Design of Constrained IIR Filters Using PSO

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Abstract: In this study, IIR (Infinite Impulse Response) filters having a null frequency in a stopband are designed using PSO (Particle Swarm Optimization). A new penalty function is introduced to an objective function in addition to a conventional penalty function which ensures a stability of IIR filters. Then, local minimums are brought to the objective function by adding such a constraint. Therefore, it is important to avoid a local minimum stagnation of PSO. In the proposed method, a particle reallocation method is applied when the stagnation has occurred. The effectiveness of the method is verified through several design examples.

1. Introduction

A design problem of IIR (Infinite Impulse Response) filters is difficult to solve because it is generally formulated as a nonlinear optimization problem. Some methods using heuristic approaches were proposed as design methods of IIR filters[1], [2], [3]. Especially, PSO (Particle Swarm Optimization)[4] is applied because of low computational cost and a strong directivity[2], [3].

In a lot of signal processing applications, special constraints are often required in addition to a normal specification like a frequency selection characteristic. Forming a null characteristic is one of constraints and useful for suppressing the noise that has large power component in a specific frequency. However, such a constraint brings the design problem a large number of local minima. As a result, it is easily expected that the number of stagnation occasions increases in comparison with the non-constraint specification. Therefore, a strategy of stagnation avoidance is extremely important and is strongly required.

In this paper, the particle reallocation method [5] is applied to design IIR filters having the null constraint in a specific frequency. In the method, the particles are shifted to another space when the stagnation occurred. Several design examples are shown to present the effectiveness of the method.

2. Design problem

The frequency response of IIR filters is described as follows,

$$H(\omega) = a_0 \prod_{n=1}^{N}(1 - z_n e^{-j\omega}) \Big/ \prod_{m=1}^{M}(1 - p_m e^{-j\omega}), \quad (1)$$

where a_0 is a scaling factor of IIR filters, N is a numerator order, M is a denominator order, $z_n (n = 1, 2, \cdots, N)$ are zeros, $p_m (m = 1, 2, \cdots, M)$ are poles, and ω is an angular frequency. In this paper, real coefficient filters are designed, and thus z_n and p_m are complex conjugates or real numbers. The design problem of IIR filters based on the Chebyshev approximation criteria can be described as

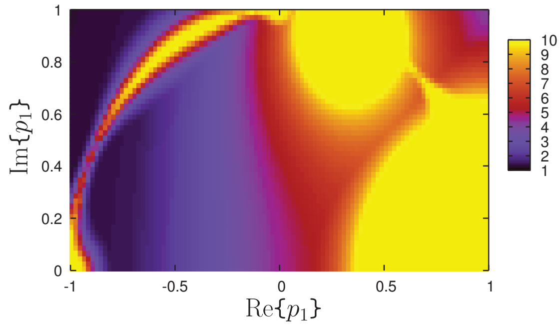

Figure 1. Objective function of IIR filter design problem

$$\min_{\boldsymbol{x}} \max_{\omega \in \Omega} |D(\omega) - H(\omega)|, \quad (2)$$

where $\boldsymbol{x} = [a_0, z_1, \cdots, z_N, p_1, \cdots, p_M]^T$ is the design parameter vector, Ω is an approximation frequency band, and $D(\omega)$ is a desired frequency response. $H(\omega)$ is the rational function. Fig.1 shows the objective function depicted over p_1. The vertical axis denotes the imaginary part of p_1 and the horizontal axis denotes the real part of p_1. The depth of color in this figure means the value of the objective function. The frequency response of IIR filters is the rational function, and all of poles must exist within the unit circle on z-plane. These conditions make the objective function a nonlinear form. Thus, the objective function of the design problem of IIR filters is a multi-modal function and is difficult to obtain the optimal solution.

