A Novel Marching-on-in-Time Algorithm for Analyzing Arbitrary-Structure Thin-Wire Antennas

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***Abstract*—This paper presents a novel marching-on-in-time algorithm for analyzing thin-wire antennas with arbitrary structures. Transient charges and currents along the thin wire are iterated explicitly in a leap-frog fashion, similarly as in the finite difference time domain method. The input impedances of different types of thin-wire antennas in frequency domain are calculated through discrete Fourier Transform (DFT) and compared with those from Method of Moments (MoM), which shows that the proposed algorithm is effective and efficient, even in the case of complex wire structures.**

Keywords—marching-on-in-time algorithm; thin-wire antenna; charge and current iterative algorithm

I. Introduction

Thin-wire antennas are widely used in various scenes especially in lower frequency band below UHF, due to the advantages of light weight, flexible geometry, ease of manufacturing and mounting, etc. Method of moments (MoM) in frequency domain has been employed to analyze and design different wire antennas in most cases [1]. However, approaches in time domain may have some additional merits. Transient response of the antenna can be easily obtained in time domain and the mechanism of the radiation can be viewed more vividly and physically. Moreover, approaches in time domain may be more efficient, especially for wideband calculation, when the antenna has a complex geometry since no time-consuming inversion of matrices is needed. As a consequence, method of moments in time domain (MoMTD) and other similar marching-on-in-time (MOT) algorithms have been applied to thin-wire antennas, but they have not been widely used because of stability and numerical convergence problems [2]-[4]. In 1988, Dalke put forward a marching-on-in-time method to analyze the coupling of the plane wave to thin-wire structures [5], in which transient charges and currents along the wires are iterated to form the time response to the incident pulse. In this paper, a similar charge and current marching-on-in-time (CCMOT) algorithm is proposed to analyze arbitrary-structure thin-wire antennas. A transient internal voltage source, instead of an incident plane wave is applied and several modifications are made to make it suitable for antenna analysis. An internal resistance is added to the source and a time averaging scheme applied to eliminate the problem of convergence. The frequency parameters of the antenna are then obtained by discrete Fourier Transform (DFT) and agree well with those obtained using MoM in frequency domain.

II. Details of the Algorithm

A. Based Equations

When a cylindrica1 thin wire satisfies the thin-wire approximation condition [6], the current and charge along the wire can be represented by a filament on the wire axis. Assuming $\tilde{\boldsymbol{A}} = \boldsymbol{A}/\mu_0$, $\tilde{\phi} = \phi/\eta_0$, $\tilde{\xi} = c\xi$, $\tilde{\rho} = c\rho$, $\tilde{\boldsymbol{E}} = \boldsymbol{E}/\eta_0$ are normalized quantity of magnetic vector potential, electric scalar potential, linear charge density, volume charge density and electric field respectively, where μ_0, η_0, c are magnetic permeability, wave impedance and velocity of light in free space, according to the potential function theory and current continuity principle, we have

$$\tilde{\boldsymbol{A}}(\boldsymbol{r},t) = \frac{1}{4\pi}\int_L \frac{I(s',t-R/c)\hat{\boldsymbol{s}}'}{R}ds' \tag{1}$$

$$\tilde{\phi}(\boldsymbol{r},t) = \frac{1}{4\pi}\int_L \frac{\tilde{\xi}(s',t-R/c)}{R}ds' \tag{2}$$

$$\tilde{\boldsymbol{E}}^s(\boldsymbol{r},t) = -\nabla\tilde{\phi}(\boldsymbol{r},t) - \frac{\partial\tilde{\boldsymbol{A}}(\boldsymbol{r},t)}{c\partial t} \tag{3}$$

$$\oint_S \boldsymbol{J}(\boldsymbol{r},t)\cdot d\boldsymbol{S} = -\frac{\partial}{c\partial t}\int_V \tilde{\rho}(\boldsymbol{r},t)dV \tag{4}$$

where $\boldsymbol{J}(\boldsymbol{r},t)$, $I(s,t)$ represent the volume current density and linear current density respectively, s' the variable along the thin-wire, R the distance between the source point and the field point. Applying electric boundary condition on the surface of the thin wire, another equation is obtained, which is,

$$\left[\boldsymbol{E}^s(\boldsymbol{r}_0,t) + \boldsymbol{E}^i(\boldsymbol{r}_0,t)\right]_{\tan} = 0 \tag{5}$$

where $\boldsymbol{E}^s$ and $\boldsymbol{E}^i$ are the scattering and incident electric fields, 'tan' indicates the tangential component. Equations (1)-(5) form the basis of the CCMOT algorithm. For simplicity, all hats on the symbols of the normalized quantities are removed later in the paper.

