Effect of the Material and Geometry Parameters on the Differential Phase Shift in the Circular Waveguide, Containing a Ferrite Cylinder with Azimuthal Magnetization and a Dielectric Toroid

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Abstract—**The influence of the material and geometry parameters on the differential phase shift, afforded by the circular waveguide with an azimuthally magnetized co-axial ferrite cylinder and a dielectric toroid, is studied for normal TE_{01} mode, provided the relative permittivity of the isotropic layer is larger than that of the anisotropic one. An iterative technique is used, employing the positive purely imaginary roots of the characteristic equation of the structure, derived by complex Kummer and real cylindrical functions, determined, varying the imaginary part of the complex first parameter of the Kummer ones. The outcomes are presented in normalized form tabularly and graphically and are debated. It is established that for certain values of the parameters of guiding line the phase shift increases, compared to the case in which the permittivities of both media are equal, considered earlier. Simultaneously, the area in which it might be produced, narrows.**

I. Introduction

The circular ferrite and ferrite-dielectric waveguides with azimuthal magnetization, sustaining normal TE_{01} mode, may give birth to differential phase shift and are suitable for the design of nonreciprocal digital phase shifters [1-5]. Important results for the quantity mentioned brought forth by a geometry of the class regarded, taking in a ferrite cylinder and a dielectric toroid of equal values of their relative permittivities, have recently been obtained [1]. For the purpose, the method of successive approximations has been harnessed, using the roots of the structure's charateristic equation, written in terms of complex Kummer and real Bessel and Neumann functions [1]. The following notations have been accepted: r_0, r_1 and $\rho = r_1 / r_0$ ($0 < \rho < 0$) – waveguide and ferrite cylinder radii, and ratio between them, resp., ε_r and ε_d – relative permittivities of the ferrite and dielectric load, α – off-diagonal element of the Polder permeability tensor of the ferrite medium ($|\alpha| < 1$), β – phase constant of the propagating wave, $\Delta\overline{\beta} = \overline{\beta}_- - \overline{\beta}_+$ – differential phase shift, $\overline{r}_{0cr}$ and $\Delta\overline{\beta}_{cr}$, and $\overline{r}_{0en-}$ and $\Delta\overline{\beta}_{en-}$ – guide radius and phase shift, relevant to cut-off and to the En_{1-} – line for fixed ρ at which the propagation for negative ferrite magnetization terminates (co-ordinates of the endpoints of the area in that the configuration operates as a phase shifter). The barred (normalized) quantities have been defined by the relations $\overline{r}_0 = \beta_0 r_0 \sqrt{\varepsilon_r}$ and $\overline{\beta} = \beta / (\beta_0 \sqrt{\varepsilon_r})$ with $\beta_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ [1], [2]. [The subscripts "+" and "–" answer to positive ($\alpha_+ > 0$) and negative ($\alpha_- < 0$) magnetization, resp.]

The object of this work is the analysis of the influence of the parameters ρ, $\overline{r}_0$, $|\alpha|$, ε_d and ε_r ($\varepsilon_d \geq \varepsilon_r$) on $\Delta\overline{\beta}$, produced by the transmission line described and on the borders of the domain in which it comes into being (on $\overline{r}_{0cr}$ and $\Delta\overline{\beta}_{cr}$, and on $\overline{r}_{0en-}$ and $\Delta\overline{\beta}_{en-}$). The aforesaid iterative approach is applied. In the computations it is assumed that $\varepsilon_d = 2\varepsilon_r$.

II. Effect of the Material and Geometry Parameters on the Differential Phase Shift

A sample of the numerical results for $\Delta\overline{\beta}$ is presented in Table I in case $\rho = 0.4$ and $\rho = 0.8$, and in Fig. 1 for $\rho = 0.8$ (see the solid curves), vs. $\overline{r}_0$ and $|\alpha|$. Table II lists some values of the quantities $\overline{r}_{0cr}$ and $\Delta\overline{\beta}_{cr}$, and $\overline{r}_{0en-}$ and $\Delta\overline{\beta}_{en-}$ for discrete $|\alpha|$, assuming $\rho = 0.4$ and $\rho = 0.8$. The pertinent to them $\Delta\overline{\beta} - LEnv_1$ ($\Delta\overline{\beta} - REnv_1$) – dashed (dotted) envelopes in Fig. 1 mark the boundaries of the area in which phase shift might be provided. Out ot it $\Delta\overline{\beta}$ does not exist (for the corresponding parameters the boxes in Table 1 are blank). The juxtaposition of the data for $\varepsilon_d = \varepsilon_r$ [1] and $\varepsilon_d = 2\varepsilon_r$ allows to reveal the influence of structure parameters on $\Delta\overline{\beta}$.

A. Impact of ρ, $\overline{r}_0$ and $|\alpha|$ at chosen ε_d and ε_r ($\varepsilon_d \geq \varepsilon_r$):

1) for all ρ, $\overline{r}_0$ and $|\alpha|$, $\Delta\overline{\beta} > 0$; 2) for specific ρ and fixed $\overline{r}_0$ ($|\alpha|$), $\Delta\overline{\beta}$ grows (diminishes) with $|\alpha|$ ($\overline{r}_0$); 3) for given $\overline{r}_0$ and $|\alpha|$ $\Delta\overline{\beta}$ increases with ρ, if $0 < \rho < 0.8$; 4) for fixed ρ $\Delta\overline{\beta}$ attains maximum (minimum) at the $\Delta\overline{\beta} - LEnv_1$ ($\Delta\overline{\beta} - REnv_1$) – boundary of the domain in that it is observed.

