An Analytical Approach for Antenna Performance Evaluation for MIMO Systems

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***Abstract*—An analytical approach for antenna performance evaluation in multiple-input–multiple-output (MIMO) systems is proposed. By considering the elevation angles of the electromagnetic rays at both the base station (BS) and mobile station (MS), a three-dimensional (3D) channel model is introduced. Then the analytical approach which evaluates the effects of antenna configurations on channel capacity and diversity performance of MIMO systems is derived. In order to verify the proposed method, a link-level simulation is implemented, in which the effects of isolation on system throughput, and the effects of envelope correlation coefficients (ECC) on equivalent diversity gain are evaluated, respectively. The simulation results validated our proposed approach.**

***Key Words*—Antenna configurations, 3D channel model, antenna gain imbalance, envelope correlation coefficients, multiple-input–multiple-output (MIMO) system**

I. INTRODUCTION

MIMO transmission can improve the spectral efficiency significantly by utilizing multi-antennas both at the transmitter and receiver [1]-[2]. The performance of MIMO system is mainly dependent on the characteristics of antenna and real propagation environment. There are two common channel models, including the spatial channel model (SCM) proposed by 3GPP [3] and the WINNER II channel model proposed by the WINNER project [4], both of which are two-dimensional (2D), only assume that rays are transmitted in the horizontal plane, without considering the vertical dimension. In order to fully investigate the channel characteristics, it is important to exploit a more accurate three-dimensional(3D) channel model [5].Base on the ITU 2D channel model, a 3D channel model was presented in [6] by adopting the distance-dependent elevation angles. In [7], the common MIMO channel models were classified. It mainly describes the geometry-based stochastic channel model (GSCM) and explains the impact of the propagation environment, such as impact of the distribution of scatters.

In addition to channel model, the performance of antenna is another important factor to the MIMO systems. To some extent, the characteristics of antenna also affect channel characteristics. For instance, the correlation among antenna elements usually leads to channel fading correlation. The correlation of the terminal antennas was studied in [8], and its impact on MIMO channel capacity was investigated. The authors of [9] proposed a designing method for mobile terminal antennas which can reduce the ECC effectively. The other parameters of antenna, for instance, antenna gain imbalance (AGI), antenna isolation, may also have significant effects on the MIMO performance. Therefore, it is necessary to propose an evaluation model and investigate the effects of various antenna configurations.

In this paper, an analytical approach for antenna performance evaluation in MIMO systems is proposed. First, a 3D channel model considering the elevation dimension is presented. Then we investigates the effects of antenna characteristics on channel capacity and diversity performance of MIMO system. Finally, a link-level simulation is implemented to verify the proposed method.

II. PROPOSED MODEL AND ANALYTICAL APPROACH

A. 3D Channel Model

Based on the WINNER II channel model, the 3D channel model could be derived from adopting elevation components of angle of arrival (AOA) and angle of departure (AOD) of the electromagnetic rays as Fig.1.

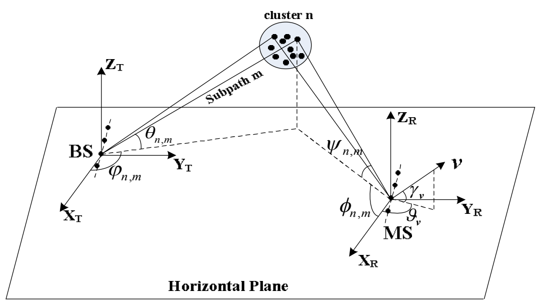

Fig. 1. 3D Channel Model.

The generation of elevation and azimuth angles is the same with [4].Vertical and horizontal angles related to thelarge-scale parameters, such as the distribution of vertical angle extension, are inferred by actual statistical data. It should be mentioned that the horizontal AOA and AOD are randomly coupled in 2D channel model. However, in 3D channel model, due to the azimuth and elevation angles are generally not independent with each other. The generation procedure of the angles is divided into

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the following three steps. Firstly, coupling randomly the horizontal departure angle $\varphi_{n,m}$ with the vertical departure angle $\theta_{n,m}$, then the 3D departure angle $(\varphi_{n,m}, \theta_{n,m})$ at the transmitter is obtained. Secondly, coupling randomly the horizontal arrival angle $\phi_{n,m}$with the vertical arrival angle $\psi_{n,m}$, then the 3D arrival angle $(\phi_{n,m}, \psi_{n,m})$at the receiver is obtained. Finally, the 3D departure angle $(\varphi_{n,m}, \theta_{n,m})$ and the arrival angle$(\phi_{n,m}, \psi_{n,m})$ should be coupled randomly.