B. Discretization of the Thin-Wire Structures and Time

The thin-wire structure should be first discretized with straight segments before being analyzed by an MOT algorithm. Curve wires should be approximated by a series of straight segments, as in Fig. 1. Current or tangential magnetic vector potential nodes (hollow circles) are set at the middle of each segment and charge or electric scalar potential nodes (solid circles) are set at the two ends of each segment, which is suitable for dealing with the junctions of multiple thin wires [7]. The quantities at the middle of each segment represent the average value on the whole segment and the quantities at the end of each segment represent average value of all the half segments connected to the node.

The maximum length of the segment should be constrained to no more than one tenth of the shortest wavelength in the frequency band to ensure the accuracy of the results. Along the time axis, the quantities at the middle of each segment and at the both ends of each segment are sampled at one half time step interval, which conveniently leads to apply a central difference of second-order accuracy to form an explicit iteration algorithm. Courant constraint should be satisfied between the space increment and time step to ensure the stability, which is

$$\Delta t \le \Delta s_{\min} / c \tag{6}$$

Δt is the time step and $\Delta s_{\min}$ is the minimum distance between any two of all positions of quantities .

C. Steps of the Algorithm

A symbol $Q_m^{n\pm0.5}(0)$ as a quantity at the middle of the mth segment at the time of $(n\pm0.5)\Delta t$ and a symbol $Q_m^n(\pm0.5)$ as a quantity at two ends of the mth segment at the time of $n\Delta t$ are defined before we derive the algorithm as below.

Step 1: replacing partial differentiation in (3) with central difference, we can write

$$A_m^{n+0.5}(0) = A_m^{n-0.5}(0) - \frac{c\Delta t}{\Delta s_m}\left[\phi_m^n(0.5) - \phi_m^n(-0.5)\right] + c\Delta t \bullet E_m^{i,n}(0) \tag{7}$$

Δs_m is the length of the mth segment and $E_m^{i,n}(0)$ is the electric field due to the excitation at the middle of the mth segment, which is usually a voltage of Gaussian pulse.

Step 2: replacing the integral with summation in (1) and solving the current at the same time and same position as the tangential magnetic vector potential, we have

$$I_m^{n+0.5}(0) = 4\pi A_m^{n+0.5}(0) - \sum_{i,i\ne m}^{MI} C_{m,i} I_m^{n+0.5-\tau_{m,i}}(0) \tag{8}$$

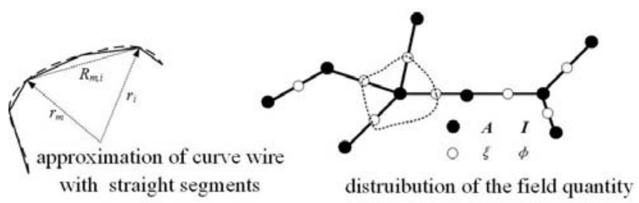

Fig. 1. Discretization of the thin-wire structure

Where MI is the total number of the segments and $C_{m,i}$ the ith coefficient before the current of ith segment, $\tau_{m,i}$ the time delay between the middles of the ith and the mth segments, which is usually not an integer and should be interpolated to the nearby integer and a half time steps.

Step 3: applying (4) on the closed surface around the junction (seen Fig. 1) , replacing the partial differentiation with central difference and solving the charge , we have

$$\xi_m^{n+1}(\pm0.5) = \xi_m^n(\pm0.5) - \frac{c\Delta t}{\sum_{j=1}^{J_m} \frac{\Delta s_{m_j}}{2}} \sum_{j=1}^{J_m} p_j I_{m_j}^{n+0.5}(0) \tag{9}$$

Where J_m is the total number of the segments connected to the mth node and $p_j = \pm1$ depends whether the current flow toward or outward the mth node.

Step 4: replacing the integral with summation in (2), we can write

$$\phi_m^{n+1}(\pm0.5) = \frac{1}{4\pi}\sum_{i=1}^{MQ} D_{m,i}\xi_m^{n+1-\tau_{m,i}}(\pm0.5) \tag{10}$$

Where MQ is the total number of the nodes and $D_{m,i}$ the ith coefficient before the charge of ith nodes, $\tau_{m,i}$ the time delay between the ith and the mth nodes, which is usually not an integer and should be interpolated to the nearby integer time steps.

The CCMOT algorithm can therefore be conducted when steps (1)-(4) are repeated until a convergence criterion is satisfied or the maximum number of time steps is arrived. Then the parameters in frequency domain can be obtained through DFT. The flow chart of the algorithm is shown in Fig. 2.

