An MMSE channel estimation algorithm based on the conjugate gradient method for OFDM systems

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Abstract: In this report, we propose a new scheme in order to implement an MMSE channel estimator for OFDM systems. We know that the time domain maximum likelihood estimator (MSE) can achieve highly accurate impulse response estimation by using a time domain long preamble of an OFDM frame. On the other hand, the impulse response estimation based on the minimum mean square error (MMSE) criterion can achieve superior channel estimation in low SNR conditions; however, it requires prior statistical information such as delay profiles of channels. In addition, the computational complexity becomes large because of the inverse matrix calculation. In order to overcome these implementation issues, we propose to estimate the statistical values based on the observed information of the channel. In addition, we show that we can suppress the increase of the computational complexity for the matrix inverse calculation by applying the conjugate gradient method while achieving the same performance with the ideal MMSE estimator.

1. Introduction

The OFDM (Orthogonal Frequency Division Multiplexing) systems have been applied to many wireless communication standards, such as the IEEE802.11a/g/n, the WiMAX and the 3GPP LTE, because of its superior frequency efficiency and robustness for frequency selective fading channels. Since one tap frequency domain equalizers are applied to the OFDM sub-carriers, it is easy to implement the coherent detection for BPSK, QPSK and M-QAM. In addition, recent semiconductor technologies make it possible to realize very high performance channel decoders such as Turbo decoders and LDPC decoders. These trends impose the receivers to operate in very low signal-to-noise ratios (SNRs), which results in the increase of the channel estimation error. Because of this phenomenon, the powerful channel decoders cannot give its maximum ability.

Generally, the frequency domain channel estimators have been applied to the OFDM receivers because of its simplicity. The scattered-pilots are the typical training signals for the frequency channel estimators [1]. On the other hand, the existence of a long preamble, that is attached on the head of the packet frame of the IEEE802.11a/g, makes it possible to carry out the channel estimation in the time domain [2]. By using the time domain long preamble, we can estimate the impulse response of the unknown channels and can obtain the frequency domain channel information by the FFT. It was shown that the time domain approach could improve the channel estimation performance by assuming to constrain the impulse response length within the guard intervals [3].

The maximum likelihood estimator (MLE) and minimum mean square error estimator (MMSEE) were proposed as the time domain channel estimation algorithms in [4]. These algorithms showed superior channel estimation performance. The MLE was the best approach from the implementation point of view because it did not require the inverse matrix calculation. On the other hand, the MMSEE showed superior BER performance in low SNRs compared with the MLE; however, the implementation of the MMSEE was difficult because it required prior information of the delay profile and the noise power in addition to the calculation of the inverse matrix.

In this report, we propose a new approach to implement the MMSE algorithm for the OFDM systems. In order to overcome the difficulties on the implementation of the MMSEE, we propose to estimate the power delay profiles of the channels based on the information from the MLE. In addition, we show that we can restrain the increase of the computational complexity by applying the conjugate gradient method for the matrix inverse calculation while achieving the same performance with the ideal MMSEE.

2. ML and the MMSE channel estimators

2.1 OFDM system structure

The transmitter and receiver structure of the OFDM system is shown in Fig.1. The data from channel coding are interleaved and digital modulation such as the BPSK is applied. The modulated signals are transformed into the time domain signals by an IFFT and transmitted after attaching a long preamble on the head of the frame. A transmitted frame structure is shown in Fig.2. The received signals are divided into the data part and preamble. The channel estimator, which is used to carry out the coherent detection for received OFDM symbols after the FFT, is obtained by using the received long preamble.

2.2 Maximum likelihood estimator

We can carry out the impulse response estimation of unknown channels by using the time domain long preamble as shown in Fig.3 [4]. The estimated impulse response is transformed into the frequency domain by the FFT to make it possible to apply the frequency domain equalizer. It is also shown that the time domain channel estimation approach can achieve superior channel estimation performance in comparison to that of the frequency domain approach[3].

