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Semi-Blind Channel Estimation for OFDM with Multiple Receive Antennas

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Abstract: This paper presents a semi-blind channel estimation scheme for OFDM systems with multiple receive antennas. Using the statistical property of the received signal, we combine the pilot-based least-squares estimator with the subspace-based blind estimator. To improve the performance of the subspace-based method, we reduce the effect on the residual error which affects the accuracy of the noise subspace. Simulation results show that the proposed method is effective and improves the performance.

1. Introduction

In wireless communication systems for OFDM, a coherent signal detection requires a reliable estimate of the fading channel. One method to estimate the channel is to insert pilot symbols among transmitted data symbols [1], but the use of pilot symbols decreases the transmission bandwidth. Blind channel estimation techniques do not require any pilot but need to have many sample blocks to obtain reliable channel information [2]. A rather promising method consists of combining pilot-based and blind-based channel estimation, which is often referred to a semi-blind channel estimation. The semi-blind approach may allow a significant reduction in the number of pilot symbols and reduce the bit error rate in severe reception conditions. Thus, a semi-blind channel estimation for OFDM systems has been studied extensively in the recent years [3]-[4].

In this paper, we propose a semi-blind channel estimation scheme for OFDM systems with multiple receive antennas. To overcome the shortcoming of the subspace-based blind channel estimation approach, the pilot-based least-squares estimator is combined with the subspace-based blind estimator. Thus, both the statistic of the received signals and the known information of pilot symbols are used to estimate the channel. In addition, to improve the performance of the subspace-based method, we reduce the effect on the residual error which affects the accuracy of the noise subspace.

2. System Model

We consider an OFDM system with N_R receive antennas. Let the number of subcarriers be N, where N_P subcarriers are reserved for pilot symbols and $N_D = N - N_P$ subcarriers are used for data symbols. The transmitted symbols of the frequency domain are denoted by $\mathbf{X}^{(i)} = [x(0), \cdots, x(N-1)]^T$ and the received symbols are denoted by $\mathbf{Y}^{(i)} = [[\mathbf{Y}^{(i)}(0)]^T, \cdots, [\mathbf{Y}^{(i)}(N-1)]^T]^T$ with $\mathbf{Y}^{(i)}(k) = [y_1^{(i)}(k), y_2^{(i)}(k), \cdots, y_{N_R}^{(i)}(k)]^T$ where $y_j^{(i)}(k)$ is the received signal by the j-th receive antenna on subcarrier k of the i-th block. Then, the transmitted symbols are fed to an inverse discrete fourier transform (IDFT) to produce the OFDM signal, and a cyclic prefix (CP) is inserted in front of the IDFT output vector, which is a cyclic extension of the IDFT output sequence in order to eliminate the inter-block interference (IBI). The guard interval is chosen to be longer than the maximum delay spread of the channel. Let us define the DFT matrix as

$$F(i) := \frac{1}{\sqrt{N}}[e^{-j2\pi i(0)/N}, \cdots, e^{-j2\pi i(N-1)/N}]^T \quad (1)$$

$$F := [F(0), \cdots, F(N-1)] \quad (2)$$

This results in the time domain signal of the i-th OFDM symbol written as

$$\mathbf{s}^{(i)} = \mathbf{F}^H \mathbf{X}^{(i)} \quad (3)$$

where $(\cdot)^H$ denotes complex conjugate transpose of $(\cdot)$. The transmitted signal passes through a dispersive channel and is corrupted by a spatially and temporally zero mean uncorrelated complex Gaussian noise with variance σ_z^2. At the receive antenna, the received signal is expressed as

$$\mathbf{r}^{(i)}(n) = \sum_{l=0}^{L} \mathbf{h}^{(i)}(l)\mathbf{s}^{(i)}(n-l) + \mathbf{z}^{(i)}(n), \quad n = 0, \cdots, N-1 \quad (4)$$

where L is the maximum channel order, $\mathbf{z}^{(i)}(n)$ is an additive white Gaussian noise vector, and

$$\mathbf{r}^{(i)}(n) = [r_1^{(i)}(n), \cdots, r_{N_R}^{(i)}(n)]^T \quad (5)$$

$$\mathbf{h}^{(i)}(l) = [h_1^{(i)}(l), \cdots, h_{N_R}^{(i)}(l)]^T \quad (6)$$

where $h_j^{(i)}(l)$ is the channel impulse response from the transmit antenna to the j-th receive antenna. Note that $\mathbf{s}^{(i)}(n)$ for $n < 0$ are symbols from the CP. For notational simplicity, we will drop the block index (i) in the sequel except where it is needed for clarification to show the explicit dependence on the block sequence.

