Performance Evaluation of Clusters in Factor Graph for Graph-Based Detection

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Abstract: Two-dimensional (2-D) interference channel with inter-symbol interference (ISI) and inter-track interference (ITI) can degrade the performance of the read channel in bit patterned media recording (BPMR) system. In this work, we propose the method to improved graph-based (GB) detector. The factor graph applies to clusters in the cluster 2×3 channel matrix on 2-D interference channel to the GB detector. The simulation results show the performance of the proposed GB detector method with the conventional GB detector and multi-SOVA (64s) detector achieve the gains of about 0.8 and 1.0 dB at bit error rate equal 10^{-4}, in the BPMR system with media noise at areal density 2 Tbits/in^2.

Keywords-- **Graph-Based Detector, BPMR System , 2-D Interference Channel , Inter-Track Interference, Media Noise**

1. Introduction

A bit patterned media recording (BPMR) system is one of expectation for the next generation technology magnetic storage. When the areal density is increased the space between islands narrows. Therefore, it has the two-dimensional (2-D) interference channel with inter-symbol interference (ISI) and inter-track interference (ITI) [1], [3], which is one of the main limitations in BPMR system. In addition, the media noise with size and location fluctuations [2], since its degrades the performance of the data recovery channel. Recently, a graph-based (GB) detector appears as an alternative detector to mitigate the effects for the 2-D interference channel. Because, it is in fact a real 2-D detector of multi-head multi-track processing [5], [6] in the BPMR system.

In this work, we propose method to improve the performance of the GB detector. Motivated by [8], equalizes the 3×3 2-D interference channel matrix into cluster 2×3 channel matrix two targets. We compare the performance between modified GB detector, the conventional GB detector and multi-SOVA (64s) detector with 64 states [7] in BPMR system with media noise at areal density of 2 Tbits/in^2. The multi-SOVA (64s) detector is based on [7], but we use three SOVA detectors to detect the bit data from three tracks separately.

The rest of the paper is organized as follows. Section II, we describes the 2-D interference channel model. The GB detector is introduced in Section III. Section IV, provides the simulation results and discussions, Finally, in Section V, we give the conclusions.

2. 2-D Interference Channel Model

The paper considers the BPMR system with media noise, the 2-D Gaussian pulse response $H(x,z)$ of the along-track and across-track direction are then expressed as [2], [4]

$$H(x,z) = (A+\Delta_A)\exp\left\{-\frac{1}{2}\left(\frac{c(x+\Delta_x)}{PW_{50x}}+\frac{c(z+\Delta_z)}{PW_{50z}}\right)^2\right\} \qquad (1)$$

where $A=1$ is the peak amplitude of the pulse response, Δ_A is the amplitude fluctuation, Δ_x is the fluctuation in along-track direction x, Δ_z is the fluctuation in across-track direction z. PW_{50x} is the PW_{50} of along-track pulse, PW_{50z} is the PW_{50} of across-track pulse, and $c = 2\sqrt{2\ln 2}$ is a constant to account for the relationship between PW_{50} and standard deviation of Gaussian pulse.

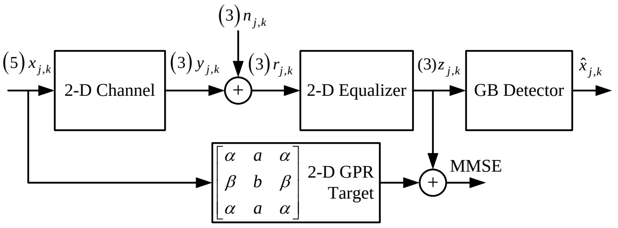

Fig. 1. Block diagram of 2-D channel with 2-D equalizer and GB detector.

In Figure 1, shows the block diagram of the proposed system, we consider a BPMR sytem with multi-head multi-track processing. Give the input bit sequences $x_{j,k-2}, x_{j,k-1}, x_{j,k}, x_{j,k+1}, x_{j,k+2} \in \{\pm 1\}$ are written on five tracks $k-2$, $k-1, k, k+1$ and $k+2$. The receiver three read-back signal sequences $r_{j,k-1}, r_{j,k}$ and $r_{j,k+1}$ are then fed to the 2-D equalizer to obtained sequences $z_{j,k-1}, z_{j,k}$ and $z_{j,k+1}$, are then sent to the GB detector, which produces the estimated input bit sequences $\hat{x}_{j,k}$.