We assume that a time domain long preamble, $l_p(t)$, is received through the channel, $h(t)$, as shown in Fig.3. Then,

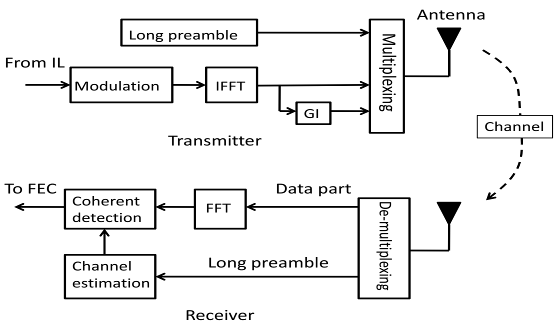

Figure 1. OFDM system structure.

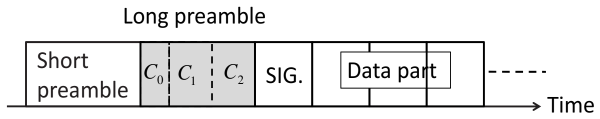

Figure 2. Frame structure.

the received signal vector can be written as

$$\mathbf{r}_{lp} = \mathbf{L}_p \mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{L}_p$, $\mathbf{h}$ and $\mathbf{n}$ denote the preamble matrix, the channel impulse response vector and the noise vector, respectively, that is,

$$\begin{aligned} \boldsymbol{r}_{lp} &= [r_{lp}(\Delta) \quad r_{lp}(\Delta+1) \quad \cdots \quad r_{lp}(L)]^T, && (2) \\ \boldsymbol{h} &= [h_1 \quad h_2 \quad \cdots \quad h_\Delta]^T, && (3) \end{aligned}$$

$$\boldsymbol{L_p} = \begin{bmatrix} l_p(\Delta) & l_p(\Delta-1) & \cdots & l_p(1) \\ l_p(\Delta+1) & l_p(\Delta) & \cdots & l_p(2) \\ \vdots & \vdots & \vdots & \vdots \\ l_p(L) & l_p(L-1) & \cdots & l_p(L-\Delta+1) \end{bmatrix}, \tag{4}$$

$$\boldsymbol{n} = [n(\Delta) \quad n(\Delta+1) \quad \cdots \quad n(L)]^T, \tag{5}$$

where L and Δ are the long preamble length and impulse response length of the channel, respectively.

The MLE of the unknown channel is written as

$$\mathbf{h}_{ml} = (\mathbf{L}_p^H \mathbf{L}_p)^{-1} \mathbf{L}_p^H \mathbf{r}_{lp} = \mathbf{T}_{ml} \mathbf{r}_{lp}. \tag{6}$$

It is noticeable that we can obtain $\mathbf{T}_{ml}$ in advance at the receivers. This means that the MLE is realized with the $O(N)$ of the computational complexity.

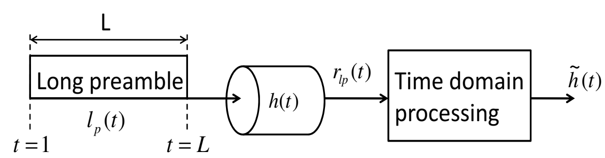

Figure 3. An impulse response estimator.

2.3 Minimum mean square error estimator

On the other hand, the MMSEE is obtained by

$$\mathbf{h}_{mmse} = (\sigma^2 \mathbf{C}_h^{-1} + \mathbf{L}_p^H \mathbf{L}_p)^{-1} \mathbf{L}_p^H \mathbf{r}_{lp}, \tag{7}$$

where $\mathbf{C}_h$ denotes the covariance matrix of the channel, that becomes a diagonal matrix if fading coefficients are statistically independent each other, that is,

$$\mathbf{C}_h = diag\{ \ E[|h_1|^2] \quad E[|h_2|^2] \quad \cdots \quad E[|h_\Delta|^2] \ \}. \tag{8}$$

From Eq.(8) the diagonal component of $\mathbf{C}_h$ is equivalent to a power delay profile of the channel. As shown in Eq.(7), $\mathbf{h}_{mmse}$ requires the prior information of the power delay profile and the noise power,σ^2.

