A Novel Periodogram-Based Frequency Offset Estimation Method for OFDM Systems

Sanghun Kim[1], Dahae Chong[1], Sun Yong Kim[2], and Seokho Yoon[1,0]

[1]School of Information and Communication Engineering, Sungkyunkwan University,
Cheoncheon-dong, Jangan-gu, Suwon, Gyeonggi-do, 440-746, Korea.
[2]Department of Electronics Engineering, Konkuk University,
1 Hwayang-dong, Gwangjin-gu, Seoul 143-701, Korea.
[0]Corresponding author (e-mail: syoon@skku.edu)

Abstract: The frequency offset estimation is one of the most important tasks in orthogonal frequency division multiplexing (OFDM) communication systems. To estimate frequency offset, a periodogram-based estimator has been proposed by Ren [6]. The method consists of three estimation steps and gives an accurate results. However, in the second and third estimation steps, the maximum frequency offset that can be estimated is limited by subcarrier spacing. This range is insufficient compared with the overall signal bandwidth, and causes high complexity in the first estimation step. In this paper, we propose a novel frequency offset estimation method with wide estimation range. The proposed method can efficiently extend the estimation ranges without loss of accuracy.

Keyword: OFDM, frequency offset, periodogram, estimation, wide range.

1. Introduction

The orthogonal frequency division multiplexing (OFDM) has been widely used for communication areas, such as European digital audio and video broadcasting (DAB/DVB), IEEE 802.11a, and European Hiper-LAN II for wireless local area network (WLAN). Recently a multi-user cellular OFDM has been adopted for IEEE standard 802.16. This is because that the OFDM gives a high spectral efficiency, simple equalizer structure, and robustness to multipath fading.

The OFDM systems, however, are very sensitive to frequency offset caused by the mismatch of the oscillators in the transmitter and receiver or the Doppler shift. In OFDM systems, the frequency offset could bring on the inter-carrier interference (ICI) and destroy orthogonality among subcarriers, resulting in significant performance degradation. Consequently, the frequency offset estimation is one of the most crucial tasks for OFDM systems [1].

To estimate the frequency offset, various methods based on a training symbol have been investigated [2]-[5]. The methods [2]-[4] exploit the time domain repetition of the training symbols, and the maximum estimation range of frequency offset is limited by its design. The literature [5] presents a frequency offset estimation method using the relationship among sub-carriers, whereas the methods [2]-[4] use the relationship among time-domain samples.

Recently, a three-step frequency offset estimation method was proposed in [6]. The method is based on the periodogram and gives an accurate estimation performance. Moreover, the estimation range of the method does not depend on the training symbol design (i.e., does not require repeated parts in the training symbol). However, the method [6] has the problem that the maximum estimation range of frequency offset in second and third steps is limited by subcarrier spacing. This range limitation causes high complexity problem in the first estimation step.

In this paper, we propose a novel frequency offset estimation method. The proposed method can efficiently extend the estimation ranges without loss of accuracy.

The remainder of this paper is organized as follows. Section 2 describes the signal model and conventional method. In Section 3, a novel frequency offset estimation method is proposed. Section 4 presents the performance comparison results. Finally, Section 5 concludes this paper.

2. Signal Model and Conventional Method

2.1 Signal Model

The n-th OFDM sample x_n is generated by the inverse fast Fourier transform (IFFT), and can be expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \text{ for } n = 0, 1, \cdots, N-1, \quad (1)$$

where X_k is a phase shift keying (PSK) or quadrature amplitude modulation (QAM) symbol in the kth subcarrier and N is the size of the IFFT.

Assuming that x_n is a constant envelope training symbol such as constant amplitude and zero autocorrelation (CAZAC) sequence [7], we can express x_n as

$$x_n = \alpha e^{j\beta_n}, \quad (2)$$

where α is a positive constant and β_n is the phase of x_n.

The constant envelope sequence is also suitable for time and frequency synchronization [8], and provides a good peak to average power ratio (PAPR) [9].