The read-back signal $r_{j,k}$ from the jth data bit on the kth track can be written as [3], [6]

$$r_{j,k} = \left(x_{j,k} \otimes h_{j,k}\right) + n_{j,k} = y_{j,k} + n_{j,k} \qquad (2)$$

or

$$r_{j,k} = \sum_{m=M}^{M}\sum_{n=N}^{N} h_{m,n}x_{j-m,k-n} + n_{j,k} \tag{3}$$

where $x_{j,k}$ is the recorded bits, $h_{m,n}$ is the 2-D interference channel response coefficients, *m* and *n* are the time indices of the bit islands in along-track and across-track directions, $\otimes$ is a convolution operator, an $n_{j,k}$ is an additive white Gaussian noise (AWGN).

The 2-D interference channel can be represented by 3×3 symmetric channel response matrix such as

$$\mathbf{H} = \begin{bmatrix} \alpha & a & \alpha \\ \beta & b & \beta \\ \alpha & a & \alpha \end{bmatrix} = \begin{bmatrix} h_{j-1,k-1} & h_{j,k-1} & h_{j+1,k-1} \\ h_{j-1,k} & h_{j,k} & h_{j+1,k} \\ h_{j-1,k+1} & h_{j,k+1} & h_{j+1,k+1} \end{bmatrix} \tag{4}$$

where α, a, β and *b* are the channel cofficients of the 2-D interference channel. $h_{j,k}$ is 2-D channel response coefficient of main track, $h_{j-1,k}, h_{j+1,k}$ and $h_{j,k-1}, h_{j,k+1}$ are the ISIs and ITIs coefficients related to the bit island on main track, respectively, and $h_{j-1,k-1}, h_{j+1,k-1}, h_{j-1,k+1}$ and $h_{j+1,k+1}$ correspond to ITI coefficients due to corner bit islands on the adjacent tracks. The read-back signal $r_{j,k}$ in equation (2) can be expanded as

$$\begin{aligned} r_{j,k} &= bx_{j,k} + \beta\left(x_{j-1,k} + x_{j+1,k}\right) + a\left(x_{j,k-1} + x_{j,k+1}\right) \\ &+ \alpha\left(x_{j-1,k-1} + x_{j-1,k+1} + x_{j+1,k-1} + x_{j+1,k+1}\right) + n_{j,k} \end{aligned} \tag{5}$$

3. Graph-Based Detector

A graph-based (GB) detector is an alternative detection in the 2-D interference channel. Because the GB detector is a real 2-D detector. In addition, the GB detector has an advantage in that the complexity is linear as a function of the number of tracks and interference lengths [5]. The GB detector applies a message-passing algorithm in the factor graph, designed based on the 3×3 2-D interference channel matrix, using *a posteriori* log-likelihood ratio (LLR) as the passing messages. The factor graph consists a factor node and bit node. The factor nodes represent the read-back signal $r_{j,k}$ and the bit nodes represent the input bits $x_{j,k}$. An edge is connected between factor nodes and bit nodes using LLR as the passing message [5], [6].

A. Improved Graph-Based Detector

The 2-D interference channel is equalized into two targets with H_{123456} and H_{456789}, respectively. We applied the clusters 2×3 channel matrix on the factor in Figure 2 [8]. In equation (4), we use the indicated six of nodes as our clusters in the cluster 2×3 channel matrix. We can define as

$$H_{123456} = \begin{bmatrix} h_{j-1,k-1} & h_{j,k-1} & h_{j+1,k-1} \\ h_{j-1,k} & h_{j,k} & h_{j+1,k} \end{bmatrix} \tag{6}$$

and

$$H_{456789} = \begin{bmatrix} h_{j-1,k} & h_{j,k} & h_{j+1,k} \\ h_{j-1,k+1} & h_{j,k+1} & h_{j+1,k+1} \end{bmatrix}. \tag{7}$$

We separates the regions so that

$$H_{456} = \begin{bmatrix} h_{j-1,k} & h_{j,k} & h_{j+1,k} \end{bmatrix} \tag{8}$$

