Simultaneous Estimation of Azimuth and Elevation Angles and Frequency of Plane Wave Signals Using a Modified Matrix Pencil Method

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Abstract: In this paper, the modified matrix pencil method [1] is extended from 2D to 3D for simultaneously estimating azimuth and elevation angles and frequency of multiple signals impinging on a volumetric array of sensors. Conventional matrix pencil method, which is used to estimate unknown parameters, needs a separate algorithm to associate the estimated components with each other to get proper groups of azimuth and elevation angles and frequencies of incoming signals [2]. The method proposed here automatically estimates unknown parameters in a group form, thereby bypassing the computationally expensive pairing operation. Moreover, simulation results show that grouping of unknown parameters using the proposed method is always correct in contrast to matrix pencil method whose results are sometimes erroneous.

1. Introduction

Many methods and techniques have been developed in the last few decades to estimate unknown parameters of plane wave signals using sensor arrays [3]. Supper resolution techniques like MUSIC [4] and ESPRIT [5] are widely used to estimate the unknown parameters by exploiting the eigen structure of the covariance matrix. These techniques are usually suitable for stationary environment and assume that the signals are not fully correlated or coherent. Another method for estimating the unknown parameters of sinusoids in noise is matrix pencil method [6]. The matrix pencil method is a direct data domain method that analyzes the data on snapshot-by-snapshot basis; consequently, non-stationary environment can be handled easily. All the techniques mentioned above are basically one dimensional techniques and find only one direction cosine of incoming signals. To find azimuth and elevation angles along with frequency, all three direction cosines need to be estimated. For estimation of other direction cosines simultaneously, these techniques are enhanced for multidimensional cases. In [7] matrix pencil method is enhanced for 2-D signals and matrix enhancement matrix pencil method (MEMP) is developed. The MEMP method is further enhanced in [2] to simultaneously estimate azimuth and elevation angles and frequency of incoming plane wave signals.

A major drawback in these enhanced techniques is that estimated parameters of different signals are not grouped automatically. To properly group these parameters, an algorithm which exploits the orthogonal property between signal and noise subspaces is used. Although this algorithm separates estimated parameters, it is computationally expensive and does not always render the correct grouping because it is a correlation maximization algorithm. Modified matrix enhancement matrix pencil (MMEMP) method [1] remedies this problem. This MMEMP method was originally developed to estimate the frequency components of a 2-dimensional signal. Here, we extend this method for 3-dimensional case and apply it on a uniform volumetric array to estimate azimuth and elevation angles and frequency of plane wave signals.

2. Problem Formulation

Consider a 3-dimensional uniform array of sensors in space with axes oriented along the cartesian coordinates. The distances between array elements are Δx, Δy and Δz along x, y and z axis, respectively, and the corresponding number of sensors are A, B and C. If I signals with azimuth and elevation angles (ϕ_i, θ_i) and wavelengths λ_i $(i = 1...I)$ arrive at the input of this array, the voltage level at the output of the sensor located at cartesian coordinates (a, b, c) is given by

$$v(a,b,c) = \sum_{i=1}^{I} M_i \exp j(\gamma_i + \frac{2\pi}{\lambda_i}\Delta x \cos\phi_i \sin\theta_i a + \frac{2\pi}{\lambda_i}\Delta y \sin\phi_i \sin\theta_i b + \frac{2\pi}{\lambda_i}\Delta z \cos\theta_i c) \quad (1)$$

where M_i's are the amplitudes and γ_i's are the phases of incoming signals. If we define f_{1i}, f_{2i} and f_{3i} as

$$f_{1i} = \tfrac{1}{\lambda_i}\Delta x \cos\phi_i \sin\theta_i, \quad f_{2i} = \tfrac{1}{\lambda_i}\Delta y \sin\phi_i \sin\theta_i$$
$$f_{3i} = \tfrac{1}{\lambda_i}\Delta z \cos\theta_i \quad (2)$$

then $v(a,b,c)$ can be written as

$$v(a,b,c) = \sum_{i=1}^{I} M_i \exp j(\gamma_i + 2\pi f_{1i} a + 2\pi f_{2i} b + 2\pi f_{3i} c). \quad (3)$$

To find direction cosines of incoming signals, we have to estimate the three frequencies of each 3-D sinusoid and group them appropriately. For further simplification we define

$$\alpha_i = M_i \exp(j\gamma_i), \quad x_i = \exp(j2\pi f_{1i})$$
$$y_i = \exp(j2\pi f_{2i}), \quad z_i = \exp(j2\pi f_{3i}) \quad (4)$$

so that (3) becomes

$$v(a,b,c) = \sum_{i=1}^{I} \alpha_i x_i^a y_i^b z_i^c. \quad (5)$$

Hence, our target is to estimate x_i, y_i and z_i, which can be used to find three direction cosines of the incoming signals.

