Blind Signal Separation Using SOBI Algorithm Under the Effect of Mutual Coupling of Array

Keita Matsubara
Dept. of Computer Science and Engineering
Nagoya Institute of Technology
Nagoya 466-8555, Japan

Nobuyoshi Kikuma
Dept. of Computer Science and Engineering
Nagoya Institute of Technology
Nagoya 466-8555, Japan
Email: kikuma@m.ieice.org

Kunio Sakakibara
Dept. of Computer Science and Engineering
Nagoya Institute of Technology
Nagoya 466-8555, Japan

Abstract—**Blind signal separation techniques using array antennas are expected to bring many advantages in the radio monitoring system. In several techniques, Independent Component Analysis (ICA) attracts much attention because of its convenience and effectiveness. In this paper, we focus on SOBI algorithm in ICA and examine the performance under the effect of mutual coupling of array elements.**

Keywords*—*Independent Component Analysis, SOBI algorithm, blind signal separation, mutual coupling, AR signal

I. Introduction

The role of radio wave monitoring systems is important to maintain regular radio environments in which we can use the radio waves effectively. Blind signal separation techniques bring many advantages in ratio monitoring system using array antennas. Particularly, Independent Component Analysis (ICA) [1] is expected to separate the incident signals and detect each signal effectively and conveniently. To perform blind signal separation of multiple signals, we normally use array antennas. However, there is a serious problem that separation accuracy is degraded by the effect of mutual coupling of array elements[2], [3]. In this paper, therefore, we focus on SOBI algorithm[5] in ICA, and we investigate in detail the immunity of ICA to the mutual coupling effect in comparison with ESPRIT algorithm[2]–[4].

II. Model for Analysis

Fig. 1 shows a K-element uniform linear array with the array element spacing of d. The array receives L incident signals whose angles of arrival are represented by θ_l $(l = 1, 2, \ldots, L)$ as shown in Fig. 1. We estimate the individual source signal $s_l(t)$ $(l = 1, 2, \ldots, L)$ from the mixed signals of L sources.

The received signals are expressed in a vector form as follows.

$$\boldsymbol{x}(t) = \tilde{\boldsymbol{A}}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \tag{2}$$

$$\tilde{\boldsymbol{A}} = [\boldsymbol{M}_1\boldsymbol{a}(\theta_1), \boldsymbol{M}_2\boldsymbol{a}(\theta_2), \cdots, \boldsymbol{M}_L\boldsymbol{a}(\theta_L)] \tag{3}$$

$$\boldsymbol{a}(\theta_l) = \begin{bmatrix} 1 & e^{j\mu_l} & e^{j2\mu_l} & \cdots & e^{j(M-1)\mu_l} \end{bmatrix}^T \tag{4}$$

$$\mu_l = -(2\pi/\lambda)d\sin\theta_l \quad (l = 1,\ 2, \ldots, L) \tag{5}$$

where $\boldsymbol{a}(\theta_l)$ is the steering vector, and $\tilde{\boldsymbol{A}}$ is the steering matrix including array error matrices $\boldsymbol{M}_l(l = 1, 2, \ldots, L)$. $\boldsymbol{s}(t)$ is the signal vector and $\boldsymbol{n}(t)$ is the noise vector.

In this paper, we assume that all sources emit first-order autoregressive (AR) signals which have following relation.

$$\boldsymbol{s}(t+\tau) = \boldsymbol{\Phi}\boldsymbol{s}(t) + \boldsymbol{e}(t+\tau) \tag{6}$$

$$\boldsymbol{\Phi} = \mathrm{diag}[\rho_1 e^{j\phi_1},\ \rho_2 e^{j\phi_2},\ \cdots,\ \rho_L e^{j\phi_L}] \tag{7}$$

where ρ_l and ϕ_l are amplitude and phase of the coefficient of l-th AR signal, and $\boldsymbol{e}(t)$ is the error vector (Gaussian) in the AR model.

Here, we restore the source signals using the separation matrix $\boldsymbol{W}$ derived by ICA. Then, the array output $\boldsymbol{y}(t)$ is represented by

$$\boldsymbol{y}(t) = \boldsymbol{W}\boldsymbol{x}(t) = \boldsymbol{W}\tilde{\boldsymbol{A}}\boldsymbol{s}(t) \tag{8}$$

From Eq.(8), if $\boldsymbol{W} = \tilde{\boldsymbol{A}}^{-1}$, then we have $\boldsymbol{y}(t) = \boldsymbol{s}(t)$ and hence the source signal vector $\boldsymbol{s}(t)$ is restored.

ICA estimates the separation matrix $\boldsymbol{W}$ by using only information that the source signals are independent. Expressing $\boldsymbol{W} = \boldsymbol{U}^H\boldsymbol{V}$, the matrix $\boldsymbol{V}$ is obtained from the whitening process[1] of the received signals, and the matrix $\boldsymbol{U}$ (unitary matrix) is obtained through SOBI algorithm[5] in this paper.