and

$$H_5 = \begin{bmatrix} h_{j,k} \end{bmatrix}. \tag{9}$$

The 3×3 2-D channel matrix H is calculated by

$$\mathbf{H} = H_{123456} + H_{456789} - H_{456} + H_5. \tag{10}$$

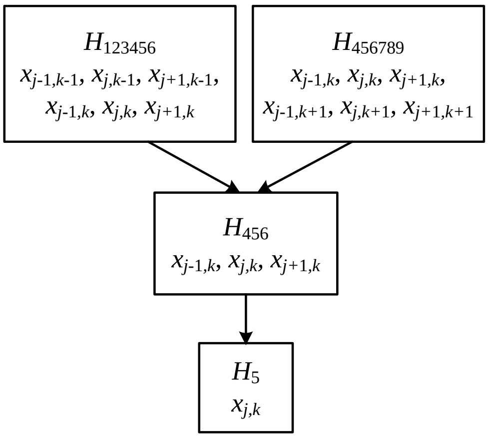

Fig. 2. The 2-D channel for clusters 2×3 channel matrix in the factor graph

From the cluster of 2×3 channel matrix of H_{123456} channel shown in Figure 2, we obtain a relation between the factor nodes and the bit nodes as shown in Figure 3. The factor node $r_{j,k}$ passes the reliability information to connected the bit nodes $x_{j-1,k-1}, x_{j+1,k}, x_{j,k+1}, x_{j,k}, x_{j+1,k-1}$, $x_{j+1,k}$, then each of these the bit nodes sends the reliability information to connected the factor nodes. For the cluster of 2×3 channel matrix of H_{456789} channel in Figure 4, the factor node $r_{j,k}$ sends the reliability information to connected the bit nodes $x_{j-1,k}, x_{j-1,k+1}, x_{j,k}, x_{j,k+1}, x_{j+1,k}$, $x_{j+1,k+1}$. After each of these the bit nodes updates and back to the reliability information to connect the factor node [6]. A graph-based detector employs belief propagation algorithm to compute LLR information in the factor nodes and bit nodes [5].

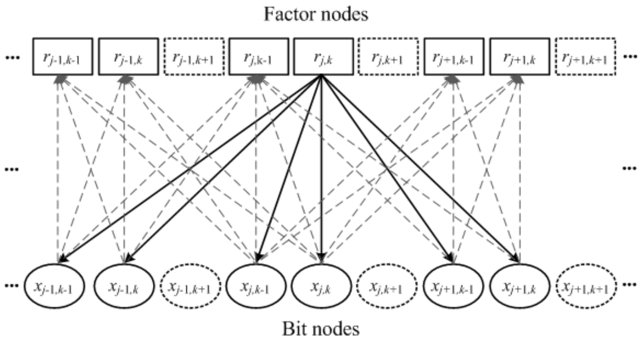

Fig. 3. The factor graph for H_{123456} channel.

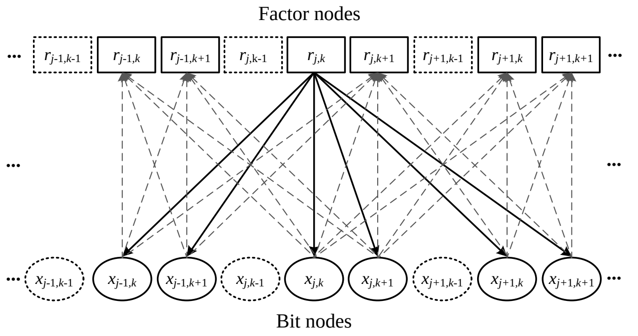

Fig. 4. The factor graph for H_{456789} channel.

The belief propagation algorithm below is explained in great details in [5]. The factor graph uses a message-passing algorithm between the factor nodes and the bit nodes. The LLR reliability information is calculated by the sum-product update rule based-on the incoming messages from other connected the bit nodes and received signal. Using the Log-MAP algorithm update message $L\left(\mu_{n\to p}\right)$ from the factor node *n* to the bit node *p* is

$$L\left(\mu_{n\to p}\right)=\log\frac{\displaystyle\sum_{x^{n}:x_{p}=+1}\exp\left(-\frac{\left(r_{n}-y_{n}\right)^{2}}{2\sigma_{0}^{2}}+\sum_{k\in x^{n}_{x_{p}},x_{k}+1}L\left(\mu_{k\to n}\right)\right)}{\displaystyle\sum_{x^{n}:x_{p}=-1}\exp\left(-\frac{\left(r_{n}-y_{n}\right)^{2}}{2\sigma_{0}^{2}}+\sum_{k\in x^{n}_{x_{p}},x_{k}-1}L\left(\mu_{k\to n}\right)\right)}\qquad(11)$$

where r_n is received signal sequences, y_n is the noiseless channel output from (5) corresponding to every combination of set for the bit nodes $k\in x^{n}_{\sim x_p}$ with x_p set to +1 or -1, and σ_0^2 is variance of an AWGN sequence. The LLR message $L\left(\mu_{k\to n}\right)$ is generated from connected the bit node *k* to the factor node *n*.

After receiving the messages from the factor nodes, all the bit nodes, in turn generate the LLR information $L\left(\mu_{p\to n}\right)$ from the bit node *p* to the factor node *n* from

$$L\left(\mu_{p\to n}\right)=\sum_{k\in C^{p}_{\sim n}}L\left(\mu_{k\to p}\right)\qquad(12)$$

where $C^{p}_{\sim n}$ represents the set of the factor nodes connected to the bit node x_p excluding the factor node r_n.