3. Matrix Pencil Approach

The 3-D matrix pencil method incorporates an enhanced data matrix D_e of dimensions LKR×(A-L+1)(B-K+1)(C-R+1) which is constructed from (5) and can be written in the form

$$D_e = F_1 A F_2 \tag{6}$$

where L, K and R are pencil parameters and matrices F_1, F_2 and A are defined as in [2]. Applying SVD, we get

$$D_e = U \Lambda V^H = U_s \Lambda_s V_s^H + U_n \Lambda_n V_n^H \tag{7}$$

where U_s and U_n span the signal and noise sub-spaces respectively. It can be seen from (6) and (7) that U_s and F_1 span the same range space provided

$$\mathtt{rank}(F_1) = \mathtt{rank}(F_2) = I. \tag{8}$$

If the above condition holds then U_s and F_1 are related as

$$U_s = F_1 T \tag{9}$$

where T is a unique $I \times I$ full rank matrix.

Now, proceeding as in [2], if we define
F_{1z}: F_1 with last LK rows deleted
F_{2z}: F_1 with first LK rows deleted
U_{1z}: U_s with last LK rows deleted
U_{2z}: U_s with first LK rows deleted
after some mathematical manipulations F_{2z} can be written as

$$F_{2z} = F_{1z} Z_d \tag{10}$$

where $Z_d = \mathtt{diag}(z_1 ... z_I)$ and (9) and (10) implies that

$$U_{1z} = F_{1z} T \ \ \mathtt{and} \ \ U_{2z} = F_{1z} Z_d T. \tag{11}$$

It can be shown that the generalized eigenvalues of matrix pencil (U_{1z}, U_{2z}) are z_i's and can be obtained as eigenvalues of $U_{1z}^\dagger U_{2z}$, where $U_{1z}^\dagger$ is the pseudo inverse of U_{1z}.

Similarly, if we define
$U_{sy} = JU_s$ and $U_{sx} = PU_s$
U_{1y}: U_{sy} with last LR rows deleted
U_{2y}: U_{sy} with first LR rows deleted
U_{1x}: U_{sx} with last KR rows deleted
U_{2x}: U_{sx} with first KR rows deleted
where J and P are exchange matrices as defined in [2], then y_i's and x_i's can be computed as the eigenvalues of $U_{1y}^\dagger U_{2y}$ and $U_{1x}^\dagger U_{2x}$ respectively. To group x_i's, y_i's and z_i's, a grouping algorithm is also required which is not discussed here.

4. Modified MP Method for 3-D Estimation

If W_z is the matrix of generalized eigenvectors of matrix pencil (U_{1z}, U_{2z}) then by definition

$$U_{2z} W_z = U_{1z} W_z Z_d \tag{12}$$
$$\Rightarrow \ F_{1z} Z_d T W_z = F_{1z} T W_z Z_d$$
$$\Rightarrow \ T^{-1} Z_d T = W_z Z_d W_z^{-1} \tag{13}$$

Since, the generalized eigenvalues and eigenvectors of matrix pencil (U_{1z}, U_{2z}) are the same as the eigenvalues and eigenvectors of $U_{1z}^\dagger U_{2z}$, we have

$$U_{1z}^\dagger U_{2z} = W_z Z_d W_z^{-1} \tag{14}$$

and from (13) and (14), we have

$$U_{1z}^\dagger U_{2z} = T^{-1} Z_d T \tag{15}$$

If W_y and W_x has generalized eigenvectors of matrix pencils (U_{1y},U_{2y}) and (U_{1x},U_{2x}) respectively, and $Y_d = \mathtt{diag}(y_1..y_I)$ and $X_d = \mathtt{diag}(x_1..x_I)$, then it can be shown that

$$U_{1y}^\dagger U_{2y} = T^{-1} Y_d T = W_y Y_d W_y^{-1} \tag{16}$$
$$U_{1x}^\dagger U_{2x} = T^{-1} X_d T = W_x X_d W_x^{-1} \tag{17}$$

Subsequently, we will use eq.s (14), (16) and (17) to determine the matrices Z_d, Y_d and X_d. More specifically, Z_d and W_z are computed from (15). However, the exact procedure to determine associated Y_d and X_d depends on whether all z_i's in Z_d are distinct or not. Both cases are considered separately.