In this channel model, the generate procedure of channel coefficients is the same as the WINNER II channel model. Assume that the propagation environment is NLOS case, then the channel coefficient between thes^{th} transmitting antenna and the u^{th} receiving antenna can be obtained from (1).

Where P_n denotes the average power of the ray cluster n. M is the number of rays per cluster. $E_{rx,u,v}(\phi_{n,m}, \psi_{n,m})$ and $E_{rx,u,h}(\phi_{n,m}, \psi_{n,m})$ are far-field patterns of vertical and horizontal polarization components for the u^{th}receiving antenna element, respectively. $\phi_{n,m}$ and $\psi_{n,m}$ are the azimuth angle and elevation angle of the arrival rays, respectively. $\Phi_{n,m}^{vv}$, $\Phi_{n,m}^{vh}$, $\Phi_{n,m}^{hv}$ and $\Phi_{n,m}^{hh}$ are the phase shifters of the corresponding polarization components, respectively. $\chi_{n,m}$ is the cross-polarization power ratio in linear scale. $E_{tx,s,v}(\varphi_{n,m}, \theta_{n,m})$ and $E_{tx,s,h}(\varphi_{n,m}, \theta_{n,m})$ are far-field patterns of vertical and horizontal polarization components for the s^{th} transmitting antenna element, respectively.$\varphi_{n,m}$ and$\theta_{n,m}$ are the azimuth angle and elevation angle of the departure rays, respectively.λ_0 is wavelength of the center frequency. $\boldsymbol{r}_s$ and $\boldsymbol{r}_u$ are the s^{th} transmitting antenna position vector and the u^{th} receiving antenna position vector, respectively. $\boldsymbol{e}_{n,m}^{AOD}$is the departure angle unit vector, and $\boldsymbol{e}_{n,m}^{AOA}$ is the arrival angle unit vector.$f_{n,m}$is the Doppler frequency-shifter that can be obtained from (2), shown at the bottom of this page.

Where $|\boldsymbol{v}|$ is the velocity of mobile station. γ_v and $\vartheta_v$are the travelling direction in the vertical and horizontal plane, respectively. Assuming the mobile station only moves in the horizontal plane, that is $\gamma_v = 0$. Then (2) can be rewritten as,

$$f_{n,m} = \frac{|\boldsymbol{v}|\cos\psi_{n,m}\cos\left(\phi_{n,m} - \vartheta_v\right)}{\lambda_0} \tag{3}$$

B. Channel Capacity

For a point-to-point MIMO system, in which the transmitter was employed with N_t transmitting antennas and the receiver was employed with N_r receiving antennas. It can be assumed that the transmitted signal experiences the flat-fading channel, and the input-output relationship can be expressed as,

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{4}$$

Where $\mathbf{y}$ is the receiving signal vector, $\mathbf{x}$ is the transmitting signal vector, $\mathbf{n}$ is the noise vector, and the channel can be represented by the complex matrix-$\mathbf{H}$. Assume that the channel state information has not been acquired at the transmitter, which means that the transmitting power is dividedon the transmitting antennas equally, and then the channel capacity can be expressed as (5) according to [1],

$$\mathrm{C} = \log_2\left[\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)\right] \tag{5}$$