TABLE I. NUMERICAL EQUIVALENTS OF THE NORMALIZED DIFFERENTIAL PHASE SHIFT $\Delta\overline{\beta}$ FOR NORMAL TE_{01} MODE IN THE CIRCULAR FERRITE-DIELECTRIC WAVEGUIDE, AS A FUNCTION OF THE PARAMETERS $\overline{r}_0$ AND $|\alpha|$ IN CASE $\varepsilon_d = 2\varepsilon_r$, ASSUMING $\rho = 0.4$ AND $\rho = 0.8$.

$\overline{r}_0$ \ $\|\alpha\|$	0.1	0.2	0.3	0.4
$\rho = 0.4$				
3	0.06238 40101	0.12570 04582	0.19085 61999	0.25862 01430
4	0.04094 16180			
$\rho = 0.8$				
4	0.07746 87188	0.15656 18011	0.23916 02627	0.32773 30027
5	0.06412 65496	0.12985 97004	0.19905 10957	0.27410 11809
6	0.05613 55137	0.11399 72098	0.17555 63812	0.24331 52760
7	0.05148 07899	0.10489 19755		
8	0.04896 86286			

TABLE II. NUMERICAL VALUES OF THE LIMITING QUANTITIES $\overline{r}_{0cr}$, $\Delta\overline{\beta}_{cr}$, $\overline{r}_{0en-}$ AND $\Delta\overline{\beta}_{en-}$ FOR NORMAL TE_{01} MODE IN THE CIRCULAR FERRITE-DIELECTRIC WAVEGUIDE AS A FUNCTION OF THE PARAMETER $|\alpha|$, ASSUMING $\rho = 0.4$ AND $\rho = 0.8$.

$\|\alpha\|$	0.1	0.2	0.3	0.4
$\rho = 0.4$				
$\overline{r}_{0cr}$	2.84201 33192	2.86235 93824	2.89743 87228	2.94912 20133
$\Delta\overline{\beta}_{cr}$	0.06706 56448	0.13393 37236	0.20019 85647	0.26492 34532
$\overline{r}_{0en-}$	4.05840 59662	3.92848 10927	3.78863 38859	3.65408 89798
$\Delta\overline{\beta}_{en-}$	0.04000 51878	0.08458 98364	0.13527 33838	0.19277 61360
$\rho = 0.8$				
$\overline{r}_{0cr}$	3.69820 07418	3.73428 40334	3.79763 07951	3.89405 35497
$\Delta\overline{\beta}_{cr}$	0.08312 83817	0.16645 67577	0.25032 37148	0.33537 97463
$\overline{r}_{0en-}$	8.33323 12925	7.65971 12645	6.97777 42800	6.33081 73456
$\Delta\overline{\beta}_{en-}$	0.04840 04472	0.10136 47428	0.16260 34506	0.23654 14490

B. Impact of ε_d at fixed ε_r ($\varepsilon_d \geq \varepsilon_r$) and specific ρ:

1) for any $|\alpha|$ and $\overline{r}_0$ $\Delta\overline{\beta}$ grows with ε_d; 2) for any $|\alpha|$ $\overline{r}_{0cr}$ and $\overline{r}_{0en-}$ lessen when ε_d increases (the existence area of $\Delta\overline{\beta}$ shrinks); 3) the total maximum of $\Delta\overline{\beta}$ enlarges with ε_d.

III. CONCLUSION

The differential phase shift for normal TE_{01} mode in the azimuthally magnetized circular ferrite-dielectric waveguide is analyzed by the complex Kummer confluent hypergeometric and real cylindrical functions formalism, combined with iterative techniques. The effect of the size of the region, occupied by ferrite, of the magnitude of off-diagonal element of its permeability tensor and of the relation of relative permittivities of the media on this quantity, is studied. It is found out that in general the increment of the relative permittivity of dielectric leads to a growth of the phase shift.

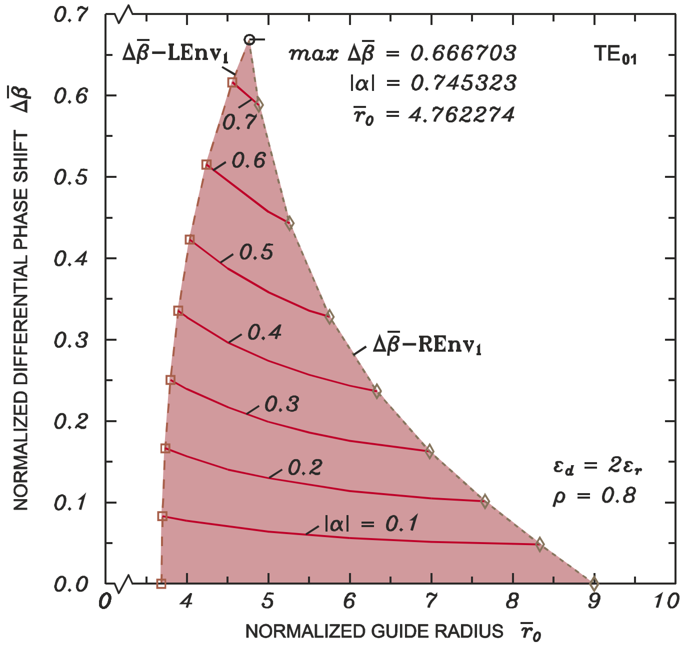

Fig. 1. Normalized differential phase shift $\Delta\overline{\beta}$ for normal TE_{01} mode in the circular ferrite-dielectric waveguide as a function of $\overline{r}_0$ and $|\alpha|$ in case $\varepsilon_d = 2\varepsilon_r$ and $\rho = 0.8$.

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