Scattering From a Finite Array of Axially Magnetized Ferrite Cylinders

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***Abstract*—The scattering of E-polarized plane wave from a finite array of axially magnetized ferrite cylinders is analysed based on the method of moments with global basis functions and Galerkin approach. To do so, the scattered wave is expressed in terms of the equivalent surface current. Furthermore, the surface impedance is introduced so that the boundary condition is imposed at the surfaces of cylinders in the unified manner as dielectric or conducting cylinders. Some numerical examples are given.**

I. Introduction

The scattering of a plane wave by an arbitrary configuration of parallel circular conductive or dielectric cylinders was studied by many researchers. The method of moments (MoM) was applied for the scattering by two parallel perfect electric conducting (PEC) circular cylinders[1]. The unknown current was expressed as a linear combination of pulse functions and the point matching technique was used. The scattering of an incident plane wave from parallel circular dielectric and PEC cylinders was analysed rigorously using a boundary value approach[2]. The scattering by conducting, lossy dielectric, ferrite cylinders was studied by using a combination of a modified iterative scattering procedure and the orthogonal expansion method[3]. The multiple scattering by finite parallel PEC circular cylinders was considered based on MoM with global basis functions and Galerkin's method[4].

In this paper, we analyse the scattering of E-polarized plane wave from a finite array of axially magnetized ferrite cylinders. We developed the technique based on the method of moments (MoM) to treat with the problem in the unified manner for conducting, dielectric or axially magnetized ferrite cylinders. To do so, we assume the equivalent surface current and introduce the surface impedance to impose the boundary conditions.

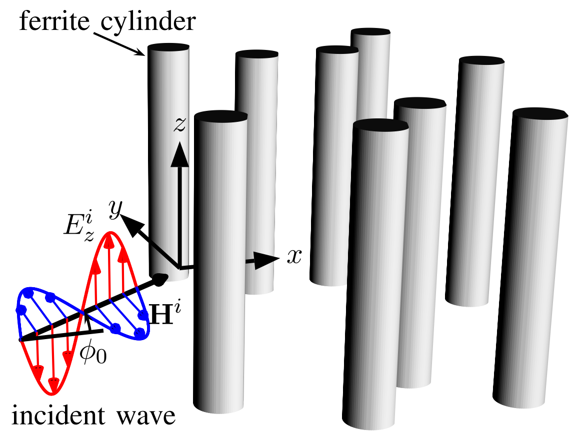


Fig. 1. Geometry of analysis

II. Computation Method

Consider N parallel infinitely long ferrite circular cylinders as shown in Fig.1. The ferrite cylinders, of which axes are parallel to the z axis, are magnetized along their axes. To analyse this two-dimensional problem in a unified manner, we developed the technique based on the method of moments. In order to treat the ferrite cylinders in the same way as perfect conductive cylinders, we assume the equivalent surface current $J_z(\vec{\rho})$ on the surface of each cylinder.

A. Scattered waves

By using Green function or a Hankel function of 0-th order $H_0^{(2)}(\cdot)$, the scattered field is given in the following integral form.

$$E_z^s(\vec{\rho}) = -\frac{k\zeta}{4}\sum_{\nu=1}^{N}\int_{C_\nu} H_0^{(2)}(k|\vec{\rho}-\vec{\rho}\,'|)J_z(\vec{\rho}\,')d\rho' \quad (1)$$

Let $\vec{\rho}_\nu$ be the center of ν-th cylinder, and introduce the local polar coordinate system (r_ν, θ_ν). Then the current

$J_z(\vec{\rho})$ on the ν-th cylinder should be a periodic function of θ_ν and its appropriate expansion is expected to be given in terms of global basis functions as

$$J_z(\vec{\rho}) = \sum_{n=-N_\nu}^{N_\nu} \chi_n^{(\nu)} e^{-jn\theta_\nu} \qquad (2)$$

The global basis functions were used to reduce the number of unknowns per cylinder and the necessary computer memory is also reduced. Substitution of (2) into (1), and usage of Graf's addition theorem[5] yields

$$E_z^s(\vec{\rho}) = -\frac{\pi\zeta}{2} \sum_{\nu=1}^{N} \sum_{n=-N_\nu}^{N_\nu} \chi_n^{(\nu)} k a_\nu J_n(k a_\nu) \times H_n^{(2)}(k|\vec{\rho} - \vec{\rho}_\nu|) e^{-jn\theta_\nu} \qquad (3)$$

B. Incident plane waves

The incident plane wave is expressed in the local coordinate system as

$$E_z^i(\vec{\rho}) = E_0 e^{-jk(x\cos\phi_0 + y\sin\phi_0)} = E_0 e^{-j\vec{k}\cdot\vec{\rho}_\nu} \sum_{n=-\infty}^{\infty} (-j)^n J_n(kr_\nu) e^{-jn(\theta_\nu - \phi_0)} \qquad (4)$$

