

Design of Equalizer based on Bernstein Polynomials under Echo Pairs

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Abstract: This paper presents design of equalizer based under echo pairs and amplitude distortion. The equalizer was used with the fourth order bernstein polynomials. The advantage of this design is to get smooth magnitude response and zero phase response. It was found possible to eliminate echo pairs and amplitude distortions. The modulated sine-squared pulse test signal was used for test the performance of proposed equalizer. This research attempted to implement the equalizer with the MATLAB software. The simulation and experimental results are in good agreement. It is likely that application of equalizer to broadcast television systems.

Keywords-- Equalizer, Bernstein, Echo Pairs

1. Introduction

The broadcast TV signal to which the receiver synchronizes its operations is called the principal signal, and the principal signal is usually the direct signal received over the shortest transmission path. Thus, the multipath signals received over other paths are usually delayed with respect to the principal signal and appear as lagging echoes signals. It is possible however, that the direct or shortest path signal is not the signal to which the receiver synchronizes. When the receiver synchronizes its operations to a (longer path) signal that is delayed respective to the direct signal, there will be a leading multipath signal caused by the direct signal, or there will a plurality of leading multipath signals caused by the direct signal and other reflected signals of lesser delay than the reflected signal to which the receiver synchronizes. In the broadcast TV art multipath signals are referred to as “echoes”. The echoes cannot be completely eliminated. It can be reduced to a greater extent by signal processing. To carry out signal processing and to make corrections, there is a pre-requisite for echo cancellation. Many researchers have proposed the method of paired echoes to study the effects of incremental amplitude distortion [1-2]. The effect of amplitude distortion, separate from phase distortion, is relatively easy to determine first by the Fourier integral.

This paper introduces an echo suppression technique from effect of amplitude distortion. The echo suppression technique is designed by equalizer based on bernstrin polynomial [3-4]. Its amplitude response is smooth and it has zero phase response. This research attempted to implement the equalizer with the MATLAB software.

2. Echo Pairs and Amplitude Distortion

This paper has a substantial contribution to the correction of the echo distortion in the broadcast color TV transmission system. The pulse test signal is the modulated sine-squared

pulse. A system with constant group delay, but with controlled amplitude distortion, can be used of symmetrical “positive” pair of echoes, i.e. the product of echo amplitude of each pair being positive, so that both are positive. The gain/frequency characteristic produced by adding the echoes to the central pulse involves a sinusoidal term, and can be defined by

$$x(t) = I(t)[1 + \cos(2\pi f_c t)] \quad (1)$$

where f_c is center frequency of sub-carrier.

$$I(t) = \frac{2}{\pi} \int_0^{\omega_s} \frac{1}{\beta} \cdot [0.3 \sin(5\beta T) + 0.4 \sin(10\beta T) + 0.3 \sin(15\beta T)] \cdot [1 - \frac{\beta}{\omega_s}] \cdot \cos(\beta t) d\beta \quad (2)$$

then

$$I(t) = I_1(t) + I_2(t) + I_3(t)$$

We can solve for $I_1(t)$, $I_2(t)$ and $I_3(t)$ from

$$I_1(t) = \frac{0.3}{2\pi} \{S_i[(5T + t)\omega_s] + S_i[(5T - t)\omega_s]\} + \frac{0.3}{4\pi} \left\{ S_i \left[\left(5T + t + \frac{2}{\tau} \right) \omega_s \right] + S_i \left[\left(5T + t - \frac{2}{\tau} \right) \omega_s \right] \right\} + \frac{0.3}{4\pi} \left\{ S_i \left[\left(5T + t + \frac{2}{\tau} \right) \omega_s \right] + S_i \left[\left(5T + t - \frac{2}{\tau} \right) \omega_s \right] \right\}$$

$$I_2(t) = \frac{0.4}{2\pi} \{S_i[(10T + t)\omega_s] + S_i[(10T - t)\omega_s]\} + \frac{0.4}{4\pi} \left\{ S_i \left[\left(10T + t + \frac{2}{\tau} \right) \omega_s \right] + S_i \left[\left(10T + t - \frac{2}{\tau} \right) \omega_s \right] \right\} + \frac{0.4}{4\pi} \left\{ S_i \left[\left(10T - t + \frac{2}{\tau} \right) \omega_s \right] + S_i \left[\left(10T - t - \frac{2}{\tau} \right) \omega_s \right] \right\}$$

$$I_3(t) = \frac{0.3}{2\pi} \{S_i[(15T + t)\omega_s] + S_i[(15T - t)\omega_s]\} + \frac{0.3}{4\pi} \left\{ S_i \left[\left(15T + t + \frac{2}{\tau} \right) \omega_s \right] + S_i \left[\left(15T + t - \frac{2}{\tau} \right) \omega_s \right] \right\} + \frac{0.3}{4\pi} \left\{ S_i \left[\left(15T - t + \frac{2}{\tau} \right) \omega_s \right] + S_i \left[\left(15T - t - \frac{2}{\tau} \right) \omega_s \right] \right\}$$

$$\text{given } S_i(u) = \int \frac{\sin(u)}{u} du, \tau = 1 \text{ MHz}$$

where T is the time interval between the echo and the main pulse. β is the frequency measured from the center frequency (f_c). ω_s is the bandwidth of main pulse, whereas the corresponding delay distortion is zero. The echo spacing is chosen so that $1/2T$ corresponds to the upper limit of the video frequency spectrum [5].