4.1 All z_i's are Distinct

If all z_i's are distinct then from Theorem 1 in [1] we have

$$W_z G = T^{-1} \tag{18}$$

where G is a diagonal matrix. Using (18) in (16) we get

$$U_{1y}^\dagger U_{2y} = W_z G Y_d G^{-1} W_z^{-1} = W_z Y_d W_z^{-1}. \tag{19}$$

Since G and Y_d are diagonal matrices, then (19) is reduced to

$$Y_d = W_z^{-1} U_{1y}^\dagger U_{2y} W_z \tag{20}$$

Similarly it can also be shown that

$$X_d = W_z^{-1} U_{1x}^\dagger U_{2x} W_z. \tag{21}$$

Since Y_d and X_d are obtained using W_z. Therefore it will be guaranteed that the corresponding diagonal entries of Y_d and X_d will be paired up correctly with Z_d.

4.2 Some z_i's are Repeated

If we have repeated z_i's, then (16) is still valid but relation between W_z and T^{-1} given by (18) does not hold. In particular if there are 'r' distinct eigenvalues in Z_d then (18) modifies to

$$W_z Q = T^{-1} \tag{22}$$

where Q is a block diagonal matrix and can be written as

$$Q = \begin{bmatrix} Q_1 & 0 & \mathbf{O} \\ 0 & \ddots & 0 \\ \mathbf{O} & 0 & Q_r \end{bmatrix} \tag{23}$$

where sub-matrices Q_ϵ (ϵ = 1,...r) correspond to each distinct eigenvalue. If q is the multiplicity of eigenvalue then size of Q_ϵ is $q \times q$. Now, if G is replaced by Q in (19) we get

$$U_{1y}^\dagger U_{2y} = W_Z Q Y_d Q^{-1} W_z^{-1} \quad (24)$$

$$\Rightarrow \quad QY_dQ^{-1} = W_z^{-1} U_{1y}^\dagger U_{2y} W_z \quad (25)$$

and similarly

$$QX_dQ^{-1} = W_z^{-1} U_{1x}^\dagger U_{2x} W_z. \quad (26)$$

Now, we have matrices QY_dQ^{-1} and QX_dQ^{-1} from which we have to find out y_i's and x_i's. Since, Y_d and X_d are diagonal matrices and Q is a block diagonal matrix, we have

$$QY_dQ^{-1} = \begin{bmatrix} Q_1Y_1Q_1^{-1} & 0 & \mathbf{O} \\ 0 & \ddots & 0 \\ \mathbf{O} & 0 & Q_rY_rQ_r^{-1} \end{bmatrix} \quad (27)$$

$$QX_dQ^{-1} = \begin{bmatrix} Q_1X_1Q_1^{-1} & 0 & \mathbf{O} \\ 0 & \ddots & 0 \\ \mathbf{O} & 0 & Q_rX_rQ_r^{-1} \end{bmatrix} \quad (28)$$

For those eigenvalues which have multiplicity 1, their corresponding sub matrices will be of size one by one and then

$$Q_\epsilon Y_\epsilon Q_\epsilon^{-1} = Y_\epsilon \texttt{ and } Q_\epsilon X_\epsilon Q_\epsilon^{-1} = X_\epsilon \quad (29)$$

If eigenvalues in Z_d have multiplicity $q > 1$, then corresponding sub matrices in (27) and (28) will be of size $q \times q$ then

$$Q_\epsilon Y_\epsilon Q_\epsilon^{-1} = A_\epsilon \quad (30)$$

$$Q_\epsilon X_\epsilon Q_\epsilon^{-1} = B_\epsilon. \quad (31)$$

From (30) and (31) we can see that due to similarity transformation the eigenvalues of A_ϵ and B_ϵ are the diagonal elements of Y_ϵ and X_ϵ respectively. Up to this stage pairing of z_i's with x_i's and pairing of z_i's with y_i's is correct but for eigenvalues of Z_d with multiplicity $q > 1$ their corresponding x_i's and y_i's which are calculated from A_ϵ and B_ϵ may not pair up correctly. So the next task is to pair up the eigenvalues of A_ϵ and B_ϵ properly. After eigenvalue decomposition, A_ϵ can be written as

$$A_\epsilon = V_\epsilon Y_\epsilon V_\epsilon^{-1}. \quad (32)$$

Next, we consider two sub cases of case 2.