3. Channel Estimation

3.1 Subspace-based Blind Estimation

In order to consider the i-th block of N transmitted symbols, the received signal $\mathbf{r}^{(i)}(n)$ in (4) can be written in a matrix-vector form as

$$\mathbf{r} = \mathcal{H}\mathbf{d} + \mathbf{z} = \mathcal{H}\mathcal{F}^H\mathbf{X} + \mathbf{z} = \mathcal{G}\mathbf{X} + \mathbf{z} \quad (7)$$

where

$$\begin{aligned}\mathbf{r} &= [[\mathbf{r}(0)]^T, \cdots, [\mathbf{r}(N-1)]^T]^T, \\ \mathbf{d} &= [s(N-L), \cdots, s(N-1), s(0), \cdots, s(N-1)]^T, \\ \mathcal{F} &= [F(N-L), \cdots, F(N-1), F(0), \cdots, F(N-1)]\end{aligned}$$

and

$$\mathcal{H} = \begin{bmatrix} \mathbf{h}(L) & \cdots & \mathbf{h}(0) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}(L) & \cdots & \mathbf{h}(0) & & \vdots \\ \vdots & & \ddots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}(L) & \cdots & \mathbf{h}(0) \end{bmatrix} \quad (8)$$

From (7), the data model is applicable to both a block stationary channel and a static channel, and does not impose any constraints on the number of CPs. Then, the statistical covariance matrix of $\mathbf{r}$ is written as

$$\begin{aligned}\mathbf{R}_r &= E\{\mathbf{r}\mathbf{r}^H\} \\ &= \mathcal{G}E\{\mathbf{X}\mathbf{X}^H\}\mathcal{G}^H + \sigma_z^2\mathbf{I}_{NN_R} \\ &= \mathcal{G}\mathbf{R}_x\mathcal{G}^H + \sigma_z^2\mathbf{I}_{NN_R} \quad (9)\end{aligned}$$

If $\mathcal{G}$ is of full column rank, the rank of $\mathcal{G}\mathbf{R}_x\mathcal{G}^H$ is N. Since the rank of $\mathbf{R}_r$ has full rank, the covariance matrix $\mathbf{R}_r$ has two mutually orthogonal subspaces, i.e., a signal subspace with dimension N and a noise subspace with dimension $d = NN_R - N$. Here, the noise eigenvectors are denoted by $\mathbf{V}_i$ where $i = 1, 2, \cdots, d$. Using the techniques in the standard subspace method [3], we have

$$\mathbf{V}_i^H\mathcal{G} = 0, \quad i = 1, 2, \cdots, d \quad (10)$$

That is, $\mathbf{V}_i$ spans the left null space of $\mathcal{G}$. To find the signal and noise subspaces, the true $\mathbf{R}_r$ is required. However, in practice, $\mathbf{R}_r$ is estimated over N_b OFDM blocks as $\hat{\mathbf{R}}_r = \frac{1}{N_b}\sum_{i=1}^{N_b}\mathbf{r}^{(i)}[\mathbf{r}^{(i)}]^H$. Thus, when a channel is estimated by the orthogonal relationship in (10), only estimate $\hat{\mathbf{V}}_i$ of the eigenvectors spanning the noise subspace, which are obtained by the eigenvalue decomposition of $\hat{\mathbf{R}}_r$, is available in practice.