A MATLAB program for obtaining the simulated results of echo patterns is shown in MATLAB Script.

```

“
clear all; close all;
clc; clf;
% Parameter setting
Fc = 4.43*10^6;
Wc = 2*pi*Fc;
T = 0.1*10^-6;
t = -5*10^-6:0.01*10^-6:5*10^-6;
Ws = 2*pi*1*10^6;
Town = 1*10^6;
% First Term
F1 = 0.3/(2*pi).*(sinint((5*T-t).*Ws)
    +(sinint((5*T+t).*Ws)));
F2 = 0.4/(2*pi).*(sinint((10*T-t).*Ws)
    +(sinint((10*T+t).*Ws)));
F3 = 0.3/(2*pi).*(sinint((15*T-t).*Ws)
    +(sinint((15*T+t).*Ws)));
% Second Term
S1 = 0.3/(4*pi).*(sinint((5*T-t-2/Town).*Ws)...
    + sinint((5*T-t+2/Town).*Ws)...
    + (sinint((5*T+t-2/Town).*Ws)...
    + (sinint((5*T+t+2/Town).*Ws)));
S2 = 0.4/(4*pi).*(sinint((10*T-t-2/Town).*Ws)...
    + sinint((10*T-t+2/Town).*Ws)...
    + (sinint((10*T+t-2/Town).*Ws)...
    + (sinint((10*T+t+2/Town).*Ws)));
S3 = 0.3/(4*pi).*(sinint((15*T-t-2/Town).*Ws)...
    + sinint((15*T-t+2/Town).*Ws)...
    + (sinint((15*T+t-2/Town).*Ws)...
    + (sinint((15*T+t+2/Town).*Ws)));
R = (F1+F2+F3) + (S1+S2+S3);
plot(t,R); grid on;
C = R.cos(2*pi*Fc*t);
Plot(t,C);
A=C+R;
plot(t,A);
p = 0*10^-6:0.01*10^-6:10*10^-6;
N = length(R);
ws = 2pi/N;
wnorm = -pi:ws:pi;
wnorm = wnorm(1:length(R));
w = wnorm*ws;
X=fft(R);
plot(w,abs(fftshift(X)));
N = length(C);
ws = 2pi/N;
wnorm = -pi:ws:pi;
wnorm = wnorm(1:length(C));
w = wnorm*ws;
X1=fft(C);
Plot(w,abs(fftshift(X1)));
N = length(A);
ws = 2pi/N;
wnorm = -pi:ws:pi;
wnorm = wnorm(1:length(A));
w = wnorm*ws;
X2=fft(A);
plot(w,abs(fftshift(X2)));
”

```

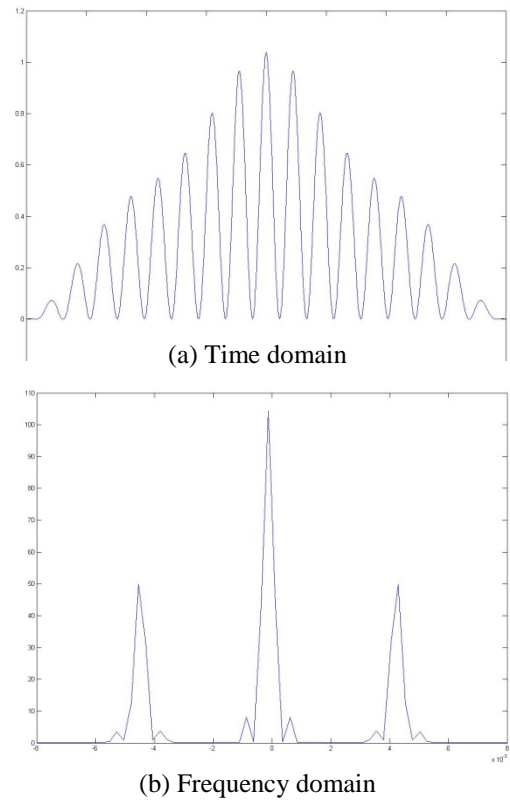


Figure 1. Simulation results of echo patterns.

