

# Pre-inverse Active Noise Control System with Auxiliary Filter

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**Abstract:** We proposed a pre-inverse active noise control (PIANC) in order to solve the unstable problem on a Filtered-x ANC. The PIANC uses the flat delayed signal instead of a filtered input signal. However, since the conventional PIANC uses a series adaptive system to estimate the inverse transfer function of the secondary path, the noise reduction ability is degraded due to bias error. Therefore, this paper introduces a bias free structure into the estimation of the inverse transfer function of the secondary path. The proposed structure prevents the bias error by taking advantage of the delay of the primary path. This paper shows that the proposed method without a power scheduling has possibility to be superior to a Filtered-x ANC with power scheduling, and evaluates the computational complexity.

**Keywords-- Active Noise Control, Adaptive Signal processing**

## 1. Introduction

Filtered-x algorithm is used as a feed-forward type active noise control [1]. However, the modeling error occurs when a secondary path characteristic is non-stationary. The modeling error then deteriorates the performance of the ANC and influences the stability of a control filter. The filtered-x ANC is unstable where the phase error is greater than  $\pi/2$  [2], [3]. Thus, the ANC injecting auxiliary noise has been proposed for reducing the modeling error [4], [5]. However, it is difficult to control an adaptive filter while keeping the phase error less than  $\pi/2$  on an implementation of the ANC.

In order to solve the modeling error problem, a pre-inverse ANC (PIANC) with bias free adaptive algorithm has been proposed [6]. The PIANC uses a flat delayed signal instead of the filtered noise for the adaptive algorithm of a control filter, therefore the primary and secondary paths are independently controlled whereas the filtered-x ANC controls a primary path and a secondary path simultaneously by an adaptive filter. Thus, the PIANC becomes stable. The PIANC uses a pre-inverse filter which has an inverse transfer function of the secondary path before the secondary path. It requires estimating the inverse filter of the secondary path modeling. If a normalized least mean square (NLMS) algorithm is used for a pre-inverse filter, it converges on a solution with bias error because the disturbance is included in a tap input of the adaptive algorithm. Therefore, the square sum of correlation function (SSCF) algorithm has been proposed as a bias free adaptive algorithm. Unfortunately, although the SSCF algorithm

improves the estimation accuracy of the inverse transfer function of the secondary path when the power of the auxiliary noise is enough, the estimation accuracy is degraded as the power of the auxiliary noise decreases.

Therefore, this paper proposes the pre-inverse ANC with an auxiliary filter. The conventional PIANC has an adaptive filter for estimating the inverse transfer function of the secondary path. On the other hand, the proposed PIANC introduces an auxiliary filter for estimating the secondary path. Then, the adaptive algorithm of the pre-inverse filter estimates the inverse transfer function of the auxiliary filter by taking advantage of the delay of the primary path. Therefore, the input signal without disturbance can be used as the tap input of the adaptive filter for estimating the inverse transfer function of the secondary path. Thus, the estimation accuracy of secondary path modeling is improved although the power of the auxiliary noise decreases.

## 2. Feed-forward Type ANC

### 2.1. Filtered-x ANC

Figure 1 shows the structure of an ANC system based on the filtered-x NLMS (Normalized Least Mean Square) algorithm with auxiliary noise [4]. In Figure 1,  $M_d$  and  $M_c$  represent detection and error microphones respectively.  $g(n)$  is the noise detected near noise source at time  $n$ .  $w(n)$  is auxiliary noise which is white [7].  $H_P(z)$  and  $H_S(z)$  represent the transfer function of the primary and secondary path respectively. The both paths are non-minimum phase system.  $p(n)$  and  $d(n)$  are respectively the output signals of the primary and secondary path.  $\tilde{H}_C(z)$  and  $\tilde{H}_S(z)$  respectively represent a control filter and secondary path modeling filter which are transversal type adaptive filters.  $g_c(n)$  represents the output signal of the control filter.  $y(n)$  and  $e(n)$  are respectively the error of adaptive algorithms for  $\tilde{H}_C(z)$  and  $\tilde{H}_S(z)$ . The secondary path modeling filter  $\tilde{H}_S(z)$  estimates  $H_S(z)$ .  $g_s(n)$  represents the input signal of the adaptive algorithm for  $\tilde{H}_C(z)$ . The control filter estimates  $-H_P(z) / H_S(z)$ . Then,  $p(n)$  is suppressed by  $d(n)$ . Since the update algorithm of the control filter uses the output signal  $g_s(n)$  of the secondary path modeling filter  $\tilde{H}_S(z)$ , the ability of noise reduction is deteriorated due to the modeling error of a secondary path when the path is non-stationary. In addition, the filtered-x ANC is unstable where the phase error is greater than the modeling error of the secondary path  $\pi/2$ .