3. Proposed scheme

3.1 Preparation

From Eq.(7),

$$\begin{aligned} \boldsymbol{h}_{mmse} &= [\sigma^2 \boldsymbol{C}_h^{-1} + \boldsymbol{L_p}^H \boldsymbol{L_p}]^{-1} \boldsymbol{L_p}^H \boldsymbol{r}_{lp}, \\ &= [\sigma^2 \boldsymbol{C}_h^{-1} + \boldsymbol{L_p}^H \boldsymbol{L_p}]^{-1} \boldsymbol{L_p}^H \boldsymbol{L_p} \\ &\cdot \ (\boldsymbol{L_p}^H \boldsymbol{L_p})^{-1} \boldsymbol{L_p}^H \boldsymbol{r}_{lp}, \\ &= [\sigma^2 \boldsymbol{I} + \boldsymbol{C}_h \boldsymbol{L_p}^H \boldsymbol{L_p}]^{-1} \boldsymbol{C}_h \boldsymbol{L_p}^H \boldsymbol{L_p} \\ &\cdot \ (\boldsymbol{L_p}^H \boldsymbol{L_p})^{-1} \boldsymbol{L_p}^H \boldsymbol{r}_{lp}, \\ &= \boldsymbol{R}^{-1} \boldsymbol{P}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{R} &= (\sigma^2 \boldsymbol{I} + \boldsymbol{X}), && (10) \\ \boldsymbol{P} &= \boldsymbol{X} \boldsymbol{h}_{ml}, && (11) \\ \boldsymbol{X} &= \boldsymbol{C}_h \boldsymbol{L_p}^H \boldsymbol{L_p}. && (12) \end{aligned}$$

These equations show that the MMSEE can be produced by using the MLE and additional information. They also suggest that the following procedure can be applied for a new MMSE channel estimation algorithm:
1) The MLE is carried out.
2) The power delay profile estimation is given by using the result from the MLE. The noise power is to be constant based on the operating point of the SNR.
3) A least square type algorithm, such as the conjugate gradient method(CGM), is applied to solve the simultaneous equation, $\boldsymbol{R}\boldsymbol{h}_{mmse} = \boldsymbol{P}$.

3.2 MMSE channel estimation algorithm using the CGM

The procedure leads us to a new MMSE algorithm shown in Table 1. Since $\boldsymbol{R}$ is the Hermitian matrix, we can solve the equation by using the CGM. Initially, we prepare $\boldsymbol{A} = \boldsymbol{L}_p^H \boldsymbol{L}_p$ and $\boldsymbol{B} = \boldsymbol{A}^{-1}$ in advance at the receiver by using the preamble matrix, $\boldsymbol{L}_p$. $\boldsymbol{h}_{ml}$ is obtained by the ML scheme before estimating $\boldsymbol{C}_h$. $f_h(\mathbf{h}_{ml})$ in Table 1 denotes to execute the power delay profile estimation. By using these results $\boldsymbol{R}$ is constructed. After that we finally obtain $\boldsymbol{h}_{mmse}$ by using the CGM. We will show that a few iterations of the CGM is enough to obtain sufficient PER performance. This contributes to restrain the increase of the computational complexity. The CGM is shown in Table 2.

Table 1. MMSE channel estimation algorithm.

1.Initialization	$\boldsymbol{L}_p \leftarrow$ $\boldsymbol{A} = \boldsymbol{L}_p^H \boldsymbol{L}_p$ $\boldsymbol{B} = \boldsymbol{A}^{-1}$
2.Long preamble	$\boldsymbol{r}_{lp} \leftarrow$
3.MLE	$\boldsymbol{h}_{ml} = \boldsymbol{B}\boldsymbol{L}_p^H \boldsymbol{r}_{lp}$
4.MMSE parameters	σ^2 is to be constant. $\boldsymbol{C}_h = f_h(\boldsymbol{h}_{ml})$
5.CGM initialization	$\boldsymbol{X} = \boldsymbol{C}_h \boldsymbol{A}$ $\boldsymbol{R} = (\sigma^2 \boldsymbol{I} + \boldsymbol{X})$ $\boldsymbol{P} = \boldsymbol{X}\boldsymbol{h}_{ml}$ $ITE \leftarrow$
6.CGM	$\boldsymbol{h}_{mmse} = CGM(\boldsymbol{R}, \boldsymbol{P}, ITE)$

Table 2. CGM.

