Quantifying the Benefits of Pulse Shaping for Equalized PAM Transmissions in VLC Systems

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Abstract: In digital data transmissions, it is known that pulse shaping can help improve the transmission performance by reducing the amount of frequency selective distortion on the transmitted signal. However, to what degree pulse shaping is needed and by how much the transmission performance is improved in the context of visible light communications (VLC) has not explicity been reported. This paper provides numerical results from simulation experiments on data transmissions using pulse amplitude moduation (PAM) over two commonly used channel models for VLC, namely the exponential channel impulse response (CIR) and the ceiling-bounced CIR. For equalization, the linear minimum mean square error (MMSE) criterion is adopted for the sake of a possible practical implementation. For unequalized PAM transmissions, numerical results indicate that pulse shaping using the square-root raised cosine pulse (SRRC) and pulse shaping using the rectangular pulse yield similar bit error rate (BER) performances. However, in the presence of linear MMSE equalization with a fixed filter length, pulse shaping using the SRRC pulse can lead to significantly lower BERs when compared to the baseline case of using the rectangular pulse.

Keywords—**visible light communications, pulse shaping, pulse amplitude modulation, bit error rate**

1. Introduction

While optical wireless communications (OWC) has been investigated for over a few decades [1], it has recently received a lot of renewed interest in the context of visible light communications (VLC) where visible light is used as carrier signals for digital modulation [2].

In a typical VLC system, light emitting diodes (LEDs) are used for both illumination and data transmissions. Since LEDs are non-coherent light sources, intensity modulation (IM) is employed. Accordingly, power detection or direct detection (DD) is used at the receiver, which typically contains a photodiode to convert the optical power of the received optical signal into a photocurrent, i.e., electrical signal.

With IM/DD, the transmitted signal must be unipolar, i.e., non-negative. While this unipolar constraint raises new challenges in data modulation for OWC in general, the constraint is not as stringent for VLC, which normally adds a DC bias to the data signal for the purpose of illumination. Hence, for VLC, standard signal sets for pulse amplitude modulation (PAM) may be employed.

While pulse shaping is known to help improve the bit error rate (BER) performance of digital transmissions over frequency selective channels, to what degree pulse shaping is needed and by how much the BER performance is improved in the context of VLC has not explicitly been reported. Hence, this paper provides numerical results from simulation experiments to quantify the benefits of pulse shaping on reducing the BER in data transmissions. In particular, PAM is assumed since it is the most basic modulation format and should be considered first.

2. System Model and Simulation Parameters

Consider an M-PAM transmission system in which each transmitted pulse is multiplied by one of M possible amplitude values. Let $p(t)$ denote the pulse shape. Commonly used pulse shapes are rectangular and square-root raised cosine (SRRC) pulses as expressed below [3]. Note that T is the pulse period, i.e., one pulse sent for each time period T.

$$p_{\text{rect}}(t) = \begin{cases} 1/\sqrt{T}, & t \in (-T/2, T/2] \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$p_{\text{SRRC}}(t) = \frac{4\alpha}{\pi\sqrt{T}} \times \left[\frac{\cos((1+\alpha)\pi t/T) + \dfrac{T\sin((1-\alpha)\pi t/T)}{4\alpha t}}{1-(4\alpha t/T)^2} \right] \tag{2}$$

The parameter $\alpha \in [0, 1]$ is referred to as the roll-off factor. While the rectangular pulse has unlimited bandwidth, the SRRC pulse has a finite bandwidth equal to $\frac{1+\alpha}{2T}$.

Two VLC channel models are considered, namely the exponential channel impulse response (CIR) and the ceiling-bounced CIR [1].

$$h_{\text{expo}}(t) = H(0)\frac{1}{D}e^{-t/D}u(t) \tag{3}$$

$$h_{\text{ceil}}(t) = H(0)\frac{6a^6}{(t+a)^7}u(t) \tag{4}$$

In the above CIR expressions, $H(0)$ is the DC gain, D denotes the delay spread, $u(t)$ is the unit-step function, while $a = 12\sqrt{\frac{11}{13}}D$.

