Design of IIR Filters Using PSO with Improved Intensification and Diversification Ability

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Abstract: In this paper, a design method for IIR (Infinite Impulse Response) filters using PSO (Particle Swarm Optimization) is studied. In our previous work, an avoidance method of the local minimum stagnation has been proposed, in which a penalty is added to an objective function when the stagnation has occurred. Then, a penalty range is determined by using multi-swarms. However, it requires high computational cost. In this study, a novel method which repeats a diversification and an intensification alternatively is proposed for a single-swarm PSO, in which a penalty range is fixed to a small value. Several design examples are shown to present the effectiveness of the proposed method.

1. Introduction

IIR (Infinite Impulse Response) filters are used in many applications, such as a communication, a measurement and a control system. However, a frequency response of IIR filters is a rational function. In addition, it is necessary to ensure the stability. Thus, a design problem falls into a non-linear optimization problem, and it makes the design a tough job [1]-[3].

The design method using PSO (Particle Swarm Optimization) [4] was proposed to solve the design problem [5]-[8]. PSO is one of stochastic multipoint search methods. PSO has a strong directivity toward a local minimum. Consequently, multiple local minimums can be enumerated rapidly by PSO. However, PSO updating tends to stagnate around such a local minimum and thus indicates a premature convergence.

In order to overcome a premature convergence, an avoidance method for the local minimum stagnation has been proposed [9]. In the method, the diversification ability is enhanced while suppressing the intensification ability.

In [10], an avoidance method for the local minimum stagnation has been proposed, in which a penalty function is added to an objective function when the stagnation has occurred. In the method, high intensification and diversification ability are utilized simultaneously. A penalty range C_w is determined by using multi-swarm PSO in [10]. However, high computational cost is required.

In this study, C_w is fixed for the single-swarm PSO. Although a false local minimum may be occurred by fixing C_w, the penalty function is further added at such a point and thus the stagnation can be avoided. As a result, many local solutions are enumerated by the proposed method for repeating the local minimum stagnation and adding the penalty.

Several examples are shown to present the effectiveness of the method.

2. Design problem of IIR filter

The frequency response $H(\omega)$ of IIR filter is described as following,

$$H(\omega) = a_0 \frac{\prod_{n=1}^{N} (1 - z_n e^{-j\omega})}{\prod_{m=1}^{M} (1 - p_m e^{-j\omega})}, \quad (1)$$

where a_0 is a scaling factor, N is a numerator order, M is a denominator order, z_n $(n = 1, 2, \cdots, N)$ are zeros, p_m $(m = 1, 2, \cdots, M)$ are poles, and ω is an angular frequency. In this paper, real coefficient filters are designed, and thus z_n and p_m are limited to complex conjugates or real numbers. In a sense of the Chebyshev approximation criteria, the design problem is described as following,

$$\min_{\boldsymbol{x}} \max_{\omega \in \Omega} W(\omega) |D(\omega) - H(\omega)|, \quad (2)$$

where $\boldsymbol{x} = [a_0, z_1, \cdots, z_N, p_1, \cdots, p_M]^T$ is the design parameter vector, Ω is an approximation frequency band, $W(\omega)$ is a weight function and $D(\omega)$ is a desired frequency response.

3. Particle Swarm Optimization

PSO is one of the optimization methods based on the swarm behavior of animals. PSO is consisted of multiple particles which are defined by a location vector $\boldsymbol{x}$ and a velocity vector $\boldsymbol{v}$. The updating procedure of u-th particle is described as following,

$$\boldsymbol{x}_u^{t+1} = \boldsymbol{x}_u^t + \boldsymbol{v}_u^{t+1} \quad (3)$$

$$\boldsymbol{v}_u^{t+1} = w^t \boldsymbol{v}_u^t + c_1 r_1 (\boldsymbol{p}_u^t - \boldsymbol{x}_u^t) + c_2 r_2 (\boldsymbol{g}^t - \boldsymbol{x}_u^t), \quad (4)$$

where t is the number of iterations, $\boldsymbol{p}_u^t$ is the best location of u-th particle, $\boldsymbol{g}^t$ is the best location among all particle locations up to t-th iteration, r_1 and r_2 are uniform random numbers in the interval of [0, 1], w is an inertia weight parameter, c_1 is a weight parameter toward the $\boldsymbol{p}_u^t$, and c_2 is a weight parameter toward the $\boldsymbol{g}^t$. Such a updating brings PSO the strong directivity toward a local minimum. Therefore, solution candidates can be enumerated rapidly by using PSO. However, due to the strong directivity, PSO tends to stagnate around a local minimum and thus indicates a premature convergence.