In order to use the Sylvester structure of $\mathcal{H}$, $\hat{\mathbf{V}}_i$ is divided into blocks as $\hat{\mathbf{V}}_i = [\hat{\mathbf{v}}_i^T(0), \cdots, \hat{\mathbf{v}}_i^T(N-1)]^T$ where $\hat{\mathbf{v}}_i(m)$ is a $N_R \times 1$ vector. Denote $\mathbf{h} = [\mathbf{h}^T(0), \cdots, \mathbf{h}^T(L)]^T$ as the $N_R(L+1)\times 1$ channel impulse response vector. Then, (10) can be expressed equivalently as

$$\hat{\mathcal{V}}_i^H\mathbf{h} = \mathbf{0}_{N\times 1} \quad (11)$$

where

$$\hat{\mathcal{V}}_i = \begin{bmatrix} \hat{\mathbf{v}}_i(0) & \cdots & \hat{\mathbf{v}}_i(N-1) \\ \hat{\mathbf{v}}_i(1) & \cdots & \hat{\mathbf{v}}_i((N)_N) \\ \vdots & & \vdots \\ \hat{\mathbf{v}}_i(L) & \cdots & \hat{\mathbf{v}}_i((N-1+L)_N) \end{bmatrix}$$

where $(k)_N$ denotes the residue of k modulo N.

By stacking across d vectors from the noise subspace, (11) is generalized as

$$\hat{\mathcal{V}}^H\mathbf{h} = \mathbf{0}_{dN\times 1} \quad (12)$$

where $\hat{\mathcal{V}} = [\hat{\mathcal{V}}_1, \cdots, \hat{\mathcal{V}}_d]$ is an $N_R(L+1)\times dN$ matrix.

When the number of OFDM blocks is insufficient to estimate the covariance matrix $\mathbf{R_r}$, it causes the residual error which affects the accuracy of the noise subspace and degrades the performance of the subspace-based blind estimation [5]. To improve the performance of the subspace-based estimation method, it is needed to reduce the residual error. In order to reduce the effect on the residual error, we present a method to improve the accuracy of $\hat{\mathcal{V}}$ composed of the noise subspace.

Then, the weighting factor $c(i)$ is defined to consider the orthogonality between vectors $\mathbf{a}$, $\mathbf{b} \in \mathbf{V}_{\mathbb{C}}$. Using the weighting factor $c(i)$, we propose a blind channel estimation algorithm based on the weighted vector set. First, we obtain an initial decision matrix $\mathcal{D}$ through the existing channel estimation method. Note that the decision matrix $\mathcal{D}$ has the same dimension as that of $\mathbf{h}$. Using the decision matrix $\mathcal{D}$ and equation (12), we define the weighting factor $c(i)$ between $\hat{\mathcal{V}}(i)$ and $\mathcal{D}$ as follow

$$\begin{aligned} c(i) &:= 1 - ||\cos\theta_{\mathbb{C}}(\hat{\mathcal{V}}(i), \mathcal{D})||_2, \quad (13) \\ \cos\theta_{\mathbb{C}}(\hat{\mathcal{V}}(i), \mathcal{D}) &= \frac{(\hat{\mathcal{V}}(i)\cdot\mathcal{D})_{\mathbb{C}}}{||\hat{\mathcal{V}}(i)||_2 \cdot ||\mathcal{D}||_2} \quad (14)\end{aligned}$$

where $\hat{\mathcal{V}}(i)$ denotes the i-th column vector of the matrix $\hat{\mathcal{V}}$ in (12). We introduce the modified $\bar{\mathcal{V}}$ as

$$\bar{\mathcal{V}} = \{\bar{\mathcal{V}}(i) \mid \bar{\mathcal{V}}(i) = c(i)\hat{\mathcal{V}}(i), \quad 1 \leq i \leq dN\} \quad (15)$$

Then, using the obtained $\bar{\mathcal{V}}$, we reconstruct equation (12) as follow

$$\bar{\mathcal{V}}^H\mathbf{h} = \mathbf{0}_{dN\times 1} \quad (16)$$

In the semi-blind estimation, we can use pilot symbols to obtain the decision matrix $\mathcal{D}$.

3.2 Semi-Blind Estimation

In this section, we propose the simple semi-blind channel estimation method for OFDM systems. The idea for devising a semi-blind channel estimation consists of the pilot-based least-squares estimator and the subspace-based blind channel estimator, and it is given as the following cost function :

$$\mathcal{C}_{semi} = \lambda||\mathbf{Y} - \mathbf{W}\mathbf{h}||^2 + (1-\lambda)\mathbf{h}^H\bar{\mathcal{V}}\bar{\mathcal{V}}^H\mathbf{h} \quad (17)$$

where λ is a scalar weight coefficient that determines the contribution of the pilot-based estimator and the subspace-based blind estimator of the cost function $\mathcal{C}_{semi}$ and $\lambda \in [0, 1]$. Taking the derivative of the cost function with respect to $\mathbf{h}^H$, the semi-blind channel estimator is obtained.