Simulation programs are developed using MATLAB. Note that discrete-time simulations are used, where we convert

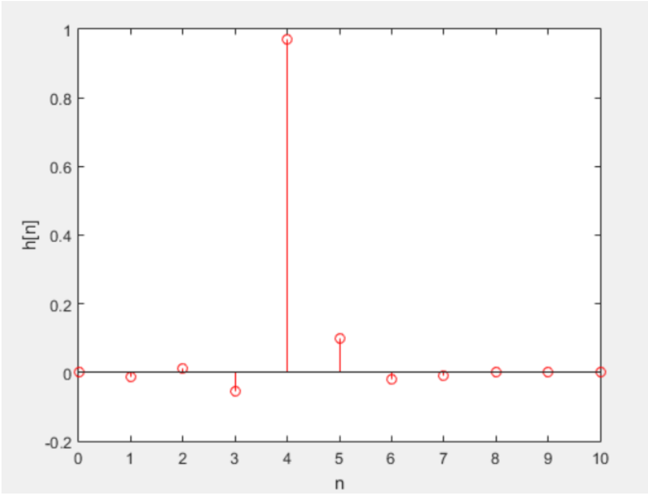

Figure 1. Discrete-time CIR of SRRC pulse shaping filter for T=100ns and the exponential CIR with H(0)=1 and D=10ns; note that the CIR is delayed by 4 symbol periods for causality.

each continuous-time CIR into a discrete-time CIR. In particular, let $h_0, \ldots, h_{L-1}$ denote the discrete-time CIR, where L is the length of CIR, i.e., $h_n \approx 0$ for $n \geq L$. In addition, let $q(t) = h(t) * p(t)$. Then, h_n's are obtained from samples of the matched filter output due to a transmitted pulse, i.e.,

$$h_n = q(t) * p(-t)|_{t=nT} \tag{5}$$

where $n \in \{0, 1, \ldots\}$. Fig. 1 shows an example of discrete-time CIR.

At the receiver, when additive white Gaussian noise (AWGN) is added to the received signal [5] , the received data signal can be expressed as

$$r_n = s_n * h_n + n_n \tag{6}$$

where s_n's are data symbols and n_n's are AWGN values.

In addition, linear minimum mean square error (MMSE) equalization is employed. In an MMSE equalizer as illustrated in Fig. 2, the goal is to choose a finite impulse response (FIR) filter denoted by $f_0, ..., f_{K-1}$ so that the equalizer output is approximately equal to the transmitted signal or its delayed version. For the receiver to compute f_n's, the prearranged training sequence $s_0, s_1, \ldots, s_{P-1}$ is assumed known at the receiver. The input-output relation of the MMSE equalizer is

$$y_n = f_n * r_n = \sum_{k=0}^{K-1} f_k r_{n-k} \tag{7}$$

The error e_n from Fig. 2 can be written as

$$e_n = s_{n-\delta} - y_n \tag{8}$$

where δ is the time delay of signal processing for equalization. The mean square error is

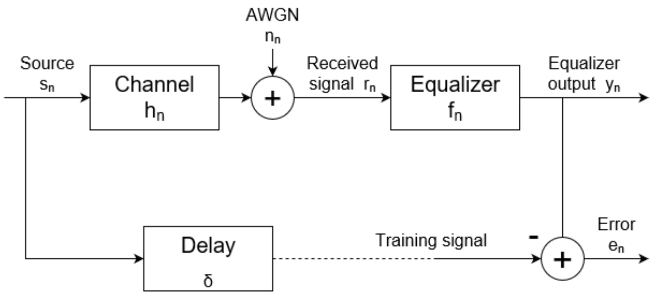

Figure 2. MMSE equalizer block diagram from [6]

$$J = \sum_{n=K-1}^{P-1} e_n^2 \tag{9}$$

For $n = K-1, K, ..., P-1$, the above equations can be written in the vector form as

$$Y = RF \tag{10}$$

$$E = S - Y = S - RF \tag{11}$$

$$J = E^T E = (S - RF)^T (S - RF) \tag{12}$$

where we define

$$Y = \begin{bmatrix} y_{K-1} \\ y_K \\ \vdots \\ y_{P-1} \end{bmatrix}, R = \begin{bmatrix} r_{K-1} & r_{K-2} & \cdots & r_0 \\ r_K & r_{K-1} & \cdots & r_1 \\ \vdots & \vdots & \ddots & \vdots \\ r_{P-1} & r_{P-2} & \cdots & r_{P-K} \end{bmatrix},$$
$$F = \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{K-1} \end{bmatrix}, S = \begin{bmatrix} s_{K-1-\delta} \\ s_{K-\delta} \\ \vdots \\ s_{P-1-\delta} \end{bmatrix} \tag{13}$$