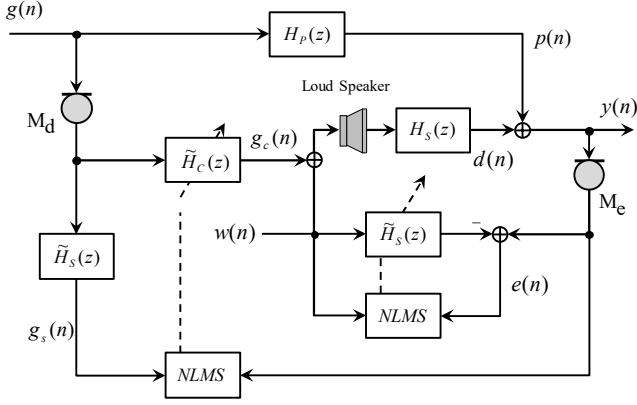


Figure 1 Filtered-x NLMS ANC with auxiliary noise.

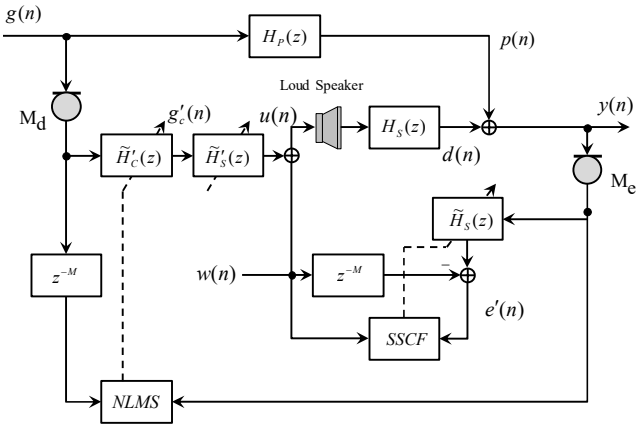


Figure 2 Conventional pre-inverse ANC.

## 2.2. Pre-inverse type ANC with SSCF algorithm

In order to solve the unstable problem, a PIANC with auxiliary noise has been proposed [6]. The PIANC introduces the filter, which has the inverse transfer function of the secondary path, before the secondary path. The PIANC controls the primary and secondary paths severally. Thus, a delayed input signal can be used as tap inputs of an adaptive algorithm for the control filter instead of a filtered input signal. Then the PIANC control the filter with stable.

Figure 2 shows the structure of the conventional PIANC.  $\tilde{H}'_c(z)$  is a control filter.  $\tilde{H}'_s(z)$  is a pre-inverse filter. At the PIANC, the delayed noise  $g(n-M)$  is used as the input signal of NLMS algorithm for the control filter.  $g'_c(n)$  and  $u(n)$  are respectively the input signals of the pre-inverse filter  $\tilde{H}'_s(z)$  and the secondary path. The error signal  $e'(n)$  is used for updating the adaptive filters  $\tilde{H}'_s(z)$ . The transfer functions of  $\tilde{H}'_c(z)$  and  $\tilde{H}'_s(z)$  are respectively defined by