Algorithm 1 Avoidance of the stagnation

if The variation of $F(\boldsymbol{x}) \leq 10^{-6}$ in J_s times successively **then**
  Adding a penalty for $F(\boldsymbol{x})$ at $\boldsymbol{g}^t$
  $i \Leftarrow 0$
  while $i \leq A_{\mathrm{p}}$ **do**
    Updating of $\boldsymbol{x}$ and $\boldsymbol{v}$ by equation (3),(4)
    if The variation of $F(\boldsymbol{x}) \leq 10^{-6}$ in J_s times successively **then**
      Adding a penalty
    end if
    $i \Leftarrow i + 1$
  end while
end if

4. Design of IIR Filters Using PSO adding a penalty

For designing IIR filters using PSO, the objective function $F(\boldsymbol{x})$ is described as following,

$$F(\boldsymbol{x}) = \delta + \varphi(p_{\max}) + \psi(\boldsymbol{x}) \quad (5)$$

$$p_{\max} = \max_{m=1,\cdots,M} |p_m|, \quad (6)$$

where δ is a maximum error, $\varphi(p_{\max})$ is the penalty function to ensure the stability of IIR filters, $p_{\max}$ is a maximum pole radius and $\psi(\boldsymbol{x})$ is the penalty function to avoid the local minimum stagnation. The design problem of IIR filters is to determine $\boldsymbol{x}$ so as to minimize $F(\boldsymbol{x})$.

4.1 Penalty function for the stability

The stability of IIR filters is ensured in necessary and sufficient condition when all of poles are within the unit circle on z-plane. However, an excess magnitude ripple appears in the transition band when the pole is close to the unit circle. In this study, a maximum pole radius is limited by following the penalty [10],

$$\varphi(p_{\max}) = \begin{cases} p_{\max}^2, & (p_{\max} > R) \\ 0, & (p_{\max} \leq R), \end{cases} \quad (7)$$

where $R(R < 1.0)$ is a maximum pole radius specified in advance.

4.2 Penalty function for avoiding the local minimum stagnation

When the stagnation has occurred, the penalty is added to the objective function for avoiding the local minimum stagnation. The Gaussian function $\psi(\boldsymbol{x})$ is added at the stagnation point $\boldsymbol{g}$. $\psi(\boldsymbol{x})$ is described as following,

$$\psi(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}C_w} \prod_{l=0}^{N+M} e^{-\left\{\frac{(\boldsymbol{x}_l - \boldsymbol{g})^2}{2C_w^2}\right\}}, \quad (8)$$

where C_w is the penalty range. The algorithm for avoiding the local minimum stagnation is shown in Algorithm 1, where J_s

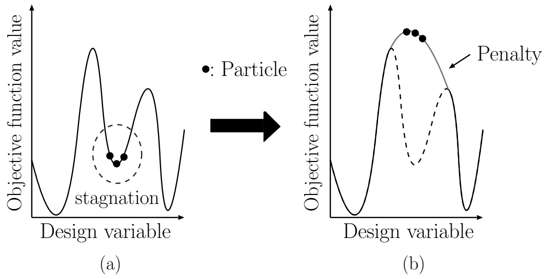

Figure 1. Avoidance of stagnation by adding the penalty

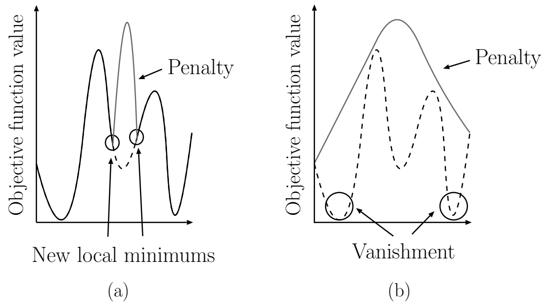

Figure 2. Difficulty for the penalty

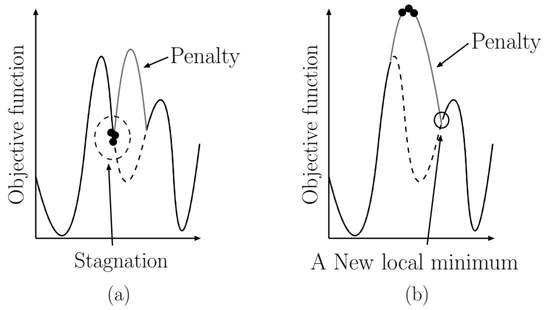

Figure 3. Avoidance of stagnation by the proposed method

is the maximum number of stagnation judgement and A_p is a period that the penalty is added. The avoidance of stagnation by adding the penalty is shown in Figure 1. In Figure 1 (a), the search stagnates by the intensification of particles. Then, the penalty is added to the objective function around the stagnation point to avoid the local minimum stagnation shown in Figure 1 (b). After recording a value of $\boldsymbol{g}^t$, $\boldsymbol{p}_u^t$ and $\boldsymbol{g}^t$ are reset to shift to a new local solution. By adding the penalty, each particle escapes from the stagnation point and shifts to a new local solution. After the search, the best solution is chosen among the recorded $\boldsymbol{g}^t$.