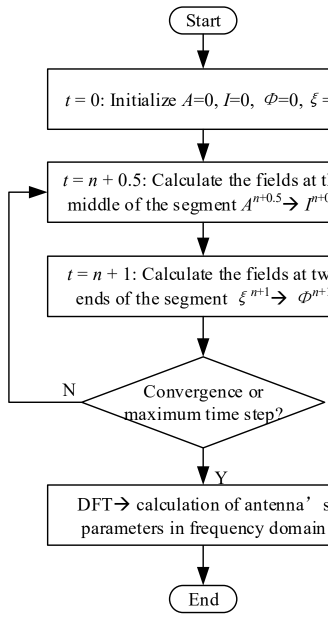

Fig. 2. The flow chart of the CCMOT algorithm

III. NUMERICAL RESULTS AND DISCUSSION

A FORTRAN code has been written based on the CCMOT algorithm and employed to several examples. The computed input currents in time domain or input impedances are compared with other methods to verify the proposed algorithm.

A. A Straight Dipole

A straight dipole is the simplest case of thin-wire antenna. The length of the dipole L=10 m, the radius 1 cm. A same Gaussian pulse is applied to excite the dipole when using CCMOT algorithm and a thin-wire FDTD algorithm. Transient input current and input impedance are shown in Fig. 3 and Fig. 4 respectively. The numbers in the parentheses in the legends of figures are the total segment numbers. It can be seen from the figures that little difference exists between the transient results form both two methods, except for the late stage. The input impedances by CCMOT agree well with those by MoM, while those by FDTD is less accurate since FDTD's cuboid cells cannot approximate the thin-wire well.

B. A Loop Antenna

A loop is a typical antenna with a closed structure, which usually makes the numerical calculation unstable. The loop with a radius of 3 meter is approximated by a polygon with 36 straight segments. When it is excited by a Gaussian pulse at the middle of any straight segment and analyzed by the CCMOT algorithm, the computed transient input current in Fig. 5 shows a sudden divergence shortly after it nearly diminishes. To keep the result convergent, we apply a time averaging scheme, which is done every time after the step 2, using

$$\tilde{I}_m^{n-1.5} = 0.125\left[I_m^{n+0.5} + I_m^{n-0.5} + 4I_m^{n-1.5} + I_m^{n-2.5} + I_m^{n-3.5}\right] \quad (11)$$

$$\tilde{I}_m^{n-0.5} = 0.25\left[I_m^{n+0.5} + 2I_m^{n-0.5} + \tilde{I}_m^{n-1.5}\right] \quad (12)$$

Then convergence is seen even after a long-time iteration. However, the input current does not converge to zero, which is a consequence of the DC component in the Gaussian pulse and can be solved by an excitation pulse without DC component, such as differentiated Gaussian pulse, as is pointed out in [4].

Another approach of removing or delaying the divergence is to use a voltage source with an internal resistance, as is shown in Fig. 5. In this case, the input current converges to zero because the DC component is dissipated by the internal resistance. Actually, the transient results before the divergence starts can be used to calculate the antennas' parameters in frequency domain through DFT

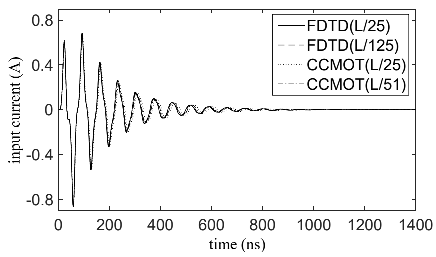

Fig. 3. Transient input current of a straight dipole

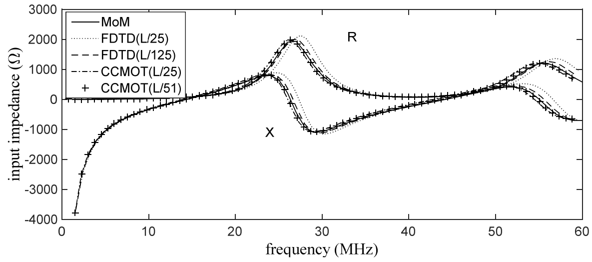

Fig. 4. Input impedance of a straight dipole

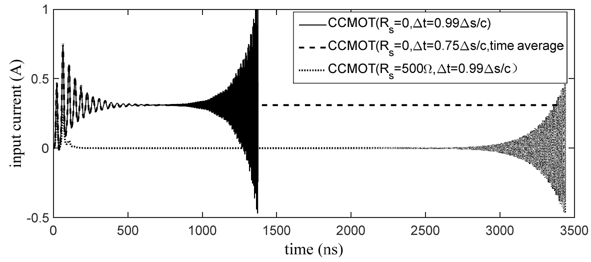

Fig. 5. Transient input current of a loop antenna

C. A Helical Antenna on A Gound Plane

More complex structure is a helical antenna on a ground plane. As an example used for verification, we assume that the diameter of the helix is 18 cm, the distance between adjacent two turns is 33.3 cm, the radius of the wire is 2 mm, and the total height of the helix is 300 cm with 9 turns. The ground plane effect can be accounted by the image principle. It is then analyzed by the CCMOT algorithm, with each turn of the helix approximated with 10 straight segments. The transient input current is shown in Fig. 6, in which complex reflection among the structure of the helix can be seen and divergence starts at the late stage if the antenna is excited by a zero-internal-resistance voltage, while few times reflection can be seen and the transient input current drops to zero quickly without any divergence if an internal resistance is added. The calculated input impedances shown in Fig. 7 are in good agreement with those from NEC2, at the both case of approximation of 10 and 20 straight segments to each turn.