After timing synchronization, the n-th received OFDM sample y_n can be obtained as

$$y_n = x_n e^{j2\pi\varepsilon n/N} + w_n, \text{ for } n = 0, 1, \cdots, N-1, \quad (3)$$

where ε represents the frequency offset normalized to the subcarrier spacing and w_n is the complex additive white Gaussian noise (AWGN) sample with zero mean and variance σ_w^2. The signal to noise ratio (SNR) ρ is defined as $\rho \triangleq \alpha^2/\sigma_w^2$

In order to make the estimation performance independent of the training symbol, the envelope equalized processing (EEP) factor f_x is used [6], which is defined as

$$f_x = \frac{x_n^*}{|x_n|^2}, \tag{4}$$

where * and $|\cdot|$ denote the complex conjugate and Euclidean norm operations, respectively.

The received signal equalized by the EEP factor can be expressed as

$$\begin{aligned} y_n^{'} &= y_n f_x \\ &= x_n e^{j2\pi\varepsilon n/N} f_x + w_n f_x \\ &= e^{j2\pi\varepsilon n/N} + w_n^{'}, \end{aligned} \tag{5}$$

where $w_n^{'} = w_n f_x$.

The mean and variance of $w_n^{'}$ are zero and σ_w^2/α^2, respectively. Therefore, the EEP does not affect SNR.

The square root of the periodogram $I(\tilde{\varepsilon})$ at frequency $\tilde{\varepsilon}$ is

$$I(\tilde{\varepsilon}) = \left|\sum_{n=0}^{N-1} y_n^{'} e^{-j2\pi\tilde{\varepsilon}n/N}\right|, \text{ for } n = 0, 1, \cdots, N-1. \tag{6}$$

2.2 Conventional Method

In the conventional method [6], a frequency offset estimate $\hat{\varepsilon}$ can be obtained via the following three estimation steps: integer, fractional, and residual frequency offset estimation.

In order to estimate the integer frequency offset, the following estimator is used.

$$\hat{\varepsilon}_I = \arg\max_{\tilde{\varepsilon}_I} \left\{\{I(\tilde{\varepsilon}_I)\}^2 + \{I(\tilde{\varepsilon}_I + 1)\}^2\right\}, \tag{7}$$

where $\tilde{\varepsilon}_I \in \{-\frac{N}{2}, \cdots, 0, \cdots, \frac{N}{2}\}$ is a trial value of an integer frequency offset estimate $\hat{\varepsilon}_I$.

The fractional frequency offset is estimated by

$$\hat{\varepsilon}_F = \frac{I(\hat{\varepsilon}_I + 1)}{I(\hat{\varepsilon}_I) + I(\hat{\varepsilon}_I + 1)}, \tag{8}$$

and the residual frequency offset is estimated by

$$\hat{\varepsilon}_R = \frac{1}{2}\frac{I(\hat{\varepsilon}_I + \hat{\varepsilon}_F + 0.5) - I(\hat{\varepsilon}_I + \hat{\varepsilon}_F - 0.5)}{\{I(\hat{\varepsilon}_I + \hat{\varepsilon}_F + 0.5) + I(\hat{\varepsilon}_I + \hat{\varepsilon}_F - 0.5)\}}. \tag{9}$$

Therefore, the frequency offset estimate is the sum of these three estimates.

$$\hat{\varepsilon} = \hat{\varepsilon}_I + \hat{\varepsilon}_F + \hat{\varepsilon}_R. \tag{10}$$

3. Proposed Methods

If we ignore the noise term $w_n^{'}$ in (5), the square root of the periodogram (6) can be rewritten as

$$\begin{aligned} I(\tilde{\varepsilon}) &= \left|\sum_{n=0}^{N-1} e^{j2\pi(\varepsilon-\tilde{\varepsilon})n/N}\right| \\ &= \left|e^{j\pi(\varepsilon-\tilde{\varepsilon})(N-1)/N} \frac{\sin\{\pi(\varepsilon-\tilde{\varepsilon})\}}{\sin\{\pi(\varepsilon-\tilde{\varepsilon})/N\}}\right|. \end{aligned} \tag{11}$$

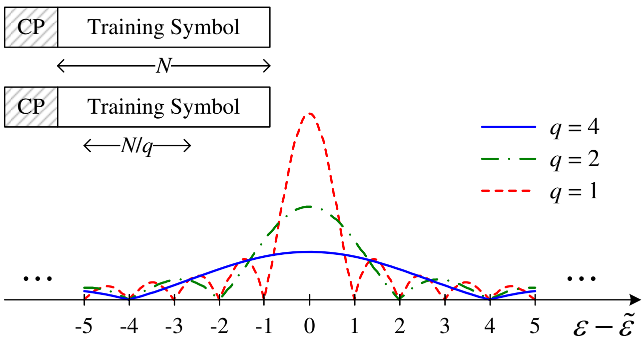

Figure 1. Width extension according to segmentation.

