

# Synthesis of a Complex Filter Using Lossy Transformers

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**Abstract:** In this paper, the synthesis of a dissipative complex filter is proposed. The proposed circuit is realized by using resistors, capacitors, and lossy transformers. As an example, a third-order complex filter is designed and its validity of the proposed method was confirmed through computer simulation.

*Keywords*—dissipative, passive, analog, complex, filter

## 1. Introduction

An analog filter is one of the important building blocks in communication systems. The most fundamental filter is an LC filter. Since it exhibits a low noise property and provides a desirable frequency response over a wide frequency range, it is widely used in the front-end circuits of radio systems.

A frequency-shifting method [1, 3-8] simply shifts the frequency response of the prototype filter along the frequency axis. As a result, the frequency response is symmetrical with respect to the shifting frequency [1]. Extended frequency transformations [2] convert the frequency axis of the prototype filter to that of a complex filter by using a bilinear function. These methods generate an imaginary valued resistor called an imaginary resistor. Its resistance is constant, pure, and imaginary, regardless of the frequency; in other words, it is not a practical component. Therefore, the most important problem is to equivalently realize an imaginary resistor by using practical components. An imaginary resistor can be equivalently realized by using a gyrator [7]. However, it is difficult to realize a gyrator using practical passive elements because of its reciprocity. An imaginary resistor can also be equivalently realized by using ideal transformers [8]. But, it is difficult to realize this circuit because the self-inductance of the ideal transformer is infinite. In order to solve this problem, the authors have proposed a technique suitable for the use of practical transformers [9]. However, this requires lossless transformers. This means that the resulting frequency response deviates from the desired one.

This paper proposes complex filters using practical passive components. The proposed frequency transformation is obtained by composing two conventional methods. One is for the real dissipative filter, and the other is for the complex filter. The resulting complex filter includes terminating resistors, capacitors, and transformers with finite self-inductances and finite  $Q$ . Further, as a design example, a third-order complex Chebyshev BPF is designed using practical passive elements, and its frequency response is examined through computer simulation.

## 2. Proposed method

### 2.1 Dissipative real filter

Conventionally, a real BPF is designed by using an LPF-BPF transformation. In this technique, the frequency axis of a normalized real LPF is converted to that of a real BPF. The proposed complex filter is also designed by using the frequency transformation proposed in [9]. In order to realize the dissipative complex filter, the prototype filter shown in Fig.1 should be prepared. As shown in this figure, the resistors are connected in parallel with the inductors and in series with the capacitors. In order to obtain this circuit, we use the following frequency transformation.

$$s_0 = \frac{s}{\varepsilon s + 1}, \quad (1)$$

where  $\varepsilon$  is constant and  $s_0$  and  $s$  are the frequency of the lossless lowpass filter and that of the dissipative lowpass filter, respectively. From Eq.(1), we have the following predistorted frequency transfer function.

$$T_d(s) = T\left(-\frac{s}{\varepsilon s - 1}\right), \quad (2)$$

where  $T(s)$  is the transfer function of the lossless lowpass filter. The reflection coefficient  $\rho(s)$  is given by

$$\rho(s)\rho(-s) = 1 - KT_d(s)T_d(-s), \quad (3)$$

where  $K$  is a constant. Factorizing the above equation gives  $\rho(s)$ . The input impedance  $Z_I$  becomes

$$Z_I = R_S \frac{1 - \rho(s)}{1 + \rho(s)}. \quad (4)$$

The element value of the prototype lowpass filter is given by expanding  $Z_I$  in a continued fraction.

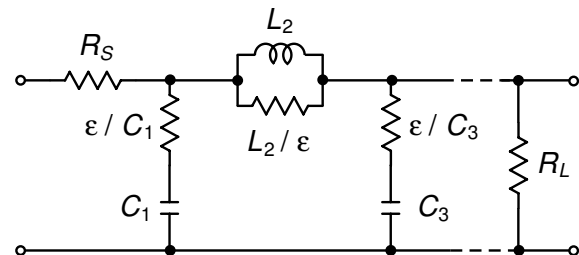


Figure 1. Dissipative real filter.