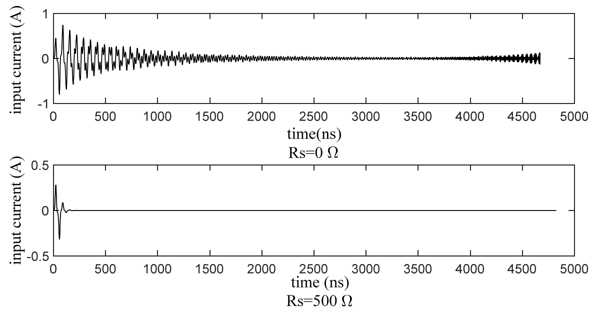

Fig. 6. Transient input current of a helical antenna on a ground plane

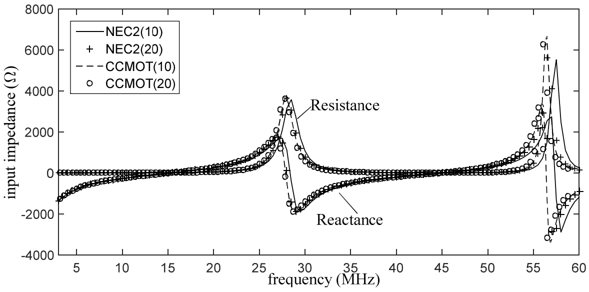

Fig. 7. The input impedance of a helical antenna on a ground plane. (the number in the parenthesis represents the number of straight segments to approximate each turn of the helix)

D. *A Caged Duo-Conical Monopole Antenna*

At last, the CCMOT algorithm is applied to a caged duo-conical monopole antenna, as in Fig. 8, which is composed of 6 wires along the generatrices of a duo-conical. It is a typical structure with junctions of multiple thin wires. Other two softwares, NEC2 and FEKO, are used to simulate the same antenna except the CCMOT algorithm. The calculated input impedances are shown in Fig. 9, in which the number in the parenthesis represents the number of segments in one minimum wavelength in the analyzed frequency band. It can be seen that the numerical results from CCMOT agree well with those from FEKO, with different lengths of segment. However, the results from NEC2 do not converge as the segment length decreases. Therefore, the CCMOT algorithm maintain a considerable accuracy even being used to analyze thin-wire antennas with junctions of multiple wires, which attributes to the accurate treatment of the junction using current continuity principle. Besides, the cpu time cost by CCMOT has no difference with the requested number of frequency while the cpu time by FEKO or any other frequency method is in proportion to the requested number of frequency.

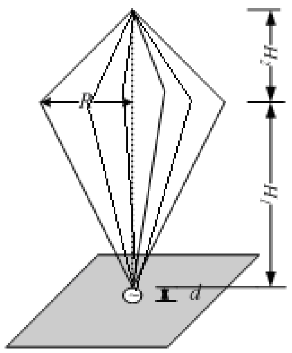

Fig. 8. The geometry of the caged duo-conical monopole antenna, H_1=4.8 m, H_2=2.4 m, R=2.3 m, d=10 cm

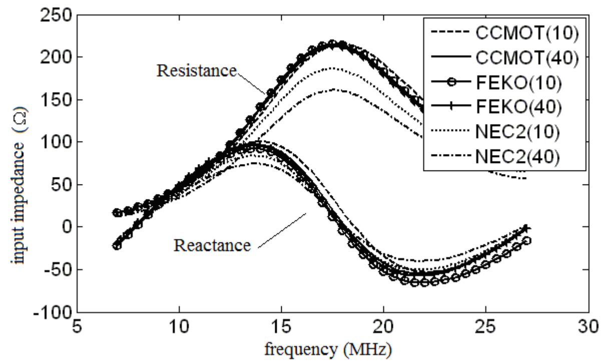

Fig. 9. The input impedance of a caged duo-conical monopole antenna.

IV. Conclusion

A novel marching-on-in-time algorithm is presented in the paper for analyzing thin-wire antennas. The divergence usually occurring in MOT algorithms can be reduced or delayed by a voltage source with an internal resistance and a time-averaging scheme. The accuracy of the CCMOT algorithm is verified by four numerical examples, which shows that the algorithm can be applied to thin-wire antennas with arbitrary structures, either simple or complex. Furthermore, as an MOT algorithm, it is of high efficiency, especially for wideband calculations, since no inversion of matrices is needed and wideband parameters can be obtained through one single calculation.

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