C. Boundary Condition

In order to impose the boundary condition at the surface of cylinders in the unified way for conducting, dielectric or axially magnetized ferrite cylinders, the surface impedance is defined by

$$\mathcal{Z}^{(\mu)}(\theta_\mu) = \sum_{n=-\infty}^{\infty} Z_n^{(\mu)} e^{-jn\theta_\mu} \qquad (5)$$

Here the coefficients are obtained by solving the Helmholtz equation with the separation of variables. The following equation holds in the ferrite cylinder.

$$\frac{1}{r_\mu}\frac{\partial}{\partial r_\mu}\left(r_\mu \frac{\partial E_z}{\partial r_\mu}\right) + \frac{1}{r_\mu^2}\frac{\partial^2 E_z}{\partial \theta_\mu^2} + k^2 \varepsilon_r^{(\mu)} \mu_{eff}^{(\mu)} E_z = 0 \qquad (6)$$

where $\varepsilon_r^{(\mu)}$ and $\mu_{eff}^{(\mu)}$ are the relative permittivity and the effective relative permeability of the μ-th ferrite cylinder and $\mu_{eff}^{(\mu)} = \left(\mu^{(\mu)2} - \kappa^{(\mu)2}\right)/\mu^{(\mu)}$. Here $\mu^{(\mu)}$ and $\kappa^{(\mu)}$ are the diagonal and off-diagonal elements of the tensor permeability of μ-th ferrite cylinder and are given by

$$\mu^{(\mu)} = 1 - \frac{\omega_M \omega_H}{\omega^2 - \omega_H^2} \qquad \kappa^{(\mu)} = \frac{\omega_M \omega}{\omega^2 - \omega_H^2} \qquad (7)$$

where $\omega_H = |\gamma| H_0$, $\omega_M = |\gamma| M_0$, and γ, H_0, and M_0 are the gyromagnetic ratio, d.c. magnetic field and the saturation magnetization of the ferrte.

The effective relative permeability $\mu_{eff}^{(\mu)}$ is positive if either $\omega < \sqrt{\omega_H(\omega_H + \omega_M)}$ or $\omega > |\omega_H + \omega_M|$ holds. Then a solution has the following form.

$$E_z(r_\mu, \theta_\mu) = \sum_{n=-\infty}^{\infty} c_n J_n\left(k_f^{(\mu)} r_\mu\right) e^{-jn\theta_\mu} \qquad (8)$$

where $k_f^{(\mu)} = k\sqrt{\varepsilon_r^{(\mu)} \mu_{eff}^{(\mu)}}$. The tangential component of the magnetic field is calculeted by using

$$H_{\theta_\mu} = \frac{1}{j\omega\mu_0 \mu_{eff}^{(\mu)}} \left(\frac{\partial E_z}{\partial r_\mu} + j\frac{\kappa^{(\mu)}}{\mu^{(\mu)}} \frac{1}{r_\mu} \frac{\partial E_z}{\partial \theta_\mu}\right) \qquad (9)$$

Hence

$$Z_n^{(\mu)} = \frac{j\zeta J_n\left(k_f^{(\mu)} a_\mu\right)}{\sqrt{\frac{\varepsilon_r^{(\mu)}}{\mu_{eff}^{(\mu)}}} \left\{J_n'\left(k_f^{(\mu)} a_\mu\right) + n\frac{\kappa^{(\mu)}}{\mu^{(\mu)}} \frac{J_n\left(k_f^{(\mu)} a_\mu\right)}{k_f^{(\mu)} a_\mu}\right\}} \qquad (10)$$

One can put $\mu^{(\mu)} = 1$ and $\kappa^{(\mu)} = 0$ for the dielectric cylinder.

In the case of $\sqrt{\omega_H(\omega_H + \omega_M)} < \omega < |\omega_H + \omega_M|$, $\mu_{eff}^{(\mu)}$ is a negative number. Then

$$Z_n^{(\mu)} = \frac{-j\zeta I_n\left(K_f^{(\mu)} a_\mu\right)}{\sqrt{\frac{\varepsilon_r^{(\mu)}}{|\mu_{eff}^{(\mu)}|}} \left\{I_n'\left(K_f^{(\mu)} a_\mu\right) + n\frac{\kappa^{(\mu)}}{\mu^{(\mu)}} \frac{I_n\left(K_f^{(\mu)} a_\mu\right)}{K_f^{(\mu)} a_\mu}\right\}} \qquad (11)$$

where $K_f^{(\mu)} = k\sqrt{\varepsilon_r^{(\mu)} |\mu_{eff}^{(\mu)}|}$.