III. SOBI Algorithm

The correlation matrix of the whitened signal vector $\bar{\boldsymbol{x}}(t) = \boldsymbol{V}\boldsymbol{x}(t)$ is defined by

$$\boldsymbol{R}(\tau) = E\left[\bar{\boldsymbol{x}}(t+\tau)\bar{\boldsymbol{x}}(t)^H\right] \tag{9}$$

SOBI algorithm obtains the unitary matrix $\boldsymbol{U}$ by means of joint diagonalization using iterative Jacobi method for multiple correlation matrices $\boldsymbol{R}(\tau)$ with different time difference τ. We express the number of matrices by L_d. By this process, we can obtain the matrix $\boldsymbol{U}$ satisfying $\boldsymbol{U}^H\boldsymbol{R}(\tau_m)\boldsymbol{U}$ $(m = 1, 2, \ldots, L_d)$ and we can determine the separation matrix $\boldsymbol{W}$ from $\boldsymbol{U}^H\boldsymbol{V}$.

IV. Computer Simulation

Under the common condition of Table I, computer simulation is carried out. For comparison, ESPRIT algorithm is employed which achieves the signal separation via DOA estimation. Figs. 2 and 3 are the results of signal separation by SOBI algorithm and ESPRIT algorithm, respectively. In both figures, the horizontal axis is the arrival angle separation of two waves, and the vertical axis is the average SIR (Signal-to-Interference Ratio) of the array output. From Fig. 2, it is confirmed that SOBI algorithm can provide high SIR regardless of the mutual coupling effect. From Fig. 3, on the other hand, it is seen that SIR of ESPRIT is degraded by the mutual coupling effect. In comparison between Fig. 2 and Fig. 3, SOBI algorithm has the higher capability of separating two AR signals with different AR parameters when the angle separation is small, resulting in the higher resolution of SOBI algorithm.

TABLE I: Simulation conditions

Array configuration	Uniform linear array
Number of elements	3
Element spacing	Half wavelength
Number of incident signals	2
DOA of signals [deg.]	(0, θ_2)
Number of snapshots	1200
Input SNR [dB]	10
Number of trials	100
Coefficient amplitude of AR signals	$\rho_1 = \rho_2 = 0.85$
Coefficient phase of AR signals	$\phi_1 = -0.2\pi$, $\phi_2 = 0.2\pi$
Variance of error vector components of AR signals	10^{-4}
L_d (SOBI)	5
Number of iterations of Jacobi method (SOBI)	10

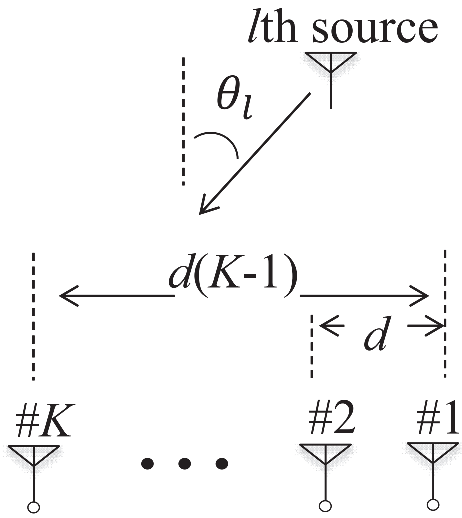

Fig. 1: Receiving model using K-element uniform linear array

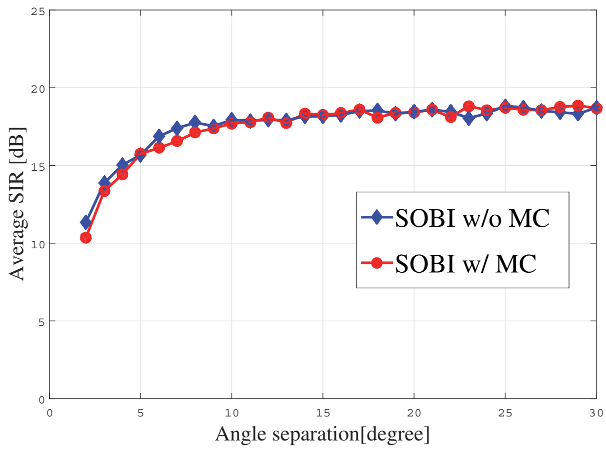

Fig. 2: Average SIR vs. angle separation of 2 waves (SOBI)

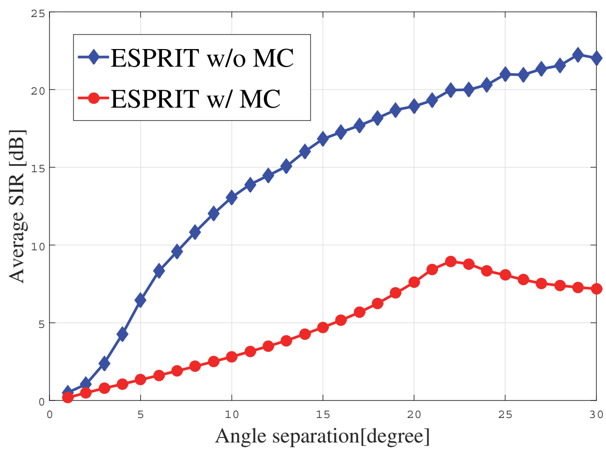

Fig. 3: Average SIR vs. angle separation of 2 waves (ESPRIT)

V. Conclusion

As a result of computer simulation, it is confirmed that SOBI algorithm which is one of ICA methods works well in blind signal separation regardless of the effect of mutual coupling of array elements. In addition, SOBI algorithm has the high resolution in separating AR signals with different AR parameters compared with ESPRIT algorithm.

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