# Parametric Representation of Radar Target Responses

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Abstract—This study describes parametric representation of radar target responses and its physical interpretation. Under a high frequency approximation, a transfer function of a scattering system is factorized into three elementary parts, and a target response with three parameters that are related to a local shape at a scattering center is derived. In order to use these parameters for radar target identification, a relationship between the parameters and the response waveform is discussed.

Keywords—radar; target identification; early-time response; high frequency approximation

# I. INTRODUCTION

In radar target identification, selection and extraction of features from radar signatures are important to achieve high identification performance. Although various kinds of features that are related to target geometry and material have already been proposed and investigated, the identification performance is still insufficient [1]. In order to improve the identification performance, we derive, in this study, a parametric representation of radar target responses, whose parameters are available and suitable for target identification [2]. Under a high-frequency approximation [3][4], we first express a scattering system of electromagnetic (EM) waves in terms of three kinds of transfer functions. Next, by using these transfer functions, we derive a radar target response with three parameters that are related to a local shape of the target [5]. These parameters can be estimated from an early-time response of the radar signature. In order to use these parameters for radar target identification, a relationship between the parameters and the response waveforms is revealed and discussed.

## II. REPRESENTATION OF THE SCATTERING SYSTEM

Let us consider radar target identification as shown in Fig. 1. In general, EM wave scattering can be interpreted as a linear system whose input and output signals are incident and received pulses, respectively. Therefore, the scattering system is expressed by a scattering transfer function (or system function)  $H(\omega)$  as shown in Fig. 2, and the target response g(t) is expressed by the inverse Fourier transform as

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) F(\omega) e^{j\omega t} d\omega$$
 (1)

with

$$H(\omega) = A(\omega)e^{j\theta(\omega)},$$
(2)

where  $F(\omega)$  is a spectrum of the incident pulse f(t),  $A(\omega)$ and  $\theta(\omega)$  are the amplitude and phase spectra, respectively. In radar target detection and identification using ordinary UWB radar systems, a high frequency approximation is usually applicable to the radar system analysis. Thus, we approximate the amplitude and phase of the scattering transfer function by using the high frequency approximation [3][4] as

$$A(\omega) \cong C\omega^{\beta} \quad (\omega > 0) \tag{3}$$

$$\theta(\omega) \cong \theta(0) + \frac{d\theta(\omega)}{d\omega}\Big|_{\omega=0} \omega = \phi - \tau \omega,$$
(4)

where C is a constant,  $\beta$  is an amplitude parameter that determines the behavior of the amplitude spectrum, and  $\phi$  and  $\tau$  are the phase shift and time shift parameters, respectively. It should be noted that Eq. (4) corresponds to a linear phase approximation. Using these approximate expressions of amplitude and phase spectra, the transfer function of the scattering system can be written as

$$H(\omega) \approx H_{\beta}(\omega) \cdot H_{\phi}(\omega) \cdot H_{\tau}(\omega)$$
(5)

where

$$\begin{cases} H_{\beta}(\omega) = |\omega|^{\beta}, & H_{\phi}(\omega) = e^{j \operatorname{sgn}(\omega) \cdot \phi} \\ H_{\tau}(\omega) = e^{-j\omega\tau}, \end{cases}$$
(6)

and we set C = 1. The approximated and factorized transfer function expressed by Eq. (5) gives the parametric representation of the response waveform with three parameters ( $\beta$ ,  $\phi$ ,  $\tau$ ). This means that if these parameters are



Fig. 1: Radar target identification.

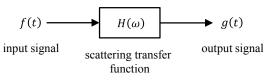


Fig. 2: Scattering transfer function of a radar system.

estimated from the target signature, we can expect that they become good features for radar target identification. Thus, we next consider the physical meaning of each transfer function and investigate the relationship between these parameters and the response waveform.

Since the transfer function  $H_{\tau}(\omega) = e^{-j\omega\tau}$  corresponds to time delay of the target response and does not make any changes to the waveform except for a pure delay, we focus on the transfer functions  $H_{\beta}(\omega)$  and  $H_{\phi}(\omega)$ . The target responses for  $H_{\beta}(\omega)$  and  $H_{\phi}(\omega)$  are given by

$$g_{\beta}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\omega|^{\beta} F(\omega) e^{j\omega t} d\omega$$
 (7)

$$g_{\phi}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j \operatorname{sgn}(\omega) \cdot \phi} F(\omega) e^{j\omega t} d\omega.$$
 (8)

From these expressions, we can see that the parameters  $\beta$  and  $\phi$  represent amplitude- and phase- distortion of the response waveform, respectively.