$$\tilde{H}'_c(z) = \sum_{i=0}^L h'_{c,i}(n)z^{-i} \quad (1)$$

$$\tilde{H}'_s(z) = \sum_{i=0}^{N'} h'_{s,i}(n)z^{-i} \quad (2)$$

where  $L$  and  $N'$  are the number of tap inputs of  $\tilde{H}'_c(z)$  and  $\tilde{H}'_s(z)$  respectively.  $h'_{c,i}(n)$  and  $h'_{s,i}(n)$  respectively represent  $i$ th tap coefficients of  $\tilde{H}'_c(z)$  and  $\tilde{H}'_s(z)$ . As the updating algorithm for the control filter, the NLMS algorithm is used. The adaptive filter  $\tilde{H}'_s(z)$  estimates  $H_s^{-1}(z)z^{-M}$ . When the adaptive filter  $\tilde{H}'_s(z)$  is converged, the output of the secondary path  $d(n)$  becomes  $u(n-M)$ . The control filter  $\tilde{H}'_c(z)$  estimates  $H_p(z)z^M$  and the noise is reduced. Since the tap inputs of  $\tilde{H}'_s(z)$  after the error microphone includes noise  $p(n)$  which is disturbance against the adaptive filter  $\tilde{H}'_s(z)$ , the estimation accuracy of inverse transfer function of the secondary path and the noise reduction ability are degraded. Therefore, the square sum of correlation function (SSCF) algorithm has been proposed as a bias free adaptive algorithm [6]. Unfortunately, the SSCF algorithm cannot improve the estimation accuracy of inverse transfer function of the secondary path when the power of the auxiliary noise decreases.

## 3. Proposed Pre-inverse type ANC

Figure 3 shows the structure of the proposed PIANC.  $\tilde{H}'_c(z)$  and  $\tilde{H}_s(z)$  are a control filter and auxiliary filter respectively.  $\tilde{H}'_s(z)$  is a pre-inverse filter. The error signals  $e(n)$  and  $e'(n)$  are used for updating the adaptive filters  $\tilde{H}_s(z)$  and  $\tilde{H}'_s(z)$ . The transfer functions of  $\tilde{H}_s(z)$  is defined by

$$\tilde{H}_s(z) = \sum_{i=0}^N h_{s,i}(n)z^{-i} \quad (1)$$

where  $N$  is the number of tap inputs of  $\tilde{H}_s(z)$ .  $h_{s,i}(n)$  represent  $i$ th tap coefficient of  $\tilde{H}_s(z)$ . The auxiliary filter  $\tilde{H}_s(z)$  estimates the secondary path  $H_s(z)$  by system identification. As the updating algorithm for adaptive filters, a NLMS algorithm is used in this paper. The NLMS algorithms for each adaptive filter are defined by

$$\mathbf{h}'_c(n+1) = \mathbf{h}'_c(n) + \mu_c \frac{y(n)\mathbf{g}(n)}{\mathbf{g}^T(n)\mathbf{g}(n)} \quad (2)$$

$$\mathbf{h}_s(n+1) = \mathbf{h}_s(n) + \mu_s \frac{e(n)\mathbf{w}(n)}{\mathbf{w}^T(n)\mathbf{w}(n)} \quad (3)$$

$$\mathbf{h}'_s(n+1) = \mathbf{h}'_s(n) + \mu'_s \frac{e'(n)\mathbf{w}_s(n)}{\mathbf{w}_s^T(n)\mathbf{w}_s(n)} \quad (4)$$

where  $\mu_c$ ,  $\mu_s$  and  $\mu'_s$  are the step size of each adaptive algorithm.  $T$  represents transposition, and

$$\mathbf{g}(n) = [g(n-M), g(n-M-1), \dots, g(n-M-L)]^T \quad (5)$$

$$\mathbf{w}(n) = [w(n), w(n-1), \dots, w(n-N)]^T \quad (6)$$

$$\mathbf{w}_s(n) = [w_s(n), w_s(n-1), \dots, w_s(n-N')]^T \quad (7)$$

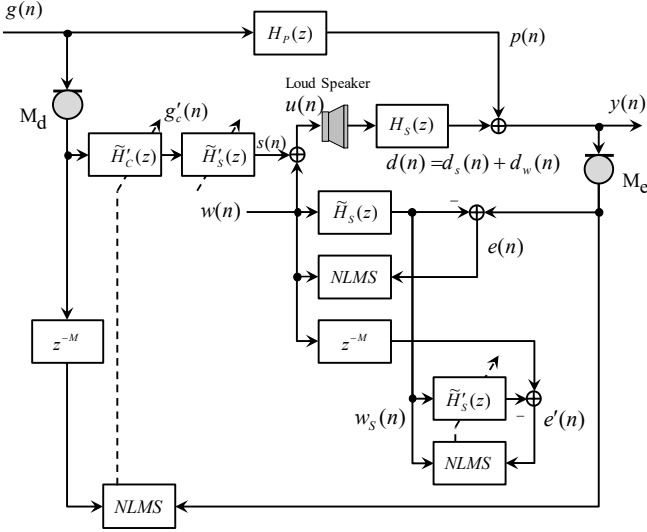


Figure 3 Proposed pre-inverse ANC.