4.2.1 A_ϵ has distinct eigenvalues

Using (30) and (32) we get

$$V_\epsilon G_\epsilon = Q_\epsilon \quad (33)$$

where G_ϵ is a diagonal matrix. Using (31) and (33) we get

$$B_\epsilon = V_\epsilon G_\epsilon X_\epsilon G_\epsilon^{-1} V_\epsilon^{-1}. \quad (34)$$

Since G_ϵ and X_ϵ are diagonal hence (34) will be reduced to

$$B_\epsilon = V_\epsilon X_\epsilon V_\epsilon^{-1} \quad (35)$$

$$\Rightarrow \quad X_\epsilon = V_\epsilon^{-1} B_\epsilon V_\epsilon. \quad (36)$$

Since, X_ϵ is obtained using V_ϵ which was obtained by EVD of A_ϵ, therefore, it will be guaranteed that corresponding diagonal entries of X_ϵ which are eigenvalues of A_ϵ will pair up correctly with the eigenvalues of B_ϵ or diagonal entries of Y_ϵ.

4.2.2 A_ϵ has repeated eigenvalues

In this case the relation (33) between V_ϵ and Q_ϵ is not valid. For the eigenvalues with multiplicity 1, corresponding columns in V_ϵ and Q_ϵ are proportional. However in the case of repeated eigenvalues, the corresponding columns in Q_ϵ are linear combination of those in V_ϵ. This relation can be described as

$$V_\epsilon S_\epsilon = Q_\epsilon \quad (37)$$

where S_ϵ is a block diagonal matrix of size $q \times q$

$$S_\epsilon = \begin{bmatrix} S_{\epsilon 1} & 0 & \mathbf{O} \\ 0 & \ddots & 0 \\ \mathbf{O} & 0 & S_{\epsilon p} \end{bmatrix} \quad (38)$$

In the above equation the sub-matrices $S_{\epsilon\psi}$ ($\psi = 1, ...p$) correspond to each distinct eigenvalue of A_ϵ, where we assume that there are 'p' distinct eigenvalues of A_ϵ. The size of $S_{\epsilon\psi}$ will be $m \times m$ where m is the multiplicity of ψ^{th} eigenvalue of A_ϵ. By combining (31) and (37) we get

$$V_\epsilon S_\epsilon X_\epsilon S_\epsilon^{-1} V_\epsilon^{-1} = B_\epsilon \quad (39)$$

$$\Rightarrow \quad S_\epsilon X_\epsilon S_\epsilon^{-1} = V_\epsilon^{-1} B_\epsilon V_\epsilon. \quad (40)$$

Since X_ϵ is diagonal and S_ϵ is block diagonal, we have

$$S_\epsilon X_\epsilon S_\epsilon^{-1} = \begin{bmatrix} S_{\epsilon 1}X_{\epsilon 1}S_{\epsilon 1}^{-1} & 0 & \mathbf{O} \\ 0 & \ddots & 0 \\ \mathbf{O} & 0 & S_{\epsilon p}X_{\epsilon p}S_{\epsilon p}^{-1} \end{bmatrix} \quad (41)$$

where for those eigenvalues of A_ϵ, which have multiplicity 1, $S_{\epsilon\psi}$ and $X_{\epsilon\psi}$ are 1×1 matrices so that

$$S_{\epsilon\psi} X_{\epsilon\psi} S_{\epsilon\psi}^{-1} = X_{\epsilon\psi}. \quad (42)$$

On the other hand for the eigenvalues of A_ϵ which have multiplicity $m > 1$, the corresponding sub-matrices in (41) will be of size $m \times m$, and can be written as

$$C_{\epsilon\psi} = S_{\epsilon\psi} X_{\epsilon\psi} S_{\epsilon\psi}^{-1}. \quad (43)$$

In this case we can obtain the diagonal elements of $X_{\epsilon\psi}$ from the eigenvalues of $C_{\epsilon\psi}$, since $X_{\epsilon\psi}$ and $C_{\epsilon\psi}$ are related through a similarity transformation. Consequently, the values of x_i's which we get from $X_{\epsilon\psi}$ are paired with repeated eigenvalues of A_ϵ which in turn are properly grouped with corresponding repeated diagonal entries in Z_d.

5. Simulation Results

In this section, we present simulation results to illustrate the performance of proposed extended modified matrix pencil method and compare it with the conventional MEMP method. The noise contaminated signal model is given by

$$v(a, b, c) = \sum_{i=1}^{I} \alpha_i x_i^a y_i^b z_i^c + w(a, b, c) \quad (44)$$

where $w(a, b, c)$ is the sensor's noise located at coordinates (a,b,c). Details of the signals are given in Table 1. The results of these two methods are shown in tables II and III.