3. Objective function

The objective function is defined as follows to apply PSO to the design problem of IIR filters having the null,

$$F(\boldsymbol{x}) = |D(\omega_d) - H(\omega_d)| + c_s\phi(\boldsymbol{x}) + c_{null}|H(\omega_{null})|, \quad (3)$$

where $\omega_d (d = 1, 2, \cdots, S)$ is the discrete angular frequency, S is the number of frequency samples, ω_{null} is the null frequency, and c_s and c_{null} are weight parameters. The second term of the right side of (3) is a penalty function for ensuring the stability of IIR filters. The third term is a penalty function for forming a null at a specified frequency.

A penalty function $\phi(\boldsymbol{x})$ is defined as follows,

$$\phi(\boldsymbol{x}) = \begin{cases} {p_{max}}^2 & , p_{max} \geq R \\ 0 & , p_{max} < R \end{cases} \quad (4)$$

where p_{max} is a maximum pole radius, and $R (R < 1.0)$ is a fixed maximum pole radius given in advance.

For the third term of the right side of (3), a null is formed at w_{null} when $H(\omega_{null}) \approx 0$. Therefore, $|H(\omega_{null})|$ becomes small if the wight parameter c_{null} is enough large.

The design problem of IIR filters is to determine $\boldsymbol{x}$ so as to minimize $F(\boldsymbol{x})$.

4. Particle swarm optimization

PSO is the multi-point searching algorithm inspired by social behavior of animals like a flock of birds. PSO has a swarm consists of some particles. Then, each particle is specified by a position vector $\boldsymbol{x}_u$ and a velocity vector $\boldsymbol{v}_u$, where $u(u = 1, 2, \cdots, P)$ is particle number and P is the number of particles, and its position is updated toward both the best solution of the swarm and the best solution of each particle. Updating of the position and the velocity of the particle u in the t-th iteration are carried out as follows,

$$\boldsymbol{x}_u^{t+1} = \boldsymbol{x}_u^t + \boldsymbol{v}_u^{t+1}, \quad (5)$$

$$\boldsymbol{v}_u^{t+1} = w^t \boldsymbol{v}_u^t + c_1 r_1 (\boldsymbol{p}_u^t - \boldsymbol{x}_u^t) + c_2 r_2 (\boldsymbol{g}^t - \boldsymbol{x}_u^t), \quad (6)$$

where $\boldsymbol{p}_u^t$ is the best solution which the u-th particle has searched before, and $\boldsymbol{g}^t$ is called the best solution among all particles up to the t-th iteration. $\boldsymbol{g}^t$ is called global best. w^t is the inertia weight parameter, c_1 is a weight parameter toward $\boldsymbol{p}_u^t$, c_2 is a weight parameter toward $\boldsymbol{g}^t$, and r_1 and r_2 are uniform random numbers in the interval of $[0, 1]$. w^t is linearly decreased using a following equation,

$$w^t = w_{max} - \frac{t}{I_{max}}(w_{max} - w_{min}), \quad (7)$$

where w_{max} is an upper bound of w, w_{min} is a lower bound of w, I_{max} is the maximum number of iterations. Equation (7) means that PSO changes from the global search to the local search. Because particles are updated using good solution information, particles are gathered around the local minimum at the end of trial. As a result, PSO can enumerate the candidates of solution rapidly. However, this characteristics often lead to the local minimum stagnation because of the strong directivity toward a local minimum.

5. Particle reallocation for avoidance of local minimum stagnation

Multi-swarm PSO is introduced and particles are reallocated using some swarms to avoid the local minimum stagnation. In this paper, each swarm searches independently. In this method, the particles belonging to a stagnated swarm are moved to the reallocation space like Fig.2. Multiple swarms are used to determine the reallocation space. Then, a convex combination of each the global best of some swarm is used for determining good reallocation space $\boldsymbol{\Gamma}$. $\boldsymbol{\Gamma}$ is defined as $[\boldsymbol{\gamma} - h\boldsymbol{e}, \boldsymbol{\gamma} + h\boldsymbol{e}]$, where $\boldsymbol{\gamma}$ is the center of $\boldsymbol{\Gamma}$, h is a width of perturbation for the reallocation, and $\boldsymbol{e} = [1, 1, \cdots, 1]^T$. $\boldsymbol{\gamma}$ can be calculated as follows,