For the first iteration, the LLR information $L\left(\mu_{k\to n}\right)$ are intialized to zero since all the input bits are assumed to be equi-probable. From the second iteration onward, the factor nodes update the information using the incoming new information $L\left(\mu_{k\to n}\right)$. These updating and exchanging information process will continue until the final iteration is reached. After the specified number of iterations, the LLR information of estimated bits data $\hat{x}_n$ at bit nodes *n* can be computed from

$$L\left(\hat{x}_{n}\right)=\sum L\left(\mu_{k\to n}\right)\qquad(13)$$

The hard estimates of the input bits are determined by

$$\hat{x}_{n}=\begin{cases}+1, & \text{if } L\left(\hat{x}_{n}\right)\geq 0\\ -1, & \text{other}\end{cases}\qquad(14)$$

4. Simulation Results and Discussions

In this paper, the 3×3 2-D generalized partial response (GRP) target and 3×7 equalizer are designed based on a minimum mean square error (MMSE) approach [3], and define the signal-to-noise ratio (SNR) from 7 to 14 decibel (dB). A SNR is defined as

$$\mathrm{SNR}=10\log_{10}(V_{p}/\sigma),\qquad(15)$$

where $V_p=1$ is normalized peak value of the read-back signal and σ is a standard deviation of AWGN. In the system, each sector consists of 4,096 bits.

We consider the BPMR system at the areal density 2 Tbits/in^2, in which both bit period and track pitch are 18 nm, PW_{50x} along-track is 19.5 nm, and PW_{50z} across-track is 24.7 nm. The 3×3 channel response matrix is generated by 2-D Gaussian function from (1) as

$$\mathrm{H}=\begin{bmatrix}0.0216 & 0.2294 & 0.0216\\ 0.0942 & 1.0000 & 0.0942\\ 0.0216 & 0.2294 & 0.0216\end{bmatrix}.$$

In Figure 5, shows the bit error rate (BER) performance of the improved GB detector in the BPMR system without media noise at areal density equal to 2 Tbits/in^2. For the improved GB detector compared with the conventional GB detector and the SOVA (64s) detector to achieve the gains of about 0.8 and 1.0 dB at BER equal 10^{-4}.

For the BPMR system with media noise due to the size fluctuation and location fluctuation of the islands, the effect of the media noise at 3%, 6% and 9% on the performance of the conventional GB detector and improved GB detector are shown in Figure 6. As the media noise with size fluctuation and location fluctuation level increases, the performance degradation is more evident.

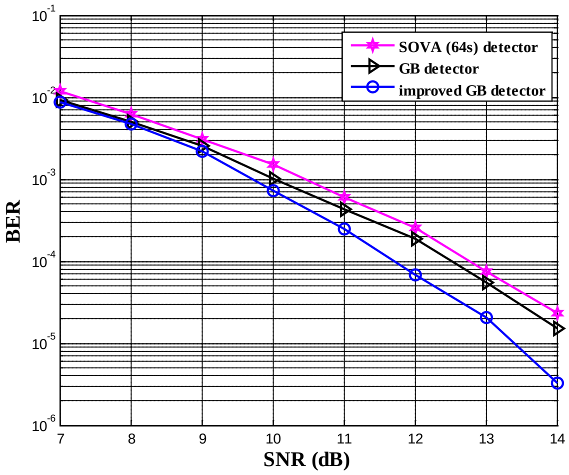

Fig. 5. Performance comparision of the improved GB detector, GB detector and SOVA (64s) detector.

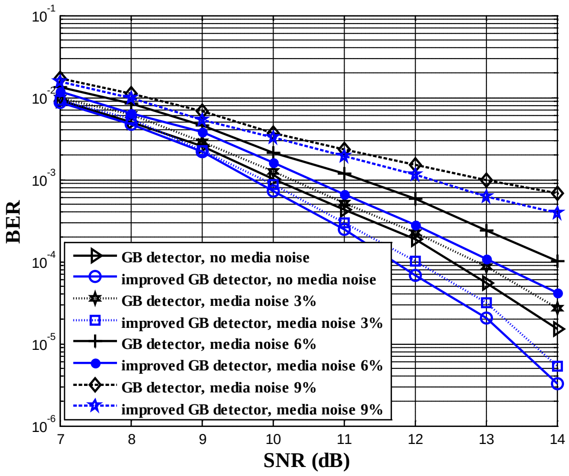

Fig. 6. BER Performance comparision between the GB detector and improved GB detector in BPMR system with media noise.

5. Conclusions

In this work, we propose the improved GB detector method to modify factor graph in the 2-D interference channel in the BPMR system with media noise. The modified GB detector method applies the clusters 2×3 channel matrix of the 3×3 2-D interference channel matrix in the factor graph for message-passing between the factor nodes and the bit odes. The simulation results show that the improved GB detector give better BER performance than the conventional GB detector and SOVA (64s) detector.

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