SNR	10dB	Number of trials	500
DOA (1st wave, 2nd wave)	$(-60°\sim60°, -60°\sim60°)$		

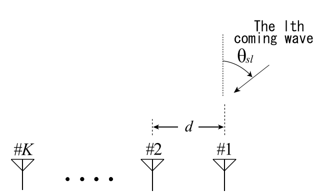

Fig. 1 K-element uniform linear array.

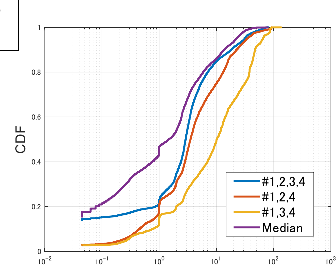

Fig. 2 CDF of RMSE.