$$\boldsymbol{\gamma} = \sum_{k=1}^{K} \lambda_k \boldsymbol{g}_k, \quad (8)$$

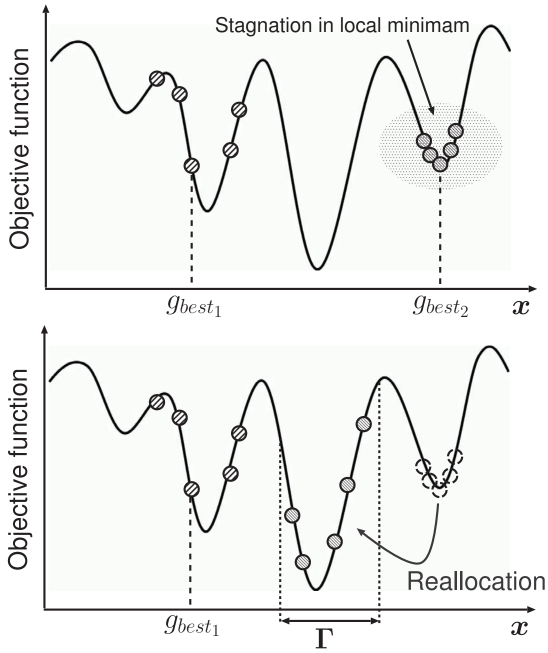

Figure 2. An example of the avoidance of local minimum stagnation by particle reallocation

where K is the number of selected swarms, $\boldsymbol{g}_k$ is the best solution of the k-th swarm, $\lambda_k \geq 0$ and $k = 1, \cdots, K$ are weight parameters, then $\sum_{k=1}^{K} \lambda_k = 1$. All swarms tend to converge with increasing of the number of updating, hence some swarms that include a stagnated swarm among all swarms are chosen randomly for successive search. Then, at least three swarms are required for determining the reallocation space on the complex plane. In [5], it was shown that it is not suitable to decrease w gradually like (7) because particles repeat the diversification and the intensification. Therefore, w is set to a constant value.

5.1 Design Procedure

The design procedure of IIR filters using multi-swarm PSO is described as follows.

step.1 Set the filter order N and M, the number of division of frequency S, the number of particles P, the number of swarms L, the maximum number of iterations I_{max}, and the maximum number of stagnation judgment α.

step.2 Initialize the position of particle $\boldsymbol{x}$ and the velocity of particle $\boldsymbol{v}$ to random value, then set $t = 1$.

step.3 Divide all particles into L swarms.

step.4 Calculate the objective function value by (3) for each particle $\boldsymbol{x}$.

step.5 Determine the personal best $\boldsymbol{p}_i^t$ and the global best $\boldsymbol{g}^t$ based on the results of step.4.

step.6 If there is a swarm that does not update the global best α times, then go to step.7. Otherwise go to step.9.

step.7 Select global bests of K swarms randomly among L swarms including a stagnated swarm.

step.8 Set the reallocation space $\boldsymbol{\Gamma}$ in the interval of

Table 1. Results of verification ($\times 10^{-2}$)

c_{null}	Best	Average	Deviation
50	1.3550	1.8670	1.8959
60	1.3281	3.0761	1.4225
70	1.7155	6.6133	17.3606
80	1.7681	2.6013	6.1152
90	1.2829	2.3737	1.2844
100	1.6198	3.5371	1.3568

$[\gamma - he, \gamma + he]$, then shift all particles belonging to a stagnated swarm to $\mathbf{\Gamma}$.

step.9 If $t = I_{max}$, finish designing. Otherwise update the position and the velocity, then go to step.4 as $t \leftarrow t + 1$.

