Calculation of Permeability Tensor of Ferrites Using HE_{111} Mode

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***Abstract*—A calculation of a permeability tensor of ferrites is presented in this paper. A ferrite resonator that is located between two parallel metal plates is introduced. Through theoretical derivation of the proposed scheme, the splitting phenomenon of the HE_{111} mode is verified. It is also found that the splitting behavior depends on the single magnetization value of the ferrite. The analysis process and calculated results are presented in detail.**

Keywords—ferrite; permeability tensor; magnetization;

I. Introduction

Recently, many microwave applications in antennas, circulators, filters and absorbers have been developed using ferrites that exhibit magnetic properties. These devices are based on changeability of a permeability tensor. Therefore, it is very important to know of the permeability tensor at a specified dc magnetic bias. The studies to find the permeability tensor using resonators have been introduced so far [1], [2]. These models assumed that the permeability tensor was independent of its frequency. In practice, however, the permeability tensor of the ferrite shows frequency dependent characteristic.

This paper demonstrates the measurement technique of the permeability tensor of ferrites. Firstly, a theoretical analysis of the cylindrical ferrite resonator that is placed between two parallel metal plates is conducted. It is derived from transcendental equations after calculating fields of the inside and outside the resonator. The magnetization value of ferrites can be obtained by satisfying transcendental equations. Based on the calculated magnetization value, the permeability tensor of the ferrite is estimated by well-known equations [3]-[5]. The proposed method considers the effect of its frequency and has accurate solutions because this configuration has an analytical solution as well. A Li-ferrite with the permittivity $\varepsilon_r = 16.5$ and a saturation magnetization $4\pi M_s = 1960$ Gauss is considered in this paper.

II. States of Ferrite

According to an applied dc magnetic bias, ferrites can be classified into three states. Ferrites in the completely demagnetized state have the scalar permeability (μ_d). In the demagnetized state, initial permeability μ_d, which depends on a saturation magnetization $4\pi M_s$ and frequency, can be written by [3]

$$\mu_d = \frac{2}{3}\left[1-\left(\frac{\gamma 4\pi M_s}{\omega}\right)^{\frac{1}{2}}\right]+\frac{1}{3} \tag{1}$$

where γ (= 2.8MHz/Gauss) is the gyromagnetic ratio and ω is an angular frequency.

In the case of the steady magnetization along z-direction, the permeability tensor $[\mu]$ of the ferrite can be represented as

$$[\mu] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \tag{2}$$

The tensor components μ, κ, and μ_z for the partially magnetized state can be estimated [4], [5] by

$$\mu = \mu_d + (1-\mu_d)\left(\frac{M}{M_s}\right)^{\frac{3}{2}}$$
$$\kappa = \gamma 4\pi M / \omega \tag{3}$$
$$\mu_z = \mu_d^{\left(1-(M/M_s)^{\frac{5}{2}}\right)}$$

where M_s is a saturation magnetization and M is magnetization. For the saturated state of ferrites, the elements of μ and κ can be predicted using equations in [7] and μ_z is always equal to 1.

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III. Theoretical Analysis

The geometry of the resonator is shown in Fig 1. A cylindrical ferrite resonator with a radius of r_0 and a height of L is located between two parallel metal plates. This structure has been popular for a theoretical analysis as its simple geometry requires no magnetic wall assumption and as such it can provide more accurate solutions. This structure has four boundary conditions;

$$E_z^{in}\Big|_{r=r_0} = E_z^{out}\Big|_{r=r_0} \quad H_z^{in}\Big|_{r=r_0} = H_z^{out}\Big|_{r=r_0}$$
$$E_\phi^{in}\Big|_{r=r_0} = E_\phi^{out}\Big|_{r=r_0} \quad H_\phi^{in}\Big|_{r=r_0} = H_\phi^{out}\Big|_{r=r_0} \tag{4}$$

Also, the boundary conditions of parallel metal plates are given by

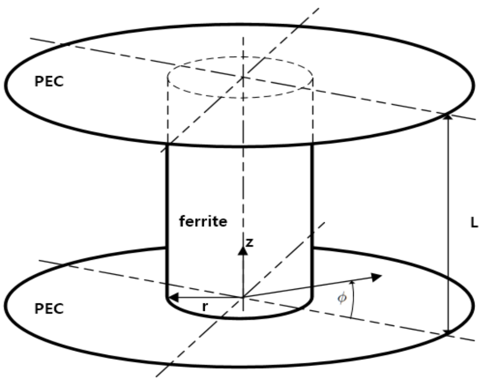

Fig. 1. Structure of ferrite resonator

$$\begin{cases} E_r\big|_{z=L} = 0 \\ E_\phi\big|_{z=L} = 0 \end{cases} \begin{cases} E_r\big|_{z=0} = 0 \\ E_\phi\big|_{z=0} = 0 \end{cases} \tag{5}$$