## **III. PHYSICAL INTERPRETATION OF THE PARAMETERS**

In order to use the parameters  $(\beta, \phi)$  for radar target identification, we next examine the relationship between the parameters and the waveform of the target response. Since the factor  $|\omega|^{\beta}$  in Eq. (7) works as a high-pass filter with low frequency suppression,  $\beta$  affects the width of the response  $g_{\beta}(t)$ . Figure 3 shows deformation of the waveform  $g_{\beta}(t)$  for monocycle pulse incidence when the parameter  $\beta$  changes from zero to two. As the incident pulse, we used a oncedifferentiated Gaussian pulse with an anti-symmetric waveform expressed as

$$f(t) = -Ate^{-\alpha t^2} \tag{9}$$

where A and  $\alpha$  are constants that determine the amplitude and the pulse width, respectively (see Fig. 3(a)). It can be seen, from Fig. 3, that the number of oscillation of the response waveform increases and the pulse width decreases as the parameter  $\beta$  increases, because the center frequency of  $g_{\beta}(t)$ shifts towards the higher frequency side by high frequency enhancement.

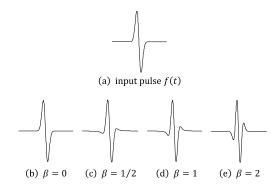


Fig. 3: Response waveform  $g_{\beta}(t)$  for  $\beta = 0, 0.5, 1, \text{ and } 2$ .

The factor  $e^{j \operatorname{sgn}(\omega) \cdot \phi}$  in Eq. (8) works as a phase shifter with the parameter  $\phi$  as a shift angle and it periodically changes the antisymmetric waveforms to asymmetric waveforms. This characteristic can be analytically shown by introducing an analytic signal. Since the transfer function  $H_{\phi}(\omega)$  can be rewritten as

$$H_{\phi}(\omega) = \cos \phi + j \sin \phi \operatorname{sgn} \omega, \qquad (10)$$

we can obtain the impulse response  $h_{\phi}(t)$  by using the inverse Fourier transform as

$$h_{\phi}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_{\phi}(\omega) e^{j\omega t} d\omega$$
  
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} (\cos \phi + j \sin \phi \operatorname{sgn} \omega) e^{j\omega t} d\omega$$
  
$$= \cos \phi \,\delta(t) - \frac{\sin \phi}{\pi t}$$
(11)

Since the target response  $g_{\phi}(t)$  corresponding to the incident pulse f(t) is expressed by a convolution integral of f(t) and h(t), we get

$$g_{\phi}(t) = \int_{-\infty}^{\infty} h_{\phi}(\tau) f(t-\tau) d\tau$$
$$= \int_{-\infty}^{\infty} \left( \cos \phi \, \delta(\tau) - \frac{\sin \phi}{\pi} \, \frac{1}{\tau} \right) f(t-\tau) d\tau$$
$$= f(t) \cos \phi - \mathcal{H}[f(t)] \sin \phi, \qquad (12)$$

where  $\mathcal{H}[f(t)]$  is the Hilbert transform of the function f(t). Equation (12) indicates that the waveform  $g_{\phi}(t)$  is expressed as a linear combination of the incident pulse and its Hilbert transform that is orthogonal to the incident pulse. Since the coefficients of two terms are  $\cos \phi$  and  $\sin \phi$ , the weights of the contributions changes as the shift parameter  $\phi$  changes. The transition of the waveform  $g_{\phi}(t)$  to the change of the parameter  $\phi$  is shown in Fig. 4. From this figure, we can see  $g_{\phi}(t)$ periodically changes its waveform between the in-phase and quadrature-phase waveforms as the shift angle  $\phi$  changes.

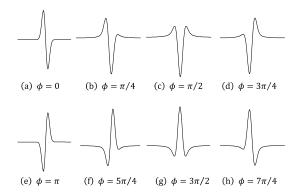


Fig. 4: Response waveform  $g_{\phi}(t)$  when  $\phi$  changes from 0 to  $2\pi$ .

In order to explain this interpretation in more detail, we introduce an *analytic signal* whose real part is the incident pulse f(t) and imaginary part is the Hilbert transform of the real part expressed as

$$f_A(t) = f(t) + j \mathcal{H}[f(t)].$$
(13)

Using the analytic signal, we can show that  $g_{\phi}(t)$  is rewritten as

$$g_{\phi}(t) = \Re[(\cos \phi + j \sin \phi) (f(t) + j \mathcal{H}[f(t)])]$$
$$= \Re[e^{j\phi}f_A(t)]$$
(14)

This equation indicates that the waveform  $g_{\phi}(t)$  is the real part of the complex-valued signal  $e^{j\phi}f_A(t)$  that is the phase shifted analytic signal. Therefore, the transfer function  $H_{\phi}(\omega)$ works as a phase shifter that changes the analytic signal  $f_A(t)$ to  $e^{j\phi}f_A(t)$ . Since deformation in anti-symmetry property of the waveform is sensitive to the shift angle  $\phi$  as shown in Fig. 4, this parameter is available and suitable for the feature for target identification.

## IV. CONCLUSIONS

We derived a parametric representation of radar target responses, whose parameters were available and suitable for target identification. Under the high-frequency approximation, we expressed a scattering system in terms of factorized transfer functions, and derived a radar target response with three parameters that were related to a local shape of the target. In order to use these parameters for radar target identification, a relationship between the parameters and response waveforms was revealed and discussed.

The proposed features are now applied to an identification problem of buried objects using subsurface radar system. The result will be shown in the near future.

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