Where ρ is the average signal noise power ratio (SNR). $\mathrm{I}_{\mathrm{N_r}}$ is an $\mathrm{N_r} \times \mathrm{N_r}$ identity matrix. As shown in formula (5), the relationship between channel capacity and antenna parameters can be demonstrated through the channel coefficients. The antenna parameters,for instance,AGI, ECC and antenna isolation,have effects on the MIMO systemcapacity. In order to analyze the effects of the antenna characteristicson MIMO system capacity, the channel matrix H in formula (5) can be modified as follow,

$$\mathbf{H} = \mathbf{\Lambda}\mathbf{H}_{eff} \tag{6}$$

Where $\mathbf{H}_{eff}$ is the actual channel matrix, the element $h_{ij}$is the channel coefficient between the j^{th}transmitting antenna and the i^{th}receiving antenna, which can be calculated according to the formula (1).$\mathbf{\Lambda}$ is a diagonal matrix that can be expressed as,

$$\mathbf{\Lambda} = diag\left(\eta_{tol,1}, \eta_{tol,2}, \cdots, \eta_{tol,N_r}\right) \tag{7}$$

$$h_{u,s,n}(t) = \sqrt{P_n}\sum_{m=1}^{\mathrm{M}}\begin{bmatrix}E_{rx,u,v}\left(\phi_{n,m},\psi_{n,m}\right)\\ E_{rx,u,h}\left(\phi_{n,m},\psi_{n,m}\right)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}exp\left(j\Phi_{n,m}^{vv}\right) & \sqrt{\chi_{n,m}}exp\left(j\Phi_{n,m}^{vh}\right)\\ \sqrt{\chi_{n,m}}exp\left(j\Phi_{n,m}^{hv}\right) & exp\left(j\Phi_{n,m}^{hh}\right)\end{bmatrix}\begin{bmatrix}E_{tx,s,v}\left(\varphi_{n,m},\theta_{n,m}\right)\\ E_{tx,s,h}\left(\varphi_{n,m},\theta_{n,m}\right)\end{bmatrix}$$
$$\cdot exp\left(j2\pi\lambda_0^{-1}\boldsymbol{r}_s\cdot\boldsymbol{e}_{n,m}^{AOD}\right)\cdot exp\left(j2\pi\lambda_0^{-1}\boldsymbol{r}_u\cdot\boldsymbol{e}_{n,m}^{AOA}\right)\cdot exp\left(j2\pi f_{n,m}t\right) \tag{1}$$

$$f_{n,m} = \frac{|\boldsymbol{v}|\cos\psi_{n,m}\cos\phi_{n,m}\cos\gamma_v\cos\vartheta_v + |\boldsymbol{v}|\cos\psi_{n,m}\sin\phi_{n,m}\cos\gamma_v\sin\vartheta_v + |\boldsymbol{v}|\sin\psi_{n,m}\sin\gamma_v}{\lambda_0} \tag{2}$$

The elements in the diagonal matrix are the total efficiency of each receiving antenna. The total antenna efficiency of the k^{th} receiving antenna can be calculated by the following expression [10],

$$\eta_{tol,k} = \eta_{r,k}\left(1-\sum_{l=1}^{N_r}\left|S_{lk}\right|^2\right), \forall k \in \{1,2,\cdots,N_t\} \tag{8}$$

Where $\eta_{r,k}$ is the radiation efficiency of the k^{th} antenna. The spatial correlation of the channel can be described bya correlation matrix of the spatial channel, which can be obtained by the following procedure.

At first, assuming the channel matrix $\tilde{\mathbf{H}}$ is generated by the proposed 3D channel model, and it can be expressed as,

$$\begin{aligned}\tilde{\mathbf{H}} &= (\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_{N_t}) \\ &= \begin{pmatrix} h_{11}(t) & h_{12}(t) & \cdots & h_{1N_t}(t) \\ h_{21}(t) & h_{22}(t) & \cdots & h_{2N_t}(t) \\ \vdots & \vdots & \cdots & \vdots \\ h_{N_r1}(t) & h_{N_r2}(t) & \cdots & h_{N_rN_t}(t) \end{pmatrix}\end{aligned} \tag{9}$$

Where $\mathbf{H}_i (i = 1,2,\cdots,N_t)$ is a $N_r \times 1$ column vector, and $\mathrm{vec}(\tilde{\mathbf{H}})$ is defined as,

$$\mathrm{vec}(\tilde{\mathbf{H}}) = \left(\mathbf{H}_1^{\mathrm{T}}, \mathbf{H}_2^{\mathrm{T}}, \ldots, \mathbf{H}_{N_t}^{\mathrm{T}}\right)^{\mathrm{T}} \tag{10}$$