1.Input parameters	$\mathbf{R}, \mathbf{P}, ITE$
2.Initialization	$\mathbf{h}_{mmse} = \mathbf{0}$
3.Initial setting	$\mathbf{r} = \mathbf{P}$, $\mathbf{p} = \mathbf{r}$
4.Start of loop	for $LOOP = 1 : ITE$
5.	$\mathbf{R}_p = \mathbf{R} \cdot \mathbf{p}$
6.	$D = \mathbf{p}^H \cdot \mathbf{R}_p$
7.	$\alpha = \frac{\mathbf{r}^H \cdot \mathbf{R}_p}{D}$
8.	$\mathbf{h}_{mmse} = \mathbf{h}_{mmse} + \alpha\mathbf{p}$
9.	$\mathbf{r}_p = \mathbf{r}$
10.	$\mathbf{r} = \mathbf{r} - \alpha\mathbf{R}_p$
11.	$\beta = \frac{\mathbf{r}^H \mathbf{r}}{\mathbf{r}_p^H \mathbf{r}_p}$
12.	$\mathbf{p} = \mathbf{r} + \beta\mathbf{p}$
13.End of loop	end

3.3 How to handle the σ^2 and $\boldsymbol{C}_h$

As shown in Eq.(7) the noise power, σ^2, and the covariance matrix, $\boldsymbol{C}_h$ are required for the MMSE channel estimation.

Concerning the noise power we give a constant value that is based on the operating point of the SNR. According to [5], giving a constant SNR value has resulted in negligible estimation error. We refer to this result.

In contrast to the noise power, we need to estimate $\boldsymbol{C}_h$ as properly as possible. When each ray of the impulse response is independently fluctuated based on the Rayleigh fading principle, $\boldsymbol{C}_h$ can be a diagonal matrix whose diagonal elements are given by

$$\boldsymbol{h}_d = \begin{bmatrix} E[|h_1|^2] & E[|h_2|^2] & \cdots & E[|h_\Delta|^2] \end{bmatrix}^T. \quad (13)$$

In order to obtain Eq.(13) we use the MLE, $\boldsymbol{h}_{ml}$, in Table 1. An MLE at k^{th} receiving frame is defined by $\boldsymbol{h}_{ml}(k)$. The mean value of the MLE, $\tilde{\boldsymbol{h}}_{ml}(k)$, is written as a low pass filter output which is given by

$$\tilde{\boldsymbol{h}}_{ml}(k) = \mu\tilde{\boldsymbol{h}}_{ml}(k-1) + (1-\mu)\boldsymbol{h}_{ml}(k). \quad (14)$$

Here we assume that the difference of the fading coefficients between neighbor frames is negligible. The power average of $\tilde{\boldsymbol{h}}_{ml}(k)$ is given by

$$\begin{aligned} P_{h,i}(k) &= \xi P_{h,i}(k-1) + (1-\xi)|\tilde{h}_{ml,i}(k)|^2, \\ i &= 1, 2, \cdots, \Delta. \end{aligned} \quad (15)$$

Finally, the estimator of $\boldsymbol{C}_h$ at k^{th} frame becomes as

$$\tilde{\boldsymbol{C}}_h(k) = diag\{P_{h,1}(k) \quad P_{h,2}(k) \quad \cdots \quad P_{h,\Delta}(k)\}. \quad (16)$$

The parameters μ and ξ will be discussed later.

4. Simulation results

4.1 Simulation conditions

In order to consider the performance of MMSE estimators at low SNR conditions, we have applied the antenna diversity reception and BPSK for the modulation. An exponentially attenuated delay profile with delay spread of 100ns was given for the channel. The normalized Doppler frequency, $\bar{f}_d = f_d \times T_f$, was $3.3 \times 10^{-3}[Hz]$. No correlations between receiving antennas were assumed. The simulation parameters are shown in Table 3.

Table 3. Simulation parameters

PHY	IEEE802.11g(OFDM)
Sample period T_s	$50[ns]$
Anntena diversity	1×2
Modulation	BPSK
Packet length	100 octets
Frame length T_f	148[μs]
FEC	Viterbi decoder $R = \frac{1}{2}$,$K = 7$
Synchronization	Ideal
Frequency offset	0
L	160
Δ	16

4.2 The effect of the noise power

The packet error rate (PER) performance of MMSE estimator when we give the ideal noise power and constant noise power are shown in Fig.4. The constant noise power was $\sigma^2 = \frac{1}{\sqrt{2}}$. The results show that the difference of these performance is negligible, and support the validity of the discussion in [5].