Applying the boundary condition (5), longitudinal wave number β is given by

$$\beta = \frac{\pi n}{L} \quad n = 0,1,2,3,\cdots\cdots \tag{6}$$

In the demagnetized state, permeability and permittivity are scalar values. Thus, the characteristic equation for the normal modes is given as [6]

$$\left[\frac{\varepsilon_d J_m{}'(\alpha)}{\alpha J_m{}'(\alpha)} + \frac{K_m'(\gamma)}{\beta K_m(\gamma)}\right]\left[\frac{\mu_d J_m{}'(\alpha)}{\alpha J_m{}'(\alpha)} + \frac{K_m'(\gamma)}{\gamma K_m(\gamma)}\right] = \left[\frac{m\gamma}{\beta_o r\sqrt{\varepsilon_d \mu_d}}\left(\frac{1}{\alpha^2}+\frac{1}{\gamma^2}\right)\right]^2 \tag{7}$$

where

$$\alpha = \frac{2\pi r}{\lambda}\left[\varepsilon_r \mu_r - \left(n\lambda/2L\right)^2\right]^{1/2}$$
$$\gamma = \frac{2\pi r}{\lambda}\left[\left(n\lambda/2L\right)^2 - 1\right]^{1/2}$$

The fields within the ferrite can be obtained from a scalar potential ψ, which satisfies a fourth-order wave equation. When the scalar potential depends on the z-axis harmonically, a fourth-order wave equation can be separated into two second-order differential equations:

$$(\nabla_\perp{}^2 + \chi_1{}^2)(\nabla_\perp{}^2 + \chi_2{}^2)\psi = 0 \tag{8}$$

Where $\nabla_\perp{}^2$ is the transverse Laplacian operator, and χ_1 and χ_2 are transverse wavenumbers which are followed as;

$$\chi_{1,2}{}^2 = (k_0{}^2\varepsilon(\frac{\mu^2-\kappa^2}{\mu} + \mu_z) - (1+\frac{\mu_z}{\mu})\beta^2) \pm\sqrt{\frac{1}{4}\left(k_0{}^2\varepsilon(\frac{\mu^2-\kappa^2}{\mu} - \mu_z) - (1-\frac{\mu_z}{\mu})\beta^2\right)^2 + \beta^2 k_0{}^2\varepsilon\mu_z\left(\frac{\kappa}{\mu}\right)^2} \tag{9}$$

Where $k_0 = \sqrt{\omega^2\mu_0\varepsilon_0}$ and ω is angular frequency. The ferrite in the magnetized state has two transverse wavenumbers. The fields inside the ferrite of the scalar potential ψ is given by

$$\vec{E} = \begin{bmatrix} jS & T & 0 \\ -T & jS & 0 \\ 0 & 0 & jW \end{bmatrix}\nabla\psi \tag{10}$$

$$\vec{H} = \frac{\delta}{\delta z}\begin{bmatrix} jM & N & 0 \\ -N & jM & 0 \\ 0 & 0 & jR \end{bmatrix}\nabla\psi \tag{11}$$

where

$$T_{1,2} = \omega^2\varepsilon\mu_t - \beta^2 - \frac{\varepsilon}{\varepsilon_z}\chi_{1,2}^2, \quad N = \omega\varepsilon\left(\frac{\varepsilon_a}{\varepsilon} + \frac{\mu_a}{\mu}\right)$$
$$W_{1,2} = -\frac{\varepsilon}{\varepsilon_z}\left(\frac{\varepsilon_a}{\varepsilon} + \frac{\mu_a}{\mu}\right)\chi_{1,2}^2, \quad R = -\frac{T\chi_{1,2}^2}{k_0\mu_z\beta^2}$$
$$M_{1,2} = \frac{1}{\mu k_0}\left[\omega^2\left(\varepsilon\mu + \varepsilon_a\mu_a\right) - \beta^2 - \frac{\varepsilon}{\varepsilon_z}\chi_{1,2}^2\right],$$
$$S_{1,2} = \frac{\varepsilon_a}{\varepsilon}\left(\omega^2\varepsilon\mu_t - \frac{\varepsilon}{\varepsilon_z}\chi_{1,2}^2\right) + \frac{\mu_a}{\mu}\beta^2$$

The scalar potential ψ for a cylindrical ferrite rod is then

$$\psi = \left[AJ_n(\chi_1 r) + BJ_n(\chi_2 r)\right]\cos\beta z \cdot e^{i(wt-n\phi)} \tag{12}$$

$$F=\begin{vmatrix} -\gamma^2K_n(\gamma a) & 0 & -iW_1\beta J_n(\chi_1 a) & -iW_2\beta J_n(\chi_2 a) \\ 0 & -\gamma^2K_n(\gamma a) & iR_1\beta^2J_n(\chi_1 a) & iR_2\beta^2J_n(\chi_2 a) \\ i\frac{n\beta}{a}K_n(\gamma a) & iw\mu_0\gamma K'_n(\gamma a) & T_1J'_n(\chi_1 a)\chi_1-S_1\frac{n}{a}J_n(\chi_1 a) & T_2J'_n(\chi_2 a)\chi_2-S_2\frac{n}{a}J_n(\chi_2 a) \\ -iw\varepsilon_0\gamma K'_n(\gamma a) & -i\frac{n\beta}{a}K_n(\gamma a) & N_1J'_n(\chi_1 a)\chi_1\beta-M_1\frac{n\beta}{a}J_n(\chi_1 a) & N_2J'_n(\chi_2 a)\chi_2\beta-M_2\frac{n\beta}{a}J_n(\chi_2 a) \end{vmatrix} \qquad (14)$$