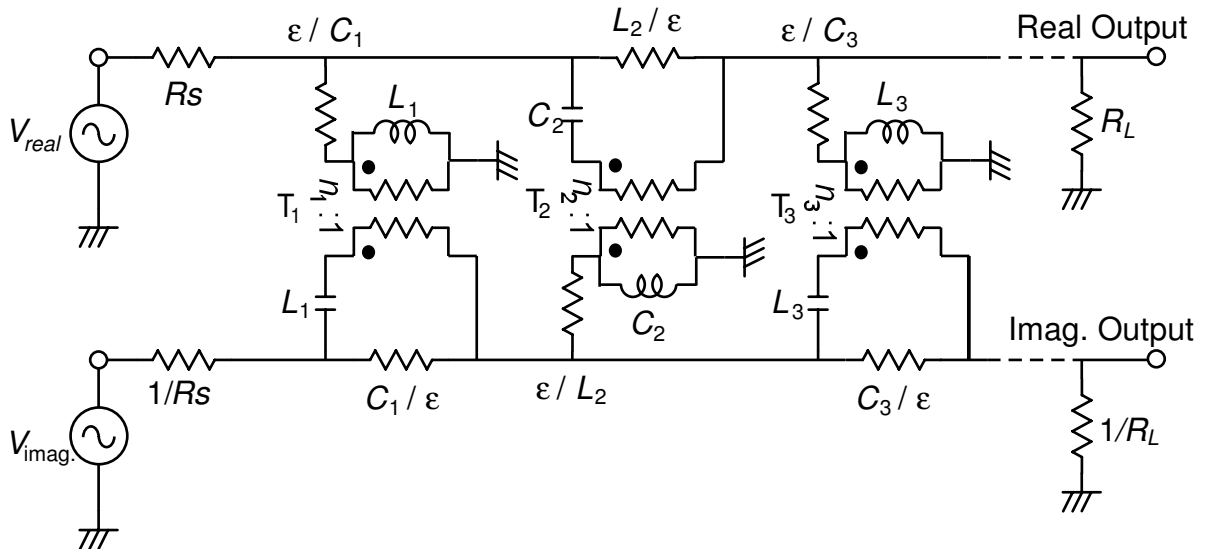


Figure 3. Proposed circuit realized by using ideal transformers.

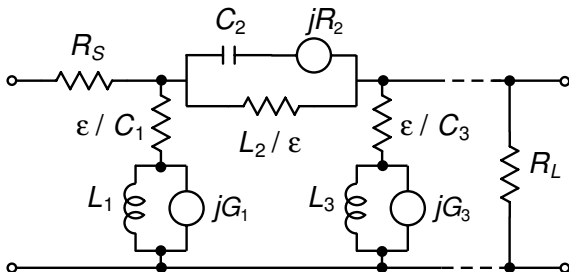


Figure 2. Complex filter including imaginary resistors.

## 2.2 Frequency transformation to complex filter

In this paper, we use the following frequency transformation [9].

$$x = f(\omega) = -\frac{a}{\omega} + x_s, \quad (5)$$

where  $x$  and  $\omega$  are the frequency of the real and that of the complex filter, respectively. The value of  $a$  and  $x_s$  are given by

$$\left. \begin{aligned} a &= \frac{\omega_0 \omega_2}{\omega_0 - \omega_2} = \frac{\omega_0 \omega_3}{\omega_3 - \omega_0} \\ x_s &= \frac{\omega_2}{\omega_0 - \omega_2} = \frac{\omega_3}{\omega_3 - \omega_0} \\ \frac{2}{\omega_0} &\equiv \frac{1}{\omega_2} + \frac{1}{\omega_3} \end{aligned} \right\}, \quad (6)$$

where  $\omega_2$  and  $\omega_3$  the passband edges of the desired complex bandpass response. The resulting complex filter is shown in Fig.2. In this circuit, the element values are given by

$$\left. \begin{aligned} L_{complex} &= 1/aC_{real} \\ jG &= jx_s C_{real} \\ C_{complex} &= 1/aL_{real} \\ jR &= jx_s L_{real} \end{aligned} \right\}, \quad (7)$$

where the subscripts *complex* and *real* denote the element values of the complex filter and those of the real filter, respectively. The imaginary resistor can be realized by using an ideal transformer [8]. Because the self-inductance of the ideal transformer is infinite, it is difficult to realize it by using the practical transformer. From Fig.4, it is found that the inductor is connected to the winding of the ideal transformer in parallel. As shown in Fig.4, they can be realized by using the transformer whose self-inductance is finite. Figure 5 shows the resulting circuit. In this figure, all the self-inductances are finite and the secondary or the primary winding has a resistor to make power dissipation. The conventional method [9] required the lossless transformer. On the other hand, the proposed method is more suitable than the conventional one from the viewpoint of its passive realization. The value of  $Q$  on the lossy winding side of the transformer near the center frequency  $\omega_0$  becomes

$$Q = \omega_0/a\varepsilon. \quad (8)$$

## 3. Design example

A complex filter which satisfies the following specifications is designed.