By L'Hopital's rule (in OFDM, $N \gg 1$),

$$\sin\{\pi(\varepsilon-\tilde{\varepsilon})/N\} \approx \pi(\varepsilon-\tilde{\varepsilon})/N. \tag{12}$$

By substituting (12) into (11),

$$\begin{aligned} I(\tilde{\varepsilon}) &= N\left|\frac{\sin\{\pi(\varepsilon-\tilde{\varepsilon})\}}{\pi(\varepsilon-\tilde{\varepsilon})}\right| \\ &\text{or} \\ &= N\left|\text{sinc}(\varepsilon-\tilde{\varepsilon})\right|. \end{aligned} \tag{13}$$

To explain the conventional method, we substitute (13) into (9) and then get (14):

$$\hat{\varepsilon}_R = \frac{1}{2}\frac{\left|\text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T-\frac{1}{2})\}\right| - \left|\text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T+\frac{1}{2})\}\right|}{\left|\text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T-\frac{1}{2})\}\right| + \left|\text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T+\frac{1}{2})\}\right|}. \tag{14}$$

where $\hat{\varepsilon}_T = \hat{\varepsilon}_I + \hat{\varepsilon}_F$.

We can remove the Euclidean norm operations of (14) for the range $-\frac{1}{2} \leq \varepsilon - \hat{\varepsilon}_I - \hat{\varepsilon}_F \leq \frac{1}{2}$, and after some manipulations,

$$\begin{aligned} \hat{\varepsilon}_R &= \frac{1}{2}\frac{\text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T-\frac{1}{2})\} - \text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T+\frac{1}{2})\}}{\text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T-\frac{1}{2})\} + \text{sinc}\{\pi(\varepsilon-\hat{\varepsilon}_T+\frac{1}{2})\}} \\ &= \varepsilon - \hat{\varepsilon}_I - \hat{\varepsilon}_F. \end{aligned} \tag{15}$$

As we can see from (14) and (15), the conventional residual frequency offset estimation methods have an estimation range of $\hat{\varepsilon}_I + \hat{\varepsilon}_F \in [\varepsilon - \frac{1}{2}, \varepsilon + \frac{1}{2}]$. Namely, the conventional residual frequency offset estimation method can give correct estimates only when the difference between actual and previously estimated offset ($\varepsilon - \hat{\varepsilon}_I - \hat{\varepsilon}_F$) is less than or equal to 0.5. According to similar process, it is easy to prove that the conventional fractional frequency offset estimation methods have an estimation range of $\hat{\varepsilon}_I \in [\varepsilon - 1, \varepsilon]$. These ranges are very narrow compared with the overall signal bandwidth, and hence, the integer frequency offset estimator (7) requires many trial values and complex operations. If we extend the estimation range of the fractional or residual frequency offset estimator, then the complexity in the integer frequency offset estimation can be reduced.

From now, we derive the method with wide estimation range. In order to expand the estimation range, from (6), we exploit the partial Fourier transform of training symbol. If we take the Fourier transform of length N/q from the training symbol, we can get q times extended sinc functions as shown in (16) and Fig. 1.

$$\begin{aligned} I_m(\tilde{\varepsilon}) &= \left| \sum_{n=m}^{m+N/q-1} e^{j2\pi(\varepsilon-\tilde{\varepsilon})n/N} \right| \\ &= \left| \sum_{n=0}^{m+N/q-1} e^{j2\pi(\varepsilon-\tilde{\varepsilon})n/N} - \sum_{n=0}^{m-1} e^{j2\pi(\varepsilon-\tilde{\varepsilon})n/N} \right| \\ &= N \left| \frac{\sin\{\pi(\varepsilon-\tilde{\varepsilon})/q\}}{\pi(\varepsilon-\tilde{\varepsilon})} \right| \qquad (16) \\ &\quad \text{or} \\ &= \frac{N}{q} \left| \mathrm{sinc}\{(\varepsilon-\tilde{\varepsilon})/q\} \right|. \end{aligned}$$