6. Design examples

6.1 Parameter verification

Parameter verification was carried out to set the value of c_{null}. Design conditions were follows, $N = 12$, $M = 6$, $\tau_d = 9$, $f_p = 0.1$, $f_s = 0.2$, $f_{null} = 0.25$, $R = 0.9$, $N_p = 90$, $I_{max} = 2.0 \times 10^4$, and the number of trials was 30. c_{null} was verified from 50 to 100. The parameters of PSO was set as follows, $c_1 = 1.0$, $c_2 = 3.0$, $w_{max} = 0.7$, and $w_{min} = 0.3$. w was decreased linearly using (7). Table.1 shows the best error, the average error, and the standard deviation every c_{null}. Fig.3 shows the magnitude response for c_{null}. From Table.1, the best error and the standard deviation of $c_{null} = 90$ is the smallest value. Furthermore, from Fig.3, it can be confirmed that a null is formed at ω_{null}. For these reasons, c_{null} was set to 90.

Our method depends on the width of perturbation h and the maximum number of stagnation judgment α. The parameter verification was carried out using h and α. Design conditions were the same as the previous verification. In this verification, $L = 5$ and $K = 3$. h was tested from 0.001 to 0.05 every 0.005. α was tested from 10 to 100 every 10. The best value of error is showed in Fig.4 every α. From Fig.4, the good design results can be obtained in the little maximum number of stagnation judgment α regardless of the width of perturbation h. Once the particles seem to converge, the better solution can be found by reallocating particles. h is needed to set a small value less than 1.0 to ensure the stability of IIR filters after the particle reallocation. Therefore, we set $\alpha = 10$ and $h = 0.01$.

6.2 Design examples

Three design examples are shown to reveal the effectiveness of the method. $D(\omega)$ was given as

$$D(\omega) = \begin{cases} e^{-j\omega\tau_d} & ,0 \leq \omega \leq 2\pi f_p \\ 0 & ,2\pi f_s \leq \omega \leq \pi \end{cases} . \quad (9)$$

Design conditions are listed in Table 2. For all examples, $S = 100$, $c_s = 100$, and $c_{null} = 90$ for all design examples. The discrete frequency points are sampled at a constant interval in a pass band and a stop band. Normal-PSO was used as the compared method. Initial value was set using a random number. The range of initial value of a_0 was set to $[-0.01, 0.01]$, the real part and the imaginary part of zeros were set to $[-3.0, 3.0]$, and poles were set to $[-R, R]$. The width of the perturbation h was set to 0.01 from the previous section. Our method used 5 swarms in total and 3 swarms were selected randomly to determine the reallocation space every stagnation. The parameter of PSO w was set to 0.4, especially w of the normal-PSO was decreased gradually from 0.7 to 0.3, c_1 was set to 1.0 and c_2 was set to 3.0. The PC having CPU: Intel(R) Core(TM) i3-2130 3.40[GHz].

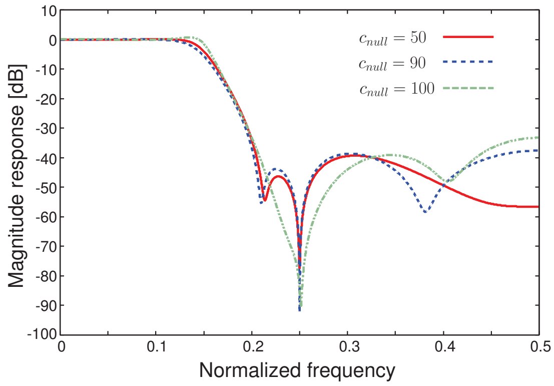

Figure 3. Magnitude response for c_{null}

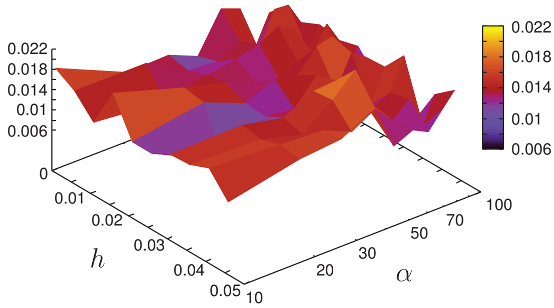

Figure 4. Result of parameter verification

Design results are listed in Table 3. In Table 3, the best design error, the average error, and the standard deviation among 50 trials are shown. Table 3 shows that our method could obtain better design results than normal-PSO in all design examples. Moreover, it can be confirmed that the standard deviation for our method could be improved. It means that our method can enumerate similar local minimums up to the final iteration in each trial and it does not depend on the random initial value. From Fig.5 to Fig.7 show the magnitude response of Ex.1, Ex.2, and Ex.3. From these results, it was shown that the appropriate null could be formed.