4.3 The difficulty of penalty adding method

The difficulty for the penalty is shown in Figure 2. When C_w is too small, a false local solution is generated shown in Figure 2 (a). When C_w is too large, the penalty is added to a local solution which is different from the stagnation point shown in Figure 2 (b). Thus, a way of setting of C_w becomes an important topics.

4.4 Proposed method

In the proposed method, C_w is fixed to a small value. Therefore, it is not necessary to determine C_w in advance such as the previous method[10]. In addition, just a single-swarm is used because the appropriate decision of C_w is not required. As a result, computational cost can be reduced relative to the previous method. In Figure 3, an example of the avoidance of stagnation by the proposed method is shown. Because C_w is fixed as shown in Figure 3 (a), a false stagnation may occur at the point that is different from a collect local minimum solution. Then, the particles are prompted to shift to a new local minimum solution by adding the penalty further as shown in Figure 3 (b). Thus, a lot of local minimum solutions can be enumerated by repeating intensification caused by the local minimum stagnation and diversification caused by adding the penalty.

5. Design example

In this section, several examples are shown to present the effectiveness of the proposed method. $D(\omega)$ is described as following,

$$D(\omega) = \begin{cases} e^{-j\omega\tau}, & (0 \leq \omega \leq 2\pi f_p) \\ 0, & (2\pi f_s \leq \omega \leq \pi), \end{cases} \tag{9}$$

where τ is a group delay, f_p is a normalized pass band edge frequency, and f_s is a normalized stop band edge frequency. Design conditions are listed in Table 1. In all designs, the design parameters were set to $W(\omega) = 1$, the number of frequency samples was 100 and the number of trials was 50. The proposed method was compared with multi-swarm PSO which is the previous method and the single-swarm IPSO (Independent-minded PSO) [11]. The number of swarms were set to 3 and 5. The PC having CPU: Intel(R) Core(TM) i3-2130 3.40[GHz], memory: 4[GByte] was used in all design examples.

PSO parameters are listed in Table 2, where P is the number of particles, and $I_{\max}$ is the maximum number of iterations. For all design methods, the weight parameters were set to $w = 0.6$, $c_1 = 1.0$ and $c_2 = 2.4$. In the proposed method, C_w was set to 0.1. The initial value of a_0 was set randomly to [-0.5,0.5], the modulus and the angle of zeros were set randomly to [0,1.5] and $[-\pi,\pi]$, and the modulus and the angle of poles were set randomly to [0,R] and $[-\pi,\pi]$.

Design results are shown in Table 3, the updating curve of Ex.3 for the proposed method is shown in Figure 4, the updating curve of Ex.3 for the previous method ($G = 5$) is shown in Figure 5, the magnitude response of Ex.1 is shown in Figure 6 and the passband magnitude response of Ex.1 is shown in Figure 7. In Table 3, δ_{best} is the best value of design error, δ_{mean} is the average design error, σ is the standard deviation and h is the computational time per a trial.

In Figure 4 and Figure 5, it was shown that the proposed method can enumerate more local minimum solutions than the previous method. Thus, it is expected that the proposed method carried out the global search than the previos method. In Table 3, it was confirmed that the proposed method could achieve better accuracy and more high-speed design than the compared methods. In Figure 6 and Figure 7, it was also confirmed that the proposed method could decrease design error on the magnitude response more than the previous method.

Table 1. Design conditions

	Ex.1	Ex.2	Ex.3
N	6	8	12
M	4	6	6
τ	4	6	9
f_p	0.20	0.20	0.20
f_s	0.30	0.27	0.25
R	0.88	0.90	0.93

Table 2. Parameters of PSO

	Ex.1	Ex.2	Ex.3
P	80	90	120
$I_{\max}(\times 10^4)$	1	2	2

Table 3. Design results

			Ex.1	Ex.2	Ex.3
δ_{best} $(\times 10^{-2})$	Prop. method		2.650	2.126	2.036
	Prev. method	$G = 3$	3.047	2.314	2.072
		$G = 5$	3.039	2.154	2.083
	IPSO		3.141	2.510	2.507
δ_{mean} $(\times 10^{-2})$	Prop. method		3.128	3.000	2.977
	Prev. method	$G = 3$	3.335	3.109	3.096
		$G = 5$	3.205	3.006	2.944
	IPSO		3.504	4.170	7.379
σ $(\times 10^{-2})$	Prop. method		0.479	0.530	0.470
	Prev. method	$G = 3$	0.549	0.549	0.491
		$G = 5$	0.498	0.605	0.452
	IPSO		0.340	1.048	4.191
h[s]	Prop. method		23.72	51.04	76.29
	Prev. method	$G = 3$	41.64	91.88	128.8
		$G = 5$	55.48	132.4	184.7
	IPSO		24.99	72.08	97.24