By letting a set of N_P pilot tones be $\mathcal{P} = \{P(1), \cdots, P(N_P)\}$, the p-th received symbol at the r-th receive antenna $Y_r(p)$ is expressed as follows

$$Y_r(p) = \sum_{l=0}^{L} x(p) e^{-j\frac{2\pi}{N} lp} h_r(l) + Z_r(p), \quad p \in \mathcal{P} \quad (18)$$

Note that $\mathbf{h} = [\mathbf{h}^T(0), \cdots, \mathbf{h}^T(L)]^T$. Thus, we obtain

$$\mathbf{Y}(p) = \mathbf{W}_p \mathbf{h} + \mathbf{Z}(p) \quad (19)$$

where $\mathbf{Y}(p) = [Y_1(p), \cdots, Y_{N_R}(p)]^T$ and $\mathbf{W}_p = \mathbf{W}_{p,L}\mathbf{W}_{p,R}$ with $\mathbf{W}_{p,L} = x(p) \otimes I_{N_R}$, $\mathbf{W}_{p,R} = [e^{-j\frac{2\pi}{N}(0)p}, \cdots, e^{-j\frac{2\pi}{N}(L)p}] \otimes I_{N_R}$. Then, we can form the $N_P N_R \times N_R(L+1)$ system of linear equations as

$$\tilde{\mathbf{Y}} = \tilde{\mathbf{W}}\mathbf{h} + \tilde{\mathbf{Z}} \quad (20)$$

where

$$\tilde{\mathbf{Y}} = \begin{pmatrix} \mathbf{Y}(P(1)) \\ \vdots \\ \mathbf{Y}(P(N_P)) \end{pmatrix}, \quad \tilde{\mathbf{Z}} = \begin{pmatrix} \mathbf{Z}(P(1)) \\ \vdots \\ \mathbf{Z}(P(N_P)) \end{pmatrix} \quad (21)$$

and

$$\tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{W}_{P(1),L} & & \\ & \ddots & \\ & & \mathbf{W}_{P(N_P),L} \end{bmatrix} \begin{bmatrix} \mathbf{W}_{P(1),R} \\ \vdots \\ \mathbf{W}_{P(N_P),R} \end{bmatrix} \quad (22)$$

Thus, the estimate of channel coefficient matrix $\mathbf{h}$ is obtained by

$$\hat{\mathbf{h}} = \arg \min_{||\mathbf{h}||=1} [\lambda ||\tilde{\mathbf{Y}} - \tilde{\mathbf{W}}\mathbf{h}||^2 + (1-\lambda)\mathbf{h}^H \bar{\mathcal{V}}\bar{\mathcal{V}}^H \mathbf{h}] \quad (23)$$

When we find $\hat{\mathbf{h}}$ satisfying $\partial \mathcal{C}_{semi}/\partial \mathbf{h}^H = 0$, the semi-blind channel estimator is obtained as

$$\hat{\mathbf{h}} = \lambda[\lambda \mathbf{I}_{N_R(L+1)} + (1-\lambda)\bar{\mathcal{V}}\bar{\mathcal{V}}^H]^{-1}\tilde{\mathbf{Y}} \quad (24)$$

With the proposed semi-blind algorithm, a complete channel estimate scheme is obtained over a block stationary channel as well as a static channel.

4. Identifiability

For the channel to be identified by the noise subspace method, the matrix $\mathcal{G}$ in (7) should have full column rank as shown in [2], [3]. Since $\mathcal{F}^H$ is a unitary matrix, rank($\mathcal{G}$) where $\mathcal{G} = \mathcal{H}\mathcal{F}^H$ is equal to rank($\mathcal{H}$). Therefore, the condition for $\mathcal{H}$ to have full column rank guarantees that $\mathcal{G}$ has full column rank. For the identifiability of blind scheme, refer to [2].

