A Study of the Message Delivery Delay with End-to-End Routing in Complex Networks

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1. Introduction

Recently, in the field of network science, communication characteristics of a large-scale networks have been mathematically clarified by modeling a large-scale network as a complex network. We have analytically derived the average message delivery delay with a typical routing strategy in a large-scale network with intermittently connected links.

Message delivery delay with end-to-end routing in complex networks is greatly affected by the processing speed of intermediate nodes as well as the existence probability of end-to-end paths, but also by the processing speed of intermediate nodes. However, it has not been fully clarified how the impact of intermediate node processing speed on message delivery delay.

In this paper, by extending our analysis [1], we therefore derive the average message delivery delay with end-to-end routing in a large network consisting of intermittent links and nodes with limited message transfer speed.

2. Analysis Model

A large network composed of many intermittent links and nodes with limited transfer speed is represented by an undirected graph $G = (V, E)$. Here, The link set E in the graph G means a set of links that are connected at a given time. The number of nodes in the graph G is denoted as $N\,(\equiv |V|)$.

Suppose that every node constructs its routing table by periodically exchanging the information about the connection status of its links with all its neighbors, and that every node sends a message to the destination node only when an end-to-end path is established. Let the routing table update interval be Δ and all message sizes equal S.

Let μ be the amount of messages that every node can transfer per unit time. For simplicity, we assume that every node attempts to send a message only once during the routing table update interval Δ. We also assume that the propagation delay of every link and the processing delay at every node are negligibly small.

3. Analysis

Let us focus on the bottleneck node $\overline{v}$ in the graph G. Suppose that only a node $\overline{v}$ is the bottleneck in the graph G and that no message discards occur at other nodes. In this case, message delivery succeeds only if (1) an end-to-end path exists between source node and destination node and (2) the sent message is not discarded at the bottleneck node $\overline{v}$.

Let D be the average message delivery delay for messages passing through the bottleneck node $\overline{v}$. The probability that an end-to-end path exists between source node and the destination node of a messages passing through the

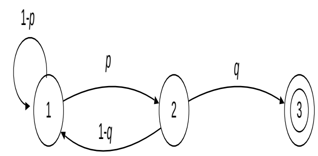

Fig. 1 3-state Markov chain

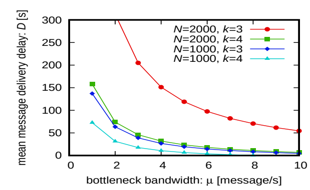

Fig. 2 Relation between the processing speed μ and the average message delivery delay D

bottleneck node $\overline{v}$ is denoted as p. The probability that message flowing into the bottleneck node $\overline{v}$ can pass through the node $\overline{v}$ is denoted as q. The average message delivery delay D is the expected number of transitions from state 1 to state 3 in a three-state Markov chain with absorbing states (Figure 1). Using $n_{1,3}(= (1 + p)/(pq))$, we obtain given by $D = \Delta\,(n_{1,3} - 1)$.

The probability p can be approximated by the probability that both source node and destination node belong to the giant cluster in the graph G.

The betweenness centrality of node v in the graph G is denoted B_v. The probability q of passing through to node that messages can pass through node $\overline{v}$ is given by The ratio of the total amount of messages arriving at node $\overline{v}$ to the amount of messages that can be transferd at node $\overline{v}$. The percentage of all messages sent out on the graph G that pass through node v is $B_v / \sum_{v \in V'} B_v$. Thus the probability q is given by

$$q = \frac{\mu\,\Delta}{S\;N'\,L\,\frac{B_v}{\sum_{v\in V'} B_v}} = \frac{\Delta\,\mu(N-1)}{B_v\,S}, \qquad (1)$$

[1]where L is the average path length of the giant cluster on the graph G and N' is the size of the giant cluster on the graph G [1].

4. Examples

Figure 2 shows the relationship between the processing speed μ of the intermediate nodes and the average message delivery delay D when the graph G is generated by the ER (Erdõs-Rényi) model. This figure shows the results when the number N of nodes and the average order $\overline{k}$ of the ER model are varied. The message size and the routing table update interval are set to $S = 1$ [packet] and $\Delta = 10$ [s].

Acknowledgements

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References

[1] 大西 美知加, 松尾 涼太郎, 大崎 博之, “リンクの通信容量が大規模ネットワークにおけるエンド-エンドルーティングの特性に与える影響に関する一考察,” 電子情報通信学会 ソサイエティ大会 講演論文集 *(B-16-14)*, p. 201, Sept. 2021.

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