4.3 The estimation of $\boldsymbol{C}_h$

The optimum values concerning the low pass filter coefficients, μ and ξ, are evaluated in this clause. Fig.5 shows ensemble averages of PER at 1dB of SNR when μ and ξ are changed. The number of ensemble average was 50. These results demonstrate that 0.97 and 0.9 are proper for μ and ξ, respectively.

4.4 Packet error rates

The PER curves of the Ideal MMSE estimator and the proposed scheme are shown in Fig.6. The number of iterations

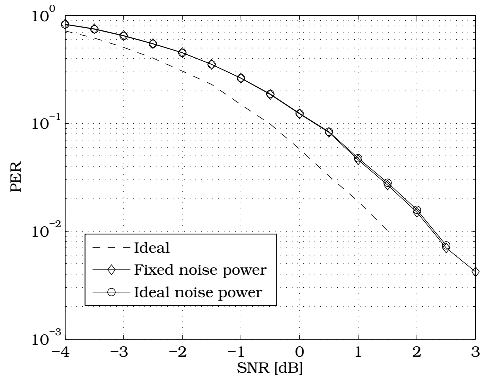

Figure 4. The effect of the noise power.

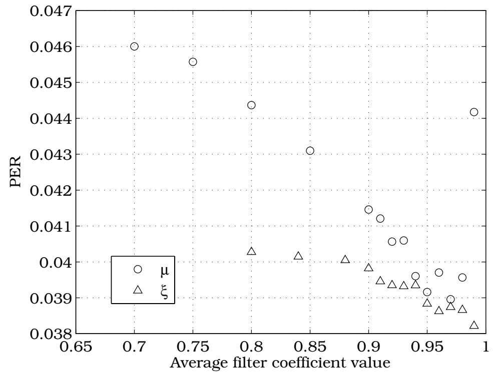

Figure 5. The effect of the low pass filter coefficients,μ and ξ.

of the CGM was five because the number was enough to obtain the proper solution while it required 16 iterations for the complete solution. The results confirm that the MMSE channel estimator show superior PER performance compared with that of the MLE, and proposed scheme achieves almost the same performance with that of the ideal MMSE estimator.

4.5 The computational complexity

Assume that $M = L - \Delta + 1$. Then, the number of total complex multiplications becomes as in Table 4. The numbers in $\{\cdot\}$ show the number of the real divisions.

The Table 4 demonstrates that small number of ITE, the number of repetition of the CGM, contributes to restrain the total computational complexity. Actually, the ITE was five in our simulation, and the total computational complexity was reduced by 45% in comparison to the full execution of the CGM.

5. Conclusion

In this report, we have proposed a new MMSE channel estimation algorithm by using the time domain long preamble for the OFDM systems. The ML channel estimation was executed before carrying out the MMSE channel estimation in order to produce the channel covariance matrix. By using the channel covariance matrix the MMSE channel estimation was

Table 4. The number of the complex multiplications

MLE (C_{MSE})	MMSEE	MMSEE($ITE = \Delta$)
$M\Delta + \Delta^2$	C_{MSE} $+ITE(7\Delta + \Delta^2)$ $+2\Delta^2$ $\{2 \cdot ITE\}$	C_{MSE} $+9\Delta^2 + \Delta^3$ $\{2\Delta\}$

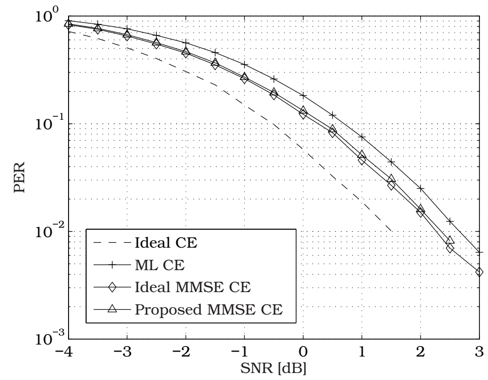

Figure 6. PER curves.

executed, and the PER performance was equivalent to that of the ideal MMSE estimator. In addition, the CGM, which was applied to solve the simultaneous equations, played an important role in restraining the computational complexity.

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