By using this surface impedance, the boundary condition is imposed as

$$E_z^i + E_z^s = \mathcal{Z}^{(\mu)} * (H_{\theta_\mu}^i + H_{\theta_\mu}^s) = \frac{1}{2\pi} \int_\alpha^{2\pi+\alpha} \mathcal{Z}^{(\mu)}(\theta_\mu - \theta) \left\{H_{\theta_\mu}^i(\theta) + H_{\theta_\mu}^s(\theta)\right\} d\theta \qquad (12)$$

The tangential component of magnetic fields is obtaind by

$$H_{\theta_\mu} = \frac{1}{j\omega\mu_0} \frac{\partial E_z}{\partial r_\mu}. \qquad (13)$$

Furthermore, the Galerkin method is used to obtain a sytem of linear equations. That is, the global basis functions were also used as weighting functions.

D. System of Linear Equations

When the field point and the source point locate on the different cylinders, the Graf's addition theorem was used again to evaluate the elements of the matrix analytically.

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \cdot & \cdot & \cdots & \cdot \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ X_N \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ B_N \end{bmatrix} \tag{14}$$

where $A_{\nu\nu}$ and $A_{\mu\nu}$ are $(2N_\nu+1)\times(2N_\nu+1)$ diagonal matrix and $(2N_\mu+1)\times(2N_\nu+1)$ matrix, respectively. X_ν and B_ν are $2N_\nu+1$ column vectors. Their elements are

$$a_{mn}^{(\nu\nu)} = -\frac{\pi^2\zeta}{k}\left\{H_n^{(2)}(ka_\nu) - \frac{Z_n^{(\nu)}}{j\zeta}H_n^{(2)\prime}(ka_\nu)\right\} \times (ka_\nu)^2 J_n(ka_\nu)\delta_{mn} \tag{15}$$

$$a_{mn}^{(\mu\nu)} = -\frac{\pi^2\zeta}{k}\left\{J_m(ka_\mu) - \frac{Z_m^{(\mu)}}{j\zeta}J_m'(ka_\mu)\right\} \times ka_\mu ka_\nu J_n(ka_\nu)H_{n-m}^{(2)}(kR_{\mu\nu})e^{-j(n-m)\alpha_{\mu\nu}} \tag{16}$$

$$b_m^{(\mu)} = -\frac{2\pi}{k}E_0 ka_\mu\left\{J_m(ka_\mu) - \frac{Z_m^{(\mu)}}{j\zeta}J_m'(ka_\mu)\right\} \times (-j)^m e^{j\{m\varphi_0 - k\rho_\mu\cos(\varphi_\mu-\varphi_0)\}} \tag{17}$$

E. Scattered Far-Field

The scattered field at any point can be computed by (3) if the system of linear equations (14) is solved numerically. Upon using the far-field approximation, $|\vec{\rho}-\vec{\rho}_\nu| \sim \rho - \vec{\rho}_\nu\cdot\vec{\rho}/\rho$ and $\theta_\nu \sim \phi$, the scattered far-field is expressed as

$$E_z^s(\vec{\rho}) \sim E_0\sqrt{\frac{2}{\pi k\rho}}e^{-j\left(k\rho-\frac{\pi}{4}\right)}f(\phi) \tag{18}$$

where $f(\phi)$ is the scattered far-field amplitude defined by

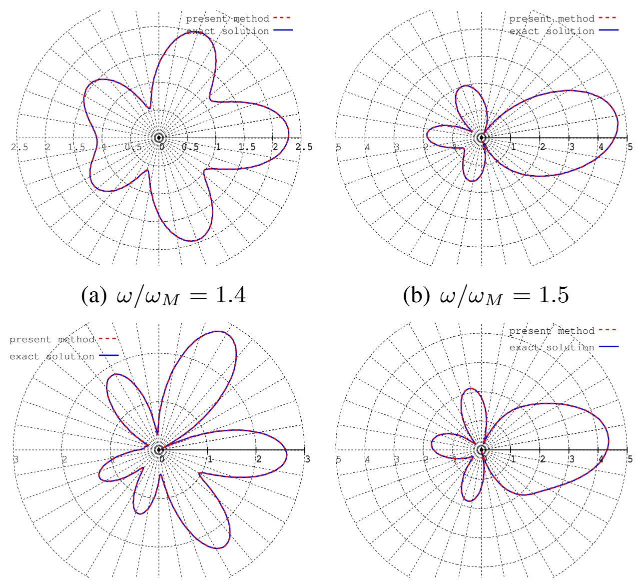
$$f(\phi) = -\frac{\pi\zeta}{2}\sum_{\nu=1}^{N} e^{jk\vec{\rho}_\nu\cdot\frac{\vec{\rho}}{\rho}} \cdot \sum_{n=-N_\nu}^{N_\nu} \chi_n^{(\nu)} ka_\nu J_n(ka_\nu)e^{-jn\left(\phi-\frac{\pi}{2}\right)} \tag{19}$$