The adaptive filter  $\tilde{H}'_s(z)$  converges on a solution so that

$$\tilde{H}_s(z)\tilde{H}'_s(z) = z^{-M} \quad (8)$$

is satisfied. When the auxiliary filter converges,  $\tilde{H}'_s(z)$  is given by

$$\tilde{H}'_s(z) = z^{-M}H_s^{-1}(z). \quad (9)$$

Therefore, the control filter  $\tilde{H}'_c(z)$  estimates  $-H_p(z)z^M$ . Assuming that the primary path  $H_p(z)$  involves delay  $z^{-M}$ , the pre-inverse ANC can reduce the noise  $p(n)$ . Since the delay  $z^{-M}$  corresponds to the distance from the detection microphone to the error microphone, this assumption is satisfied without less of generality.

Since the input signal  $w_s(n)$  of adaptive algorithm (4) does not involve the noise  $p(n)$ , the adaptive algorithm (4) converges on a solution with bias free. Therefore, the proposed PIANC improves the estimation accuracy of the inverse transfer function of the secondary path and the noise reduction ability when the power of the auxiliary noise is decreased.

#### 4. SIMULATION RESULTS

The performance of the proposed system was evaluated by computer simulations. As a primary and secondary path model, the impulse responses of the primary and secondary paths in a duct for a ventilating fan were used. We detected the impulse responses by generating the impulse signal. Figure 4 shows the duct model to detected the impulse responses. The secondary path model involves the characteristic of the loud speaker. The primary path  $H_p(z)$  and secondary path  $H_s(z)$  are non-minimum-phase systems. In order to evaluate the tracking performance for non-stationary paths, the impulse responses of the secondary paths are multiplied by -1 at iteration number  $l = 15,000$ . All sound data prepared in simulations were sampled by 1 kHz with 16bit resolution. The auxiliary

noise  $w(n)$  is a white signal whose mean is zero. The auxiliary noise to noise power ratio (ANR) is defined by

$$ANR = 10\log_{10} \frac{\sum_{n=1}^{M'} w^2(n)}{\sum_{n=1}^{M'} g^2(n)} [dB] \quad (10)$$

where  $M'$  is the number of samples. In this simulation, ANR was set to -20dB. In this paper, the proposed PIANC is compared with the Akhtar's method [1] and conventional PIANC with SSCF Algorithm (PISSCF) [2]. Akhtar's method controls the ANR by a power scheduling. Table 1 shows each parameter of adaptive filters which are used in these simulations. The adaptive algorithms, which were used in proposed and conventional PIANC, were NLMS algorithm except  $\tilde{H}'_s(z)$  on the PISSCF. For  $\tilde{H}'_s(z)$  on PISSCF, the SSCF algorithm was used [2]. Akhtar's method uses LMS algorithm.

$EV(l)$ , which is defined as follows, is used in order to evaluate the ability of noise reduction.

$$EV(l) = 10\log_{10} \frac{E \left[ \sum_{n=lL'}^{(l+1)L'-1} \{p(n) + d(n) - d_w(n)\}^2 \right]}{E \left[ \sum_{n=lL'}^{(l+1)L'-1} p^2(n) \right]} [dB] \quad (11)$$

$$d_s(n) = d(n) - d_w(n) \quad (12)$$

where  $L'$  is the length of a block and block number.  $L'$  was set to 256.  $d_s(n)$  and  $d_w(n)$  represent  $z$  noise and auxiliary noise component in  $d(n)$ .

Figure 5 shows the  $EV(l)$  performance. From the simulation results, it is too small power of the auxiliary noise for the conventional PIANC

Table 1 Simulation conditions.