Then the spatial channel correlation matrix $\mathbf{R}$ can be calculated by $\mathrm{vec}(\tilde{\mathbf{H}})$ according to the following expression,

$$\begin{aligned}\mathbf{R} &= \mathrm{cov}\left(\mathrm{vec}(\tilde{\mathbf{H}})\right) \\ &= \mathrm{E}\left\{\mathrm{vec}(\tilde{\mathbf{H}})\mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{H}})\right\}\end{aligned} \tag{11}$$

Where $\mathrm{vec}^{\mathrm{H}}(\cdot)$ denotes conjugate transpose of $\mathrm{vec}(\cdot)$. It can be assume that $h_{mp}(t)$ and $h_{nq}(t)$ are elements of $\tilde{\mathbf{H}}$. $h_{mp}(t)$ denotes the channel coefficient of the p^{th} transmitting antenna with the m^{th} receiving antenna, and $h_{nq}(t)$ denotes the channel coefficient of the q^{th} transmitting antenna with the n^{th} receiving antenna. Then the cross-correlation between $h_{mp}(t)$ and $h_{nq}(t)$ can be expressed as,

$$r_{mp,nq}(t) = \mathrm{E}\left\{h_{mp}(t)h_{nq}^*(t)\right\} \tag{12}$$

Where $r_{mp,nq}(t)$ is the element of $\mathbf{R}$. According to formula (11)-(12), the spatial channel correlation matrix $\mathbf{R}$ can be obtained. Therefore the real channel matrix that takes the antenna correlation into account can be expressed as,

$$\mathbf{H}_{eff} = \mathrm{vec}^{-1}\left(\mathbf{R}^{\frac{1}{2}}\mathrm{vec}(\mathbf{H}_w)\right) \tag{13}$$

Where $\mathbf{H}_w \in \mathbb{C}^{N_rN_t\times 1}$, and its elements are independent identically distributed with complex Gaussian distribution. $\mathrm{vec}^{-1}(\cdot)$ denotes inverse operation of $\mathrm{vec}(\cdot)$. Once $\mathbf{H}_{eff}$ is obtained, the modified channel matrix $\mathbf{H}$ can be obtained according to formula (6). Substituting $\mathbf{H}$ into formula (5), then the effects of antenna characteristics on the performance of MIMO systems can be obtained.

C. Diversity Performance

The diversity performance of MIMO system is usually evaluated by ECC, mean effective gain (MEG), and effective diversity gain [11].The effective diversity gain can be defined as the improvement of the average SNR of the diversity combined signal at a given probability as follow [12],

$$G_{div}(\mathrm{dB}) = \left[\gamma_0 - \gamma_{ref}\right]_{given\ probability} \tag{14}$$

Where γ_0 is the instantaneous SNR of the diversity combined signal, γ_{ref} is the average SNR of a reference antenna. And the given probability can be measured by the cumulative density function (CDF) curves of SNR as follow,

$$\mathrm{CDF}(x \le \gamma_0) = \int_0^{\gamma_0} p(x)dx \tag{15}$$

Where $p(x)$ is the probability density function of the SNR.In this section, diversity combining technique with respect to maximum ratio combining (MRC) and selection combining (SC) of the receiving antenna are used to evaluate the diversity performance. For a Rayleigh fading channel, the CDF of MRC and SC can be obtained from (16) shown at the bottom of this page and (17), respectively.