Table 1. Summary Of Signals Incident On The Sensor Array

No.	Frequency	ϕ	θ	f_1	f_2	f_3
1.	2000Hz	50^o	28^o	0.0754	0.0899	0.2207
2.	2000Hz	25.6^o	28^o	0.1058	0.0507	0.2207
3.	1828Hz	59^o	15^o	0.0305	0.0507	0.2207
4.	1800Hz	50^o	40^o	0.0930	0.1108	0.1724

It may be noted that the estimation accuracy of both methods is almost same but the modified method's computational complexity is less than that of conventional method. This is because the fact, that modified method does not requires an additional computationally expensive grouping algorithm which is required by the conventional method. To calculate frequency, θ and ϕ of each signal, proper grouping of x_i's, y_i's and z_i's is must. Figure1 shows the errors in calculating θ and ϕ because of wrong grouping in conventional method. Improvement in θ and ϕ estimates using modified method is obvious from figure2.

Table 2. Estimates Using Conventional Matrix Pencil Method

No.	Mean			Variance $(\times 10^{-4})$		
	f_1	f_2	f_3	f_1	f_2	f_3
1.	0.0757	0.0899	0.2195	0.0332	0.1966	0.0339
2.	0.1060	0.0502	0.2207	0.0041	0.0038	0.0005
3.	0.0305	0.0511	0.2225	0.0034	0.0024	0.0653
4.	0.0928	0.1108	0.1726	0.0413	0.0292	0.3237

Table 3. Estimates Using Modified Matrix Pencil Method

No.	Mean			Variance $(\times 10^{-5})$		
	f_1	f_2	f_3	f_1	f_2	f_3
1.	0.0756	0.0899	0.2195	0.0388	0.2016	0.0339
2.	0.1060	0.0503	0.2207	0.0041	0.0040	0.0005
3.	0.0306	0.5011	0.2225	0.0053	0.0025	0.0653
4.	0.0928	0.1108	0.1726	0.0415	0.0255	0.3237

6. Conclusion

We have extended the modified MEMP method to deal with 3D frequency estimation problem. Using this enhancement, we can simultaneously estimate the azimuth and elevation angles of arrival along with the frequency of multiple plane wave signals without facing the grouping problem. We have shown that proposed method like modified 2D method always group the corresponding parameters in correct manner in contrast with the MEMP method which often fails to form the correct groups. Computationally, this method is also superior from conventional method for two reasons. First is that it does not require an additional grouping algorithm and second is that for distinct eigenvalues it requires only one eigenvalue decomposition.

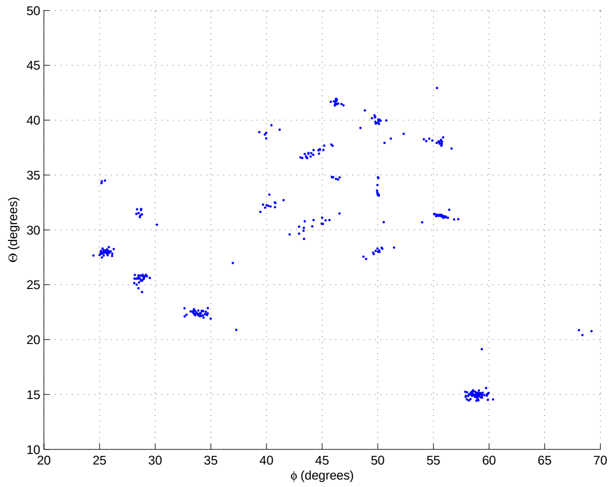

Figure 1. Hundred independent estimates of Azimuth and Elevation angles using conventional MEMP method

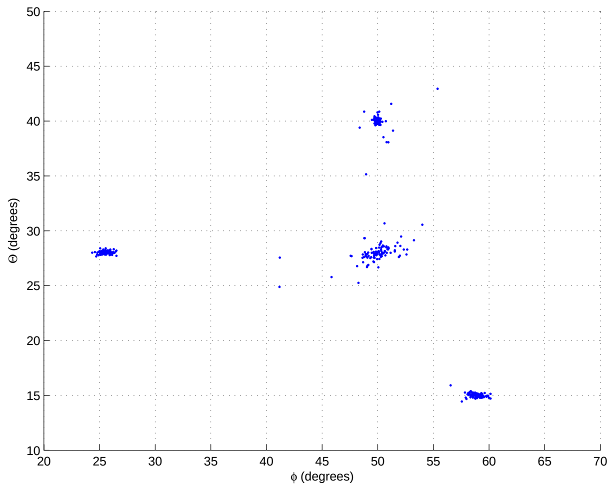

Figure 2. Hundred independent estimates of Azimuth and Elevation angles using modified MEMP method

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