	<i>Akhtar's method</i>	<i>Pre-inverse SSCF</i>	<i>Proposed Pre-inverse ANC</i>
The number of Taps of control filter	150	150	150
Step size for control filter	max0.01 min0.00 1	0.015	0.003
The number of Taps on $\tilde{H}_s(z)$	150		150
Step size for $\tilde{H}_s(z)$	0.00005		0.003
The number of taps on $\tilde{H}'_s(z)$		150	150
Stepsize for $\tilde{H}'_s(z)$		1.0	0.003
Delay $M$		11	11

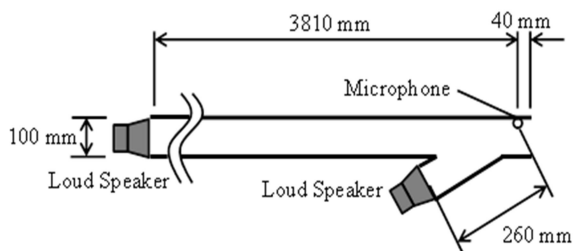


Figure 4 Duct model.

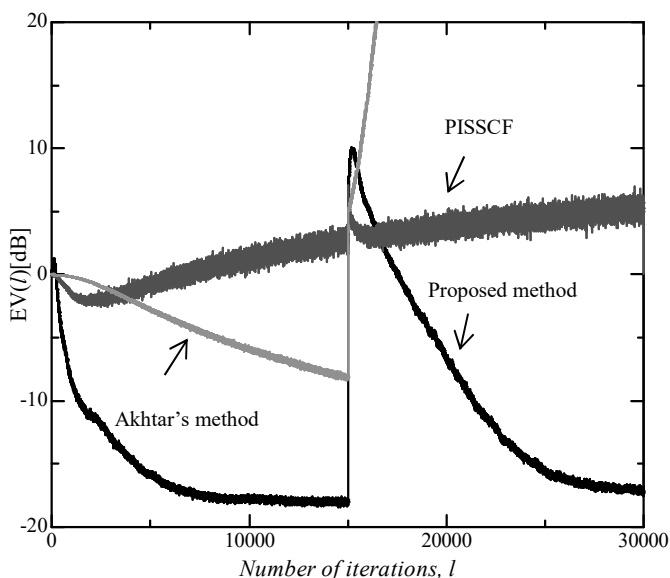


Figure 5 Noise reduction performance.

Table 2 Computational complexity.

	<i>Akhtar's method</i>	<i>PISSCF</i>	<i>Proposed ANC</i>
The number of addition per sample	764	1204	1053
The number of multiplication per sample	757	1195	1046
The number of division per sample	1	3	3

to reduce the noise. Comparing the proposed PIANC with the Akhtar's method using a power scheduling, the proposed PIANC can improve the noise reduction ability than the Akhtar's method before  $l = 15,000$ . After changing characteristics of the primary and secondary paths ( $l \geq 15,000$ ), the only proposed PIANC can track the non-stationary path. This is because of introducing the bias free structure and delay  $z^{-M}$  instead of a secondary path modeling filter. Therefore, the inverse transfer function of the secondary path is estimated accurately although the power of the auxiliary noise decreases.

Table 2 shows the computational complexity of the ANC system. The parameters shown in Table 1 are used for calculation of the computational complexity. Comparing the proposed system with Aktar's method, the proposed

system suppresses the increase of addition and multiplication by about 38%. The proposed system can improve the noise reduction ability in spite of decreasing the computational complexity.

## 5. CONCLUSIONS

The filtered-x ANC has a possibility that the modeling error of the secondary path degrades the noise reduction ability and makes the ANC unstable. Also, the filtered-x ANC tracks the non-stationary path slow because the primary and secondary paths are controlled simultaneously and dependently. In order to avoid the problem due to the modeling error, a pre-inverse ANC with the SSCF algorithm has been proposed. The PIANC controls the primary path and secondary path independently. Therefore, the performance of ANC is improved. However, the conventional PIANC converges on a solution with bias when the power of auxiliary noise is decreased. Thus, this paper proposes the bias free structure for estimating the inverse transfer function of the secondary path. The proposed PIANC takes advantage of the delay of the primary path and the input signal without disturbance is used as the tap input of an adaptive algorithm. From the simulation results, it is verified that the proposed PIANC has potential to track the non-stationary path faster while keeping the noise reduction ability. In a future work, we will research the DSP or FPGA implementation of the proposed PIANC.

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