Now, we consider the identifiability conditions for the pilot-based least-squares channel estimation. From (24), $\mathbf{h}$ is determined uniquely when $\tilde{\mathbf{W}}$ is full column rank, which is given as

$$N_P \geq (L+1) \quad (25)$$

Let $\mathbf{x}$ denote the space spanned by the solution of $\arg \min_{||\mathbf{h}||=1} ||\tilde{\mathbf{Y}} - \tilde{\mathbf{W}}\mathbf{h}||^2$ and the null space of $\bar{\mathcal{V}}^H$ denote null$[\bar{\mathcal{V}}^H]$. For the semi-blind identifiability, we have

$$\dim(\text{null}[\bar{\mathcal{V}}^H] \cap \mathbf{x}) = 1 \quad (26)$$

Because of the existence of pilot symbols, the ambiguity caused by the null space of $\bar{\mathcal{V}}^H$ is eliminated.

5. Simulation Results

To demonstrate the performance of the proposed semi-blind channel estimation algorithm, we compare the performance with the work of [6] (which we refer to as the Muquet's algorithm in the results that follow) and the work of [5] (which we refer to as the Ding's algorithm). We consider a multipath channel with order $L = 2$. The number of subcarriers N is set to 16 and a OFDM system with the 3 receive antennas is considered. Data symbols are chosen from a quadrature phase shift keying (QPSK) constellation and a guard interval is chosen to be longer than the maximum delay spread of the channel in order to eliminate the inter-block interference. In our simulations, the covariance matrix $\mathbf{R}_r$ is estimated to find signal and noise subspaces as $\mathbf{R}_r = \frac{1}{N_b}\sum_{i=1}^{N_b} \mathbf{r}^{(i)}[\mathbf{r}^{(i)}]^H$ where N_b is the number of OFDM blocks. Note that $N_b = 1$ in a block stationary channel and $N_b = 30$ in a static channel. For the comparison purpose with a block stationary channel, the pilot symbols are transmitted at the first OFDM block in a static channel. Figures 1 and 2 show the bit error rate (BER) to compare the performance with the existing semi-blind channel estimation schemes, in which we consider a block stationary channel and a static channel. As shown in Figs. 1-2, the performance of the proposed semi-blind algorithm is better than those of the existing estimation methods. The reason of this improvement is that the proposed semi-blind algorithm considers the orthogonality for the noise eigenvectors by using the weighting factor. To evaluate the estimation error, the normalized root mean squared error (NRMSE) [3] is used and defined as

$$\text{NRMSE} = \sqrt{\frac{1}{N_s(L+1)N_R}\sum_{i=1}^{N_s}\frac{||\hat{\mathbf{h}}^{(i)} - \mathbf{h}^{(i)}||_2^2}{||\mathbf{h}^{(i)}||_2^2}} \quad (27)$$

where N_s is the number of simulation times, $\hat{\mathbf{h}}^{(i)}$ and $\mathbf{h}^{(i)}$ are the estimated channel and the true channel (without ambiguity) respectively. To obtain the NRMSE performance as shown in Fig. 5, we execute the simulation of 10000 times to obtain the averaged results and set the E_b/N_o to 10 dB.

6. Conclusion

In this paper, we proposed a semi-blind channel estimation algorithm for OFDM systems with multiple receive antennas, which is characterized by its simplicity

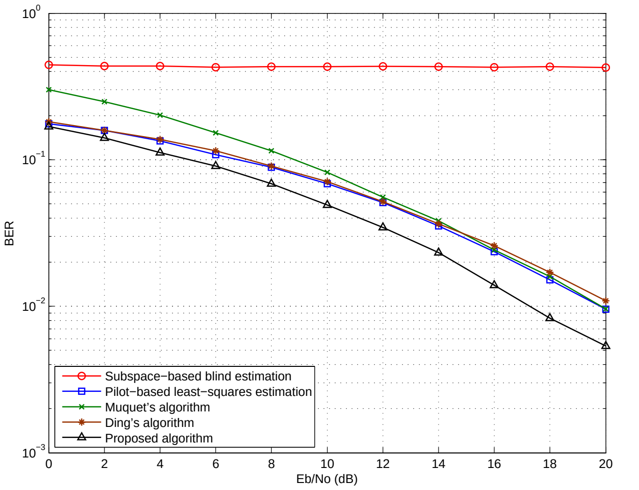

Figure 1. BER performance of channel estimation schemes for block stationary channel