6. Conclusion

In this paper, the penalty function was added to the objective function when the stagnation has occurred to avoid the local minimum stagnation. Then, C_w was fixed to the small value. From several examples, it was shown that the proposed method could achieve better accuracy and more high-speed design than the compared methods.

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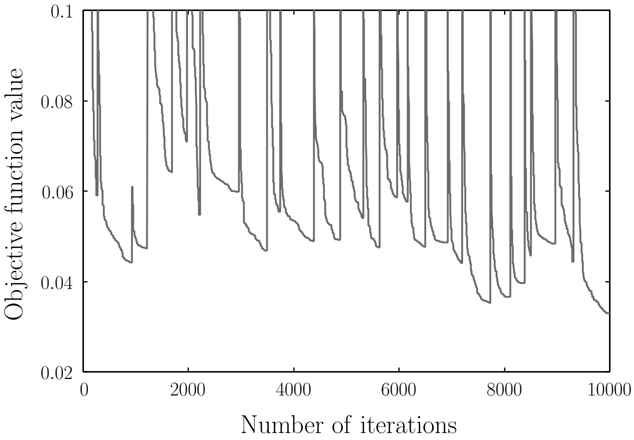

Figure 4. Updating curve of Ex.3 for the proposed method

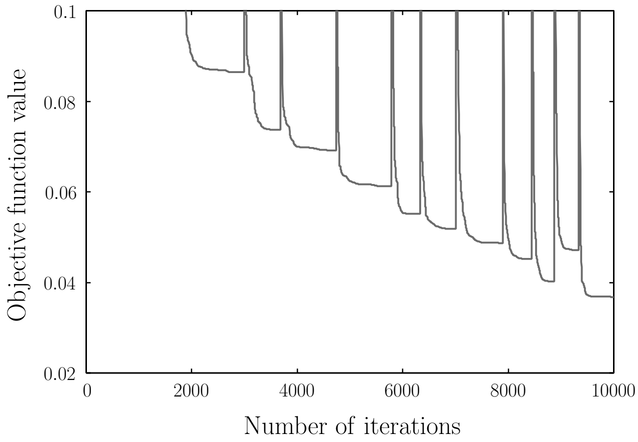

Figure 5. Updating curve of Ex.3 for the previous method(G=5)

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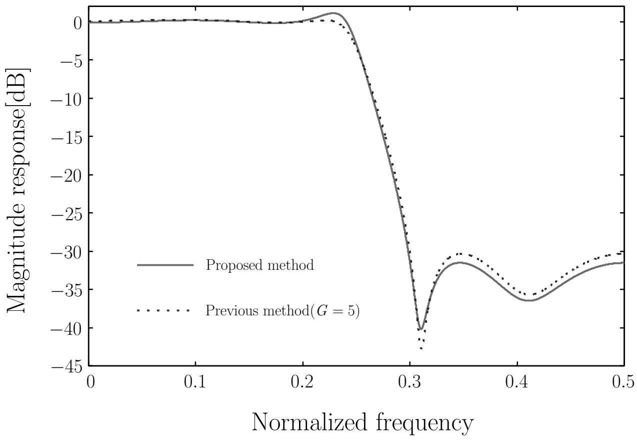

Figure 6. Magnitude response of Ex.1

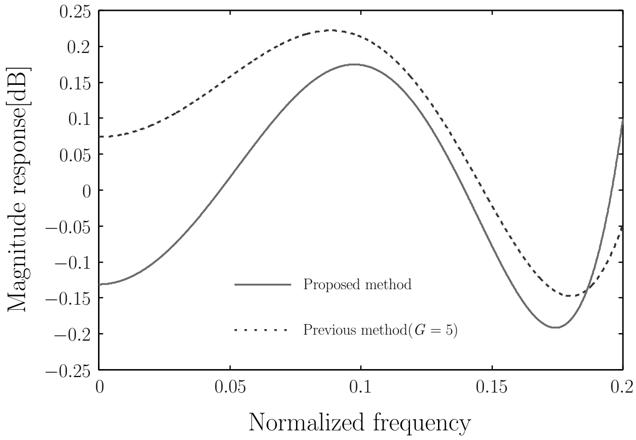

Figure 7. Passband magnitude response of Ex.1