The scattering width $\sigma(\phi)$ of the multiple cylinders is given by $4|f(\phi)|^2/k$. Thus, the total scattering width σ_{tot} is given by

$$\sigma_{tot} = \int_0^{2\pi}\sigma(\phi)d\phi = \frac{4}{k}\int_0^{2\pi}|f(\phi)|^2 d\phi \tag{20}$$

III. Numerical Examples

The scattering from a ferrite circular cylinder can be solved exactly by using the separation of variables. The following numerical values are used for the computation: $\omega_M/2\pi = 4.9$GHz; $\omega_H/2\pi = 7.84$GHz; the relative permittivity of ferrite, $\varepsilon_r = 15$; the radius of ferrite cylinder, $a = 19.13$mm. A comparison of the scattered far-field pattern is shown in Fig.2 for several frequencies. The results by separation of variables are plotted with blue solid line and those obtained by the present method are indicated by red dashed line. It is seen that the two results are almost identical.



(a) $\omega/\omega_M = 1.4$ (b) $\omega/\omega_M = 1.5$

(c) $\omega/\omega_M = 1.7$ (d) $\omega/\omega_M = 1.8$

Fig. 2. The scattered far-field pattern for a ferrite cylinder.

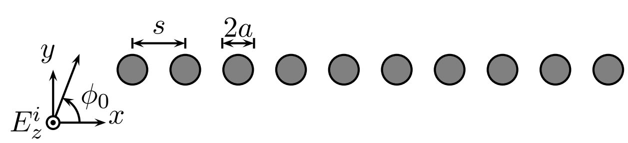
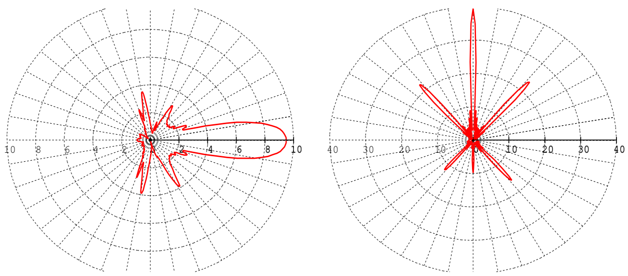


Fig. 3. A finite array of axially magnetized ferrite cylinders.

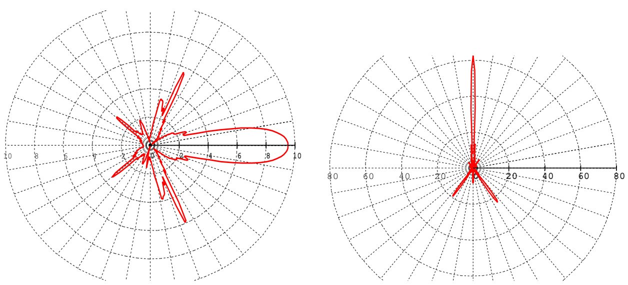
Let's consider the scattering by a finite linear array consisting of 10 axially magnetized circular cylinders shown in Fig.3. Suppose that each cylinder has a radius of $a = 18.37$ mm and the spacing s between centers of adjacent cylinders is 61.22 mm. And the other numerical values are the same as used in Fig.2. Assume that the E-polarized wave is at grazing incidence ($\phi_0 = 0°$) or normally incidence ($\phi_0 = 90°$). The scattered far-field patterns at $\omega/\omega_M = 1.5$ and $\omega/\omega_M = 1.7$ are shown in

Fig.4 and Fig.5, respectively. The energy errors from the optical theorem were at most orders of 10^{-15}.



(a) $\phi_0 = 0°$ (b) $\phi_0 = 90°$

Fig. 4. The scattered far-field pattern for the finite array at $\omega/\omega_M = 1.5$.



(a) $\phi_0 = 0°$ (b) $\phi_0 = 90°$

Fig. 5. The scattered far-field pattern for the finite array at $\omega/\omega_M = 1.7$.

IV. Conclusion

The scattering of E-polarized plane wave by an arbitrary configuration of axially magnetized ferrite cylinders was analysed based on the mehod of moments with global basis functions and the Galerkin method. The surface impedance was introduced so that the boudary condition was imposed at the surfaces of all of cylinders in the unified manner so as to treat other kind of circular cylinders such as conducting and/or dielectric ones. The elements of the coefficient matrix were calculated analytically. The method is easily applicapable to the problems of the scattering from the structure consisting of all kind of circular cylinders by using the proposed surface impedance.

References

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