7. Conclusion

In this paper, IIR filters having a null frequency in the stopband were designed by PSO with particle reallocation strategy. In the method, particles were reallocated when the local minimum stagnation occurred. Furthermore, the new penalty

Table 2. Design conditions

	Ex.1	Ex.2	Ex.3
Numerator order N	8	12	14
Denominator order M	6	6	10
Desired group delay τ_d	5	9	10
Passband edge frequency f_p	0.175	0.1	0.2
Stopband edge frequency f_s	0.25	0.2	0.25
Null frequency f_{null}	0.35	0.3	0.3
Fixed pole radius R	0.92	0.93	0.94
Number of particles N_p	150	200	250
Number of iterations I_{max}	5000	10000	30000

Table 3. Design results ($\times 10^{-2}$)

		Ex.1	Ex.2	Ex.3
Our method	Best	3.073	0.5906	1.606
	Average	10.35	1.074	2.745
	Deviation	2.900	0.3549	1.125
Normal-PSO	Best	4.968	1.749	4.539
	Average	10.95	3.752	12.75
	Deviation	2.030	1.613	18.05

function was introduced to form the null. Design results showed that our method could achieve the better design even for the constrained filter.

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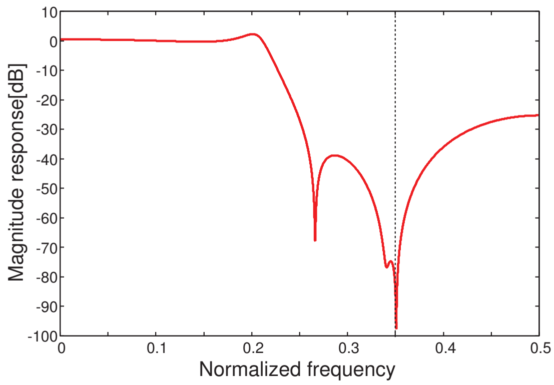

Figure 5. Magnitude response of Ex.1

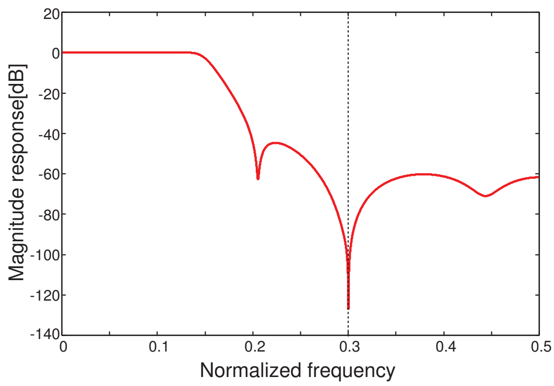

Figure 6. Magnitude response of Ex.2

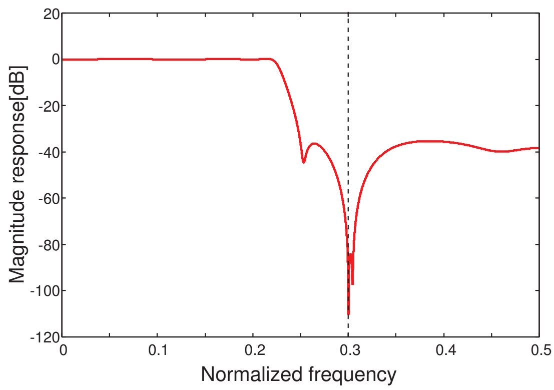

Figure 7. Magnitude response of Ex.3