A third-order complex BPF ( $n = 3$ )

Passband ripple 1 dB

Passband edges  $\omega_2 = 9$  rad/s and  $\omega_3 = 11$  rad/s

Value of  $\varepsilon$  0.01

First, the transfer function  $T(s)$  is given by

$$T(s) = \frac{1}{s^3 + 0.98840s^2 + 1.23845s + 0.491341}. \quad (9)$$

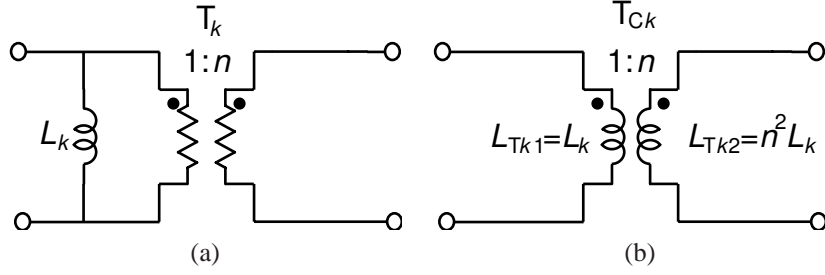


Figure 4. (a) Ideal transformer  $T_k$  and inductor  $L_k$ . (b) Equivalent circuit.

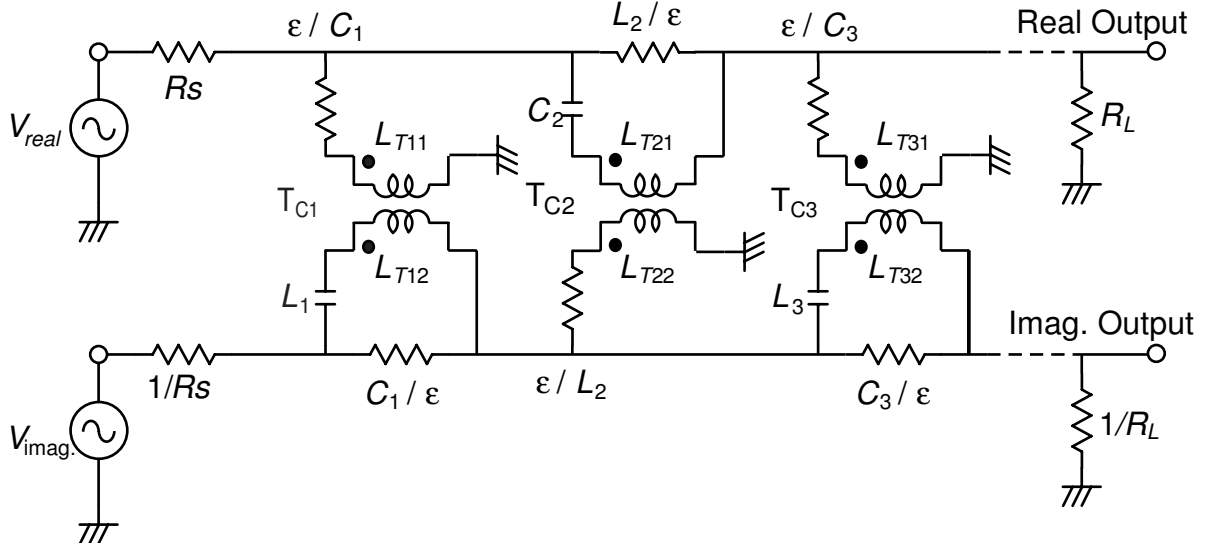


Figure 5. Proposed circuit.

From Eq.(2), we have

$$T_d(s) = \frac{(1 - 0.01s)^3}{\left\{ \begin{array}{l} 0.990239s^3 + 0.963778s^2 \\ +1.22371s + 0.491341 \end{array} \right\}}. \quad (10)$$

Substituting the above equation into Eq.(3) leads to

$$\rho(s)\rho(-s) = 1 - \frac{4K^2(1 + 0.01s)^3(-1 + 0.01s)^3}{\left\{ \begin{array}{l} 0.980574s^6 + 1.49466s^4 \\ +0.550374s^2 + 0.241416 \end{array} \right\}}. \quad (11)$$

The maximum value of  $K$  which can make the right side of the above equation be factorized into the left side form is given by

$$K = 0.238021. \quad (12)$$

The resulting  $\rho(s)$  becomes

$$\rho(s) = \pm \frac{(s + 0.158584)(s^2 + 0.774709)}{s^3 + 0.973278s^2 + 1.23577s + 0.496184}. \quad (13)$$