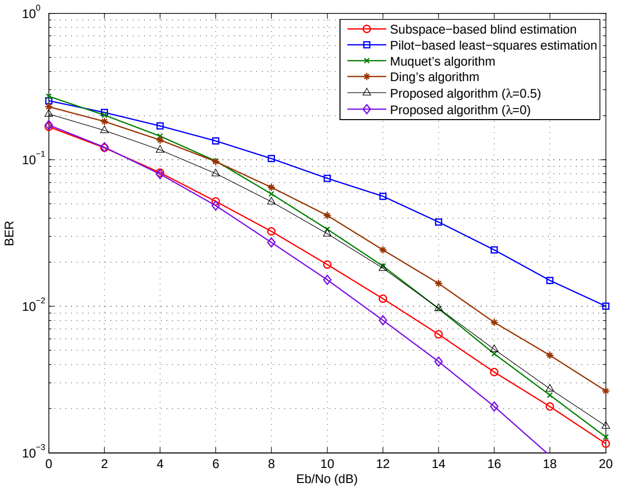

Figure 2. BER performance of channel estimation schemes for static channel (number of OFDM blocks = 30)

and better performance. In order to reduce the residual error for the noise eigenvectors, we consider the orthogonality by using the weighting factor which is obtained by the Hermitian angle. Thus, our proposed semi-blind channel estimation algorithm is robust to the limited amount of available data. Simulation results show that in a block stationary channel and a static channel, the proposed semi-blind algorithm is able to achieve better performance as compared with other algorithms.

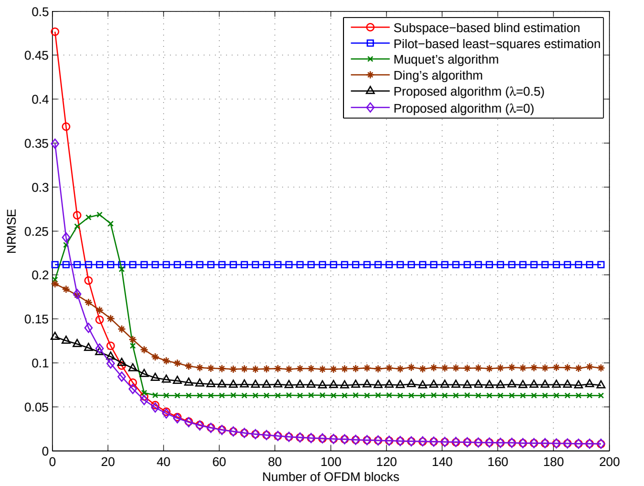

Figure 3. Convergence rate vs. number of OFDM blocks N_b for channel with $E_b/N_o = 10$ dB

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References

[1] Y. Li, "Pilot-Symbol-Aided Channel Estimation for OFDM in Wireless Systems," *IEEE Trans. on Veh. Technol.*, vol. 49, no. 4, pp. 1207-1215, July 2000.

[2] S. Roy and C. Li, "A Subspace Blind Channel Estimation Method for OFDM Systems without Cyclic Prefix," *IEEE Trans. on Wireless Commun.*, vol. 1, no. 4, pp. 572-579, Oct. 2002.

[3] C. Shin, R. W. Heath, and E. J. Powers, "Blind Channel Estimation for MIMO-OFDM Systems," *IEEE Trans. on Veh. Technol.*, vol. 56, no. 2, pp. 670-685, Mar. 2007.

[4] Y. Zeng and T. Ng, "A Semi-Blind Channel Estimation Method for Multiuser Multiantenna OFDM Systems," *IEEE Trans. on Signal Processing*, vol. 52, no. 5, pp. 1419-1429, May 2004.

[5] Z. Ding and D. B. Ward, "Subspace Approach to Blind and Semi-Blind Channel Estimation for Space-Time Block Codes," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 2, pp. 357-362, Mar. 2005.

[6] B. Muquet, M. de Courville, and P. Duhamel, "Subspace-Based Blind and Semi-Blind Channel Estimation for OFDM Systems," *IEEE Trans. on Signal Processing*, vol. 50, no. 7, pp. 1699-1712, July 2002.