It follows that the F that minimizes the mean square error becomes [6]

$$F = (R^T R)^{-1} R^T S \tag{14}$$

yeilding the minimum J equal to

$$J_{\min} = S^T (I - R(R^T R)^{-1} R^T) S \tag{15}$$

This $J_{\min}$ and F are for specific values of K and δ [6]. Fig. 3 shows the overall system structure.

The bit error rate (BER) will be plotted against a varying signal-to-noise ratio (SNR) defined as the energy per bit (E_b) divided by the one-sided power spectral density of AWGN (N_0), i.e., SNR $= E_b/N_0$. Table 1 lists key system parameters used in the simulation experiments. The delay spread for VLC is typically between 7 and 13 ns while the channel DC gain should be 10^{-6} for VLC. Without loss of generality, the

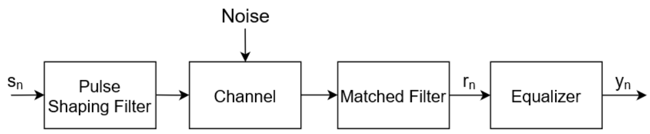

Figure 3. System overview diagram

Table 1. Parameters used in simulation experiments

Parameter	Value
Bit rate	20 Mbps
Modulation format	2-PAM, 4-PAM
Sampling rate	1000 sample/bit period
No. transmitted bits	10^6
No. training bits for equalizer	1000
Channel model	exponential, ceiling-bounced
Noise model	AWGN
Channel DC gain (H_0)	1
Channel delay spread (D)	10 ns
No. taps for linear equalizers	12

channel DC gain is set to 1, which means there is no attenuation for the channel because pulse dispersion is the main consideration in this research.

3. Numerical Results and Discussions

Fig. 4 and 5 show example transmitted, received, and equalized signals for the exponential CIR with 2-PAM using rectangular and SRRC pulses, respectively. It can be seen in both figures that the equalized signals are more similar to the transmitted signals than received signals. In addition, it is observed that the equalized signal values obtained from SRRC pulse shaping are closer to the transmitted signal values than using rectangular pulse shaping.

The construction of a linear MMSE equalizer is as follows. First, for each equalizer length (K), we vary the delay length (δ) until the minimum MSE ($J_{\min}$) is obtained. Then, we vary the value of K to get the overall minimum MSE of the equalizer. Table 2 shows different lengths of the MMSE equalizer and its MSEs for the exponential CIR with SRRC pulse shaping and 2-PAM. According to the results in table 2, $K = 12$ (with $\delta = 11$) is selected since it is better than lower values and there is no significant improvement at higher values.

Finally, the impulse response of the equalizer (f_n) obtained from $K = 12$ with $\delta = 11$ are used to transmit 1 million bits. The impulse response of the equalizer varies from one system to another according to pulse shape, the channel type and the modulation method.

Table 2. Mean-square errors of different equalizer lengths

Equalizer length (K)	Optimal delay (δ)	Average $J_{\min}$
10	9	36.8884
11	8	36.2986
12	11	35.8702
13	11	36.0701
14	11	35.9804

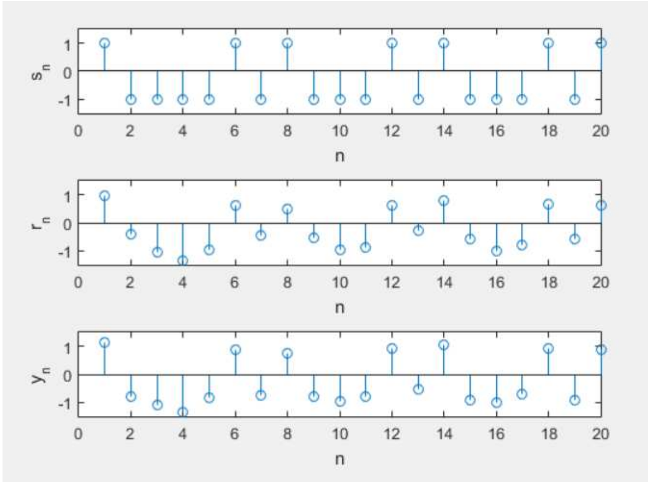

Figure 4. Transmitted signal s_n, received signal r_n and equalized signal y_n, using rectangular pulse shaping with 2-PAM