$$\mathrm{CDF_{SC}}(x \le \gamma_0) = \prod_{i=1}^{K}\left(1 - exp\left(-\frac{\gamma_0}{\lambda_i}\right)\right) \tag{17}$$

$$\mathrm{CDF_{MRC}}(x \le \gamma_0) = \sum_{i=1}^{K}\frac{1}{\prod_{j=1,j\ne i}^{K}\left(1-\frac{\lambda_j}{\lambda_i}\right)}\cdot\left(1-exp\left(-\frac{\gamma_0}{\lambda_i}\right)\right), \quad \forall \lambda_j \ne \lambda_i \tag{16}$$

$$\mathbf{\Lambda} = \begin{pmatrix} G_{e1} & 0 & \cdots & 0 \\ 0 & G_{e2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & G_{eN_r} \end{pmatrix}^{\frac{1}{2}} \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1N_r} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2N_r} \\ \vdots & \vdots & \cdots & \vdots \\ \rho_{N_r1} & \rho_{N_r2} & \cdots & \rho_{N_rN_r} \end{pmatrix} \begin{pmatrix} G_{e1} & 0 & \cdots & 0 \\ 0 & G_{e2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & G_{eN_r} \end{pmatrix}^{\frac{1}{2}} \tag{18}$$

Where $\lambda_{i,j}$ are the non-zero singular values of antenna correlation coefficient matrix $\mathbf{\Lambda}$, and can be calculated from $\mathbf{\Lambda}$by singular value decomposition (SVD), SVD($\mathbf{\Lambda}$).K is the number of non-zero singular values. $\mathbf{\Lambda}$ can be obtained from (18) shown at the bottom of this page. Where $\mathbf{G}_{el}(l = 1, 2, \cdots, N_r)$ and $\rho_{mn}(m, n \in \{1, 2, \cdots, N_r\})$ are the MEGs and ECCs of the receiving antennas, respectively, which can be obtained from the antenna far-field patterns and the channel characteristics, and the calculation method is presented in [11].

III. Simulation Results

The propagation scenario is set as urban macro-cell (UMa). Dual-antenna elements are employed at both the BS and MS side, and the electromagnetic wave signal is transmitted from the BS to the MS. Dipole antennas are adopted at the BS, and practical dual-elements antennas are used at the MS. System bandwidth is set to 10MHz, and 16QAM is used.

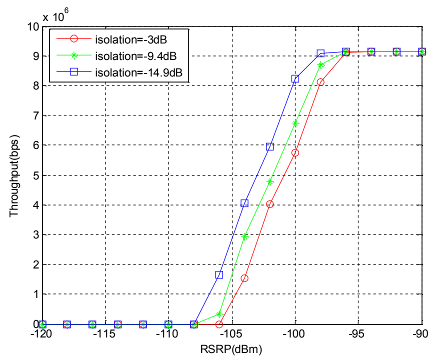

Fig. 2.Effects of antenna isolation on system throughput with RSRP

Fig.2 illustrates the effects of antenna isolation on system throughput. The antenna S-parameters S_{21} was considered. There are three groups of receiving antenna with different isolation, which are 3dB, 9.4 dB and 14.9dB. Correlation between the antenna elements is improved due to the increase of antenna isolation, and the element mutual coupling is alleviated.

Fig. 3 illustrates the effects of antenna correlation on diversity performance of MIMO system.It can be seen that the correlation deteriorates the effective diversity gain. For a MRC receiving diversity, the diversity gain of 4.8 dB at 1% CDF level is decrease due to the effects of antenna correlation. For a SC receiving diversity, the diversity gain decrease by 4.6 dB accordingly.

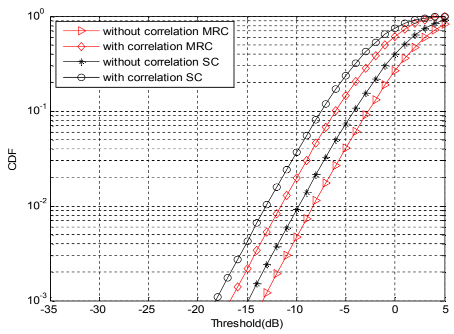

Fig. 3. Effects of antenna correlation on diversity performance

IV. Conclusion

An analytical approach of evaluating effects of the antenna characteristics on the performance of MIMO systems is proposed in this paper. Based on the WINNER II channel model, a 3D channel model is proposed by adding the elevation dimension, which describes the real propagation environment more accurately.The close-form expressions of antenna parameters to channel capacity and effective diversity gain are derived. Antenna characteristics in terms of isolation and ECC are studied through a link-level simulation, and the simulation results verify the validity of the proposed method. In practice, it can be used to support the antennas design for MIMO systems.

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