Taking the minus sign of  $\rho(s)$  leads to the following input impedance

$$Z_I = \frac{0.814694s^2 + 0.461061s + 0.373327}{2s^3 + 1.13186s^2 + 2.01048s + 0.619041}. \quad (14)$$

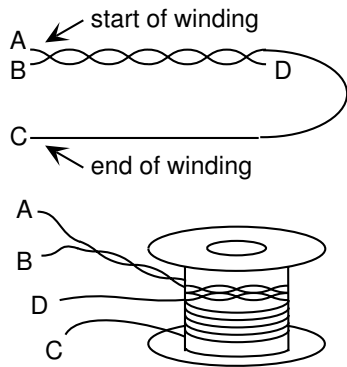
Expanding  $Z_I$  in a continued fractions leads to the following the element values.

$$\left. \begin{array}{l} R_S = 1 \\ C_1 = 2.45491 \\ L_2 = 0.744697 \\ C_3 = 2.93039 \\ R_L = 1.65817 \end{array} \right\} \quad (15)$$

Secondly, substituting  $\omega_2 = 9$  and  $\omega_3 = 11$  into equation set (6) leads to  $a = 99$ ,  $x_s = 10$  and  $\omega_0 = 9.9$ . The resulting element values of Fig. 5 become

$$\left. \begin{array}{l} R_S = 1 \\ L_{T11} = L_1 = C_1 = 0.00411462\text{H} \\ L_{T12} = 2.47971\text{H} \\ \varepsilon/C_1 = 0.00407347\Omega \\ L_{T21} = 0.752216\text{H} \\ L_{T22} = L_2 = C_2 = 0.0135639\text{H} \\ \varepsilon/L_2 = 0.0134283\Omega \\ L_{T31} = L_3 = C_3 = 0.00344698\text{H} \\ L_{T32} = 2.95993\text{H} \\ \varepsilon/C_3 = 0.00341250\Omega \\ R_L = 1.65817\Omega \end{array} \right\} \quad (16)$$

Substituting  $\omega_0 = 9.9$ ,  $a = 99$  and  $\varepsilon = 0.01$  into Eq.(8)



TDK H5AP26/16T-32H (Core)  
 BP26/16-612 (Bobbin)  
 FP26/16-6C (Holder)  
 Wire : 0.25 $\phi$  UEW

Figure 6. Transformer used in the conventional circuit [9].

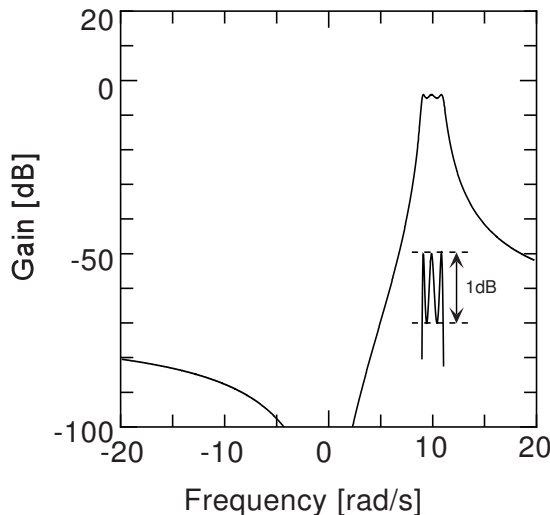


Figure 7. Frequency response.

gives  $Q = 10$ . As shown in Fig.5, the primary winding or the secondary winding of each transformer has a resistor inserted to make power dissipation. Unfortunately, this means that the value of  $Q$  on the other winding side is infinite now. The conventional circuit proposed by the authors [9] was realized by using a ferrite pot core as shown in Fig.6. The lowest value of  $Q$  was 37. If the value of  $Q$  of the manufactured transformer is higher than that of the designed circuit, the value of  $Q$  can be equivalently decreased by adding resistor to the transformer. From the above discussion, it is expected that the frequency response of the proposed method becomes better than that of the conventional one.

Figure 8 shows the simulated frequency characteristics. As shown in this figure, the proposed circuit has complex band-pass characteristics.

## 4. Conclusions

In this paper, the synthesis of a dissipative complex filter is proposed. The proposed circuit is realized by using resistors, capacitors and lossy transformers. The primary winding or the secondary winding of each transformer has a resistor inserted to make power dissipation. It is expected that the frequency response of the proposed method becomes better than that of the conventional one. The frequency response was examined through computer simulation.

Further investigation is required to confirm the validity of the proposed method through experiment.

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