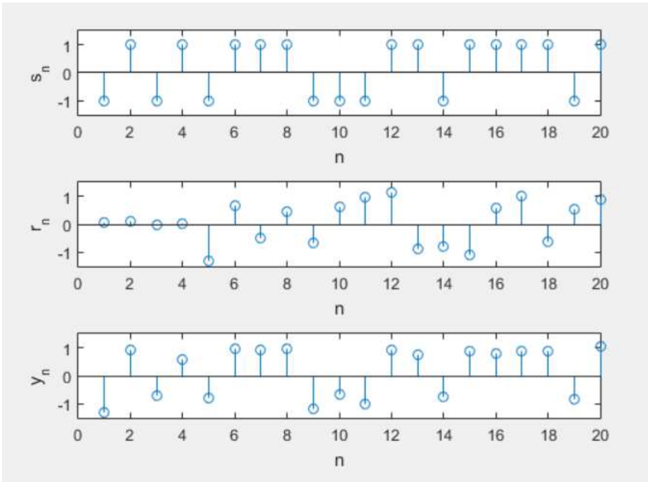

Figure 5. Transmitted signal, received signal and equalized signal using SRRC pulse shaping with 2-PAM

Fig. 6 and 7 show the BER plots for 2-PAM with the exponential and ceiling-bounced CIRs, respectively. It can be observed in each case that, without equalization, the BERs obtained from using the rectangular pulse and from using the SRRC pulse are not significantly different. However, with equalization, the BERs from using the SRRC pulse are significantly better.

Fig. 8 and 9 show similar information but for 4-PAM instead of 2-PAM. Like for 2-PAM, it is observed that the SRRC pulse can lead to lower BERs when equalization is applied at the receiver.

4. Conclusion

We investigated through computer simulation to what extent pulse shaping helps in reducing the BERs for 2-PAM and