With J_n is n-th order Bessel functions. The fields within air can be expressed by a superposition of TE and TM field equations. At this condition, TM and TE scalar potential are represented by

$$\begin{aligned}\psi_{TE}&=CK_n[\gamma r]\cos\beta z\cdot e^{i(wt-n\phi)}\\ \psi_{TM}&=DK_n[\gamma r]\sin\beta z\cdot e^{i(wt-n\phi)}\end{aligned} \qquad (13)$$

where $\gamma^2=\beta^2-k_0^{\ 2}$. After substituting expressions for the inside and outside fields into boundary conditions at $r=r_0$, we obtain a system of homogeneous linear equations for coefficients A, B, C, and D. As a result, the system can be written in the form of matrix as (14).

In order to obtain the non-trivial solution of the system, the determinant of the matrix F should be zero.

$$\det(F(M))=0 \qquad (15)$$

Therefore, the magnetization of ferrite can be calculated at a specified magnetic bias. Computing numerical calculations was performed to find solutions satisfying the aforementioned requirement (14).

IV. Calculation

Eq. (7) is used to calculate the resonant frequencies of the normal modes in the demagnetized state and eq. (15) is used to calculate the resonant frequencies in the bias states. A Li ferrite is considered in this study The ferrite material has a saturation magnetization ($4\pi M_s$) of 2000 Gauss and the permittivity of 16.5. The radius and the height of the resonator are 3.66 mm and 8.5 mm, respectively. The calculated results of the first two modes are illustrated in Fig. 2. The HE_{111} mode (the first mode) and TE_{011} mode (the second mode) in the demagnetized state are observed at 8.44 GHz and 9.71 GHz, respectively. However, when a static magnetic bias is applied, the HE_{111} mode is split into two resonant modes whereas the variation of the TE_{011} mode is small. It is found that the HE_{111} mode is split into the HE_{+111} and HE_{-111} modes. The red line corresponds to the frequency shift of the HE_{-111} mode while the blue long dash line corresponds to the frequency shift of the HE_{+111} mode. As the magnetization of the ferrite increases up to 580 Gauss, the gap between two split resonant frequencies also increases. When the value of the magnetization exceed 580 Gauss, the HE_{+111} mode disappears because one of the transverse wavenumbers becomes the imaginary value. We cannot observe the mode splitting phenomenon in the TE_{011} mode because it does not have the field component in the Φ-direction. As increasing the static dc magnetic bias, the resonant frequency of the TE_{011} mode is slightly shifted to the lower frequency because the effective permeability of the ferrite is slightly increased. The calculated resonant frequencies versus the magnetization values are listed in Table. I.

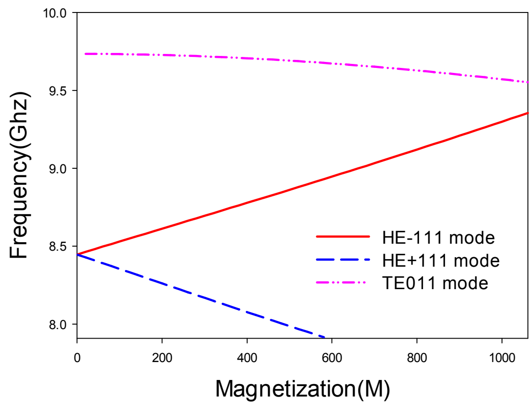

Fig. 2. Calculated resonant frequencies versus the magnetization values.

Based on the calculated magnetization values, the permeability tensor of the ferrite is estimated with the consideration of frequency dependent characteristics. For instance, if the resonant frequencies of the HE_{+111} mode is observed at 8.17 GHz, the magnetization value M is 300 Gauss. These parameters can be used for calculating the permeability tensor (using eq. (3)). The result is then

$$[\mu]=\begin{bmatrix} 0.8293 & j0.1028 & 0 \\ -j0.1028 & 0.8293 & 0 \\ 0 & 0 & 0.8202 \end{bmatrix}$$

If the value of the magnetization is higher than the saturation magnetization, the tensor components can be modeled using well-known equations called Polder's equations.

V. CONCLUSION

In order to estimate a permeability tensor of ferrites, the cylindrical ferrite resonator between two parallel metal plates is analyzed in this paper. Using the example of a Li ferrite resonator we calculate the resonant frequencies and estimate the permeability tensor. Future work is still required to verify the proposed method experimentally.

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