According to this property, a new estimator is obtained as

$$\hat{\varepsilon}_R^{\text{new}} = \frac{1}{2} \sum_{p=0}^{q-1} \frac{\{I_{mN/q}(\hat{\varepsilon}_T + \frac{q}{2}) - I_{mN/q}(\hat{\varepsilon}_T - \frac{q}{2})\}}{\{I_{mN/q}(\hat{\varepsilon}_T + \frac{q}{2}) + I_{mN/q}(\hat{\varepsilon}_T - \frac{q}{2})}. \qquad (17)$$

From similar process with (14) and (15), it is easy to find that the new estimator has q times extended estimation range.

The Cramér-Rao lower bound (CRLB) of (9) is given in [6], [10] by

$$\text{CRLB}_{\text{Conventional}}(\rho) = \frac{\pi^2}{64N\rho}, \qquad (18)$$

where ρ is the SNR of the received signal.

From (18) and [11], the CRLB of (17) is obtained as follows

$$\text{CRLB}_q(\rho) = \frac{(\pi q)^2}{64N\rho}. \qquad (19)$$

In spite of the wider estimation range, the performance of the new estimator (17) is degraded as the value of q increases. However, this problem can be solved by cascade estimation which uses multiple values of q. The proposed integer frequency offset estimation method is described in Table 1. The CRLB of the proposed method is the same as (18).

The proposed method can be used for various purposes (for example, low-complexity or high accuracy application). The proposed method can cover a wide range of frequency offset, hence, the number of trial values in integer frequency offset estimator (7) can be reduced. Consequently, significant complexity reduction can be obtained. Furthermore, in the case of using the same number of trial values, the proposed method may give more reliable estimates than those of the conventional estimator.

4. Simulation Results

In this section, the proposed frequency offset estimation method is compared with the conventional method in terms of the estimation range and accuracy. We consider N=64 and an AWGN channel.

Table 1. Pseudocode of the proposed method

$\hat{\varepsilon}_t = \hat{\varepsilon}_I$

`while` $q \geq 1$

$\quad \hat{\varepsilon}_t = \hat{\varepsilon}_t + \frac{1}{2}\sum_{m=0}^{q-1} \frac{I_{mN/q}(\hat{\varepsilon}_t+q/2)-I_{mN/q}(\hat{\varepsilon}_t-q/2)}{I_{mN/q}(\hat{\varepsilon}_t+q/2)+I_{mN/q}(\hat{\varepsilon}_t-q/2)}$

$\quad q = q/2$

`end`

$\hat{\varepsilon}_{\text{proposed}} = \hat{\varepsilon}_I + \hat{\varepsilon}_t$

Fig. 2 shows the estimated ($\hat{\varepsilon}_R^{\text{new}}$) versus the remaining frequency offset ($\varepsilon - \varepsilon_I - \varepsilon_F$) of (17) without noise. As we can see from the figure, the conventional method (in the case of q = 1) has linear unbiased estimates only when the remaining frequency offset is ranged from -0.5 to 0.5, which is very small compared to the overall signal bandwidth. However, in the proposed method (q = 2 and 4), the range yielding linear unbiased estimates is q times wider than that of the conventional method. In other words, the proposed method has wider estimation range than that of the conventional method.

Fig. 3 shows the mean squared error (MSE) of the proposed (in the case of q = 2) and conventional methods, as a function of the remaining frequency offset ($\varepsilon - \varepsilon_I - \varepsilon_F$). In the figure, ○, ×, and △ denote the SNR of 0, 5, and 10 dB, respectively. Also the blue dash-dotted, red solid, and black dotted line denote the conventional, proposed, and CRLB, respectively. From the figure, we can clearly see that the conventional method gives close-to-CRLB performance when the remaining frequency offset is near zero ($\varepsilon - \varepsilon_I - \varepsilon_F \approx 0$). However, in the proposed method, close-to-CRLB range is more wider than that in the conventional method. In other words, the proposed method gives more improved performance over the conventional method in terms of both estimation range and accuracy.

Fig. 4 also shows the MSE of the methods, as a function of the remaining frequency offset. In the figure, the proposed method is in the case of q = 4. As we can see from the figure, the close-to-CRLB range is more widened than that in the case of q=2 and conventional method.

5. Conclusion

We have found out that the estimation range can be widen by using partial Fourier transform for training symbol. Based on this fact, we have proposed a novel frequency offset estimation method, where the estimation range has been efficiently extended by taking a partial Fourier transform on the training symbol. The simulation results have shown that the proposed method gives a wider estimation range without loss of accuracy, compared with the conventional method.

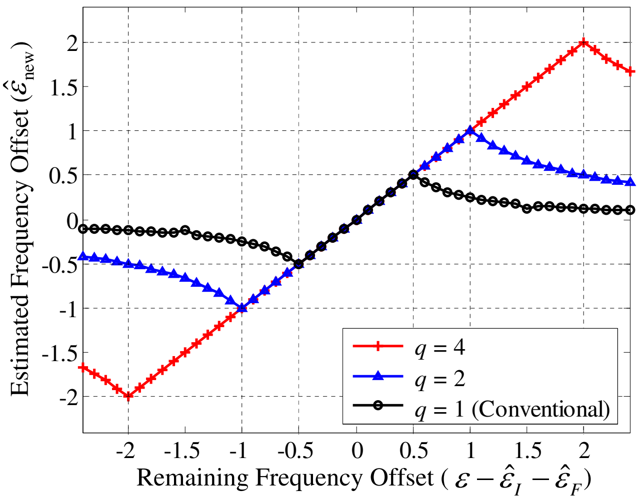

Figure 2. Estimated versus actual frequency offset.

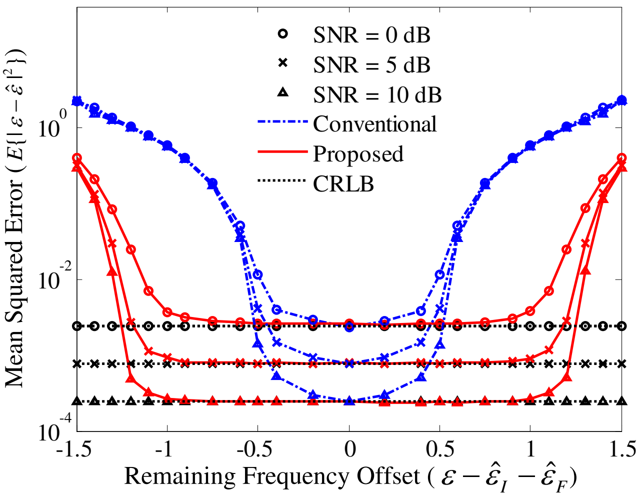

Figure 3. MSE of the conventional and proposed method according to frequency offset (q=2).

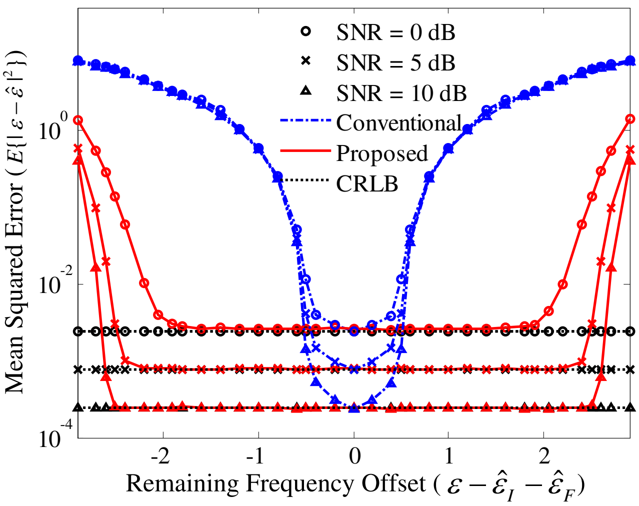

Figure 4. MSE of the conventional and proposed method according to frequency offset (q=4).

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