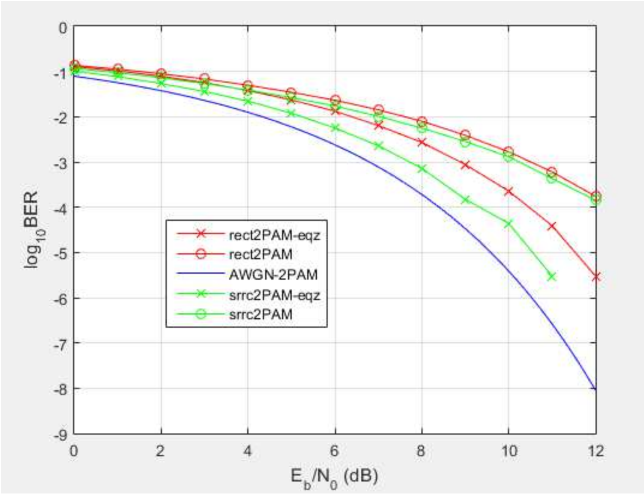

Figure 6. BER vs. SNR for 2-PAM with the exponential CIR

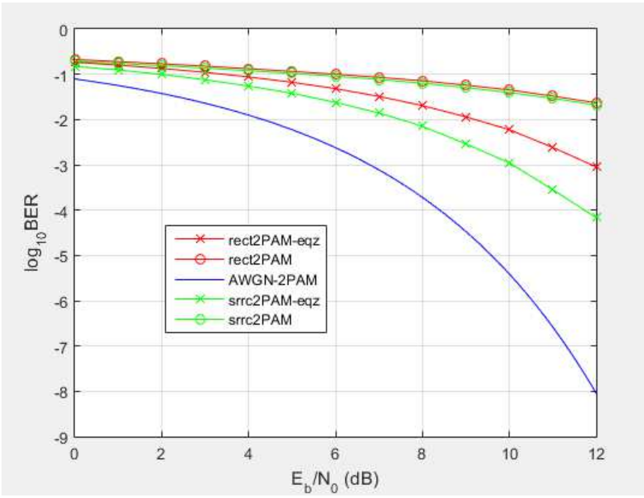

Figure 7. BER vs. SNR for 2-PAM with the ceiling-bounced CIR

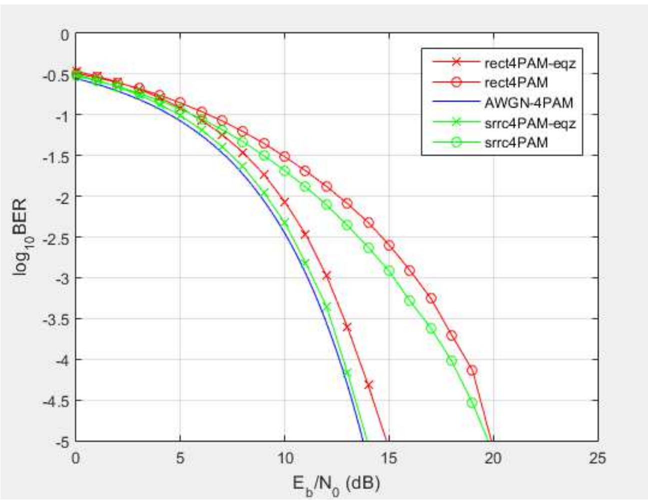

Figure 8. BER vs. SNR for 4-PAM with the exponential CIR

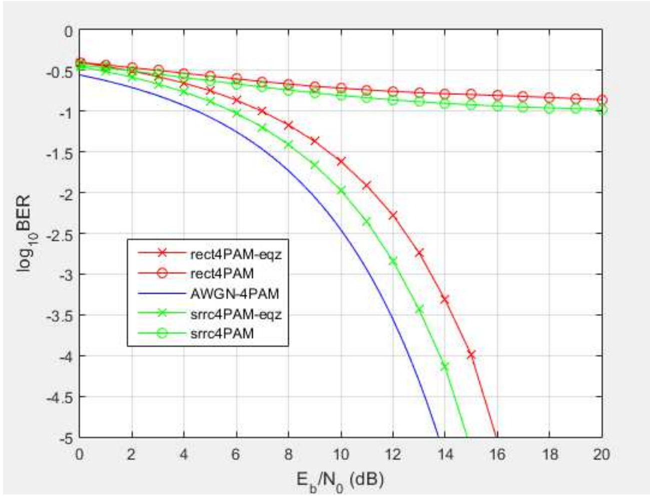

Figure 9. BER vs. SNR for 4-PAM with the ceiling-bounced CIR

4-PAM transmissions over VLC channels. Rectangular and SRRC pulse shapes are considered. For unequalized transmissions over both exponential and ceiling-bounced channels, the two pulse shapes do not lead to significantly different BER performances. However, in the presence of linear MMSE equalization with the same number of equalization filter taps, using the SRRC pulse can lead to significantly lower BERs when compared to using the rectangular pulse. Thus, pulse shaping should be employed when linear MMSE equalization is applied.

References

[1] J.M. Kahn and J.R. Barry, "Wireless infrared communications," *Proc. of IEEE*, vol. 85, no. 2, pp. 265−298, Feb. 1997.

[2] IEEE, *IEEE Std 802.15.7: Short-Range Wireless Optical Communication Using Visible Light*, 2011.

[3] E.A. Lee and D.G. Messerschmitt, *Digital Communications*, Kluwer Academic, 1994.

[4] MathWorks, "FPGA Design and SoC Codesign," available at www.mathworks.com/solutions/fpga-design/

[5] S. S. Haykin, *Digital communication*, pp. 445-446. New York: Wiley, 1988.

[6] C. R. Johnson, W. A. Sethares, and A. G. Klein, *Software Receiver Design: Build Your Own Digital Communication System in Five Easy Steps.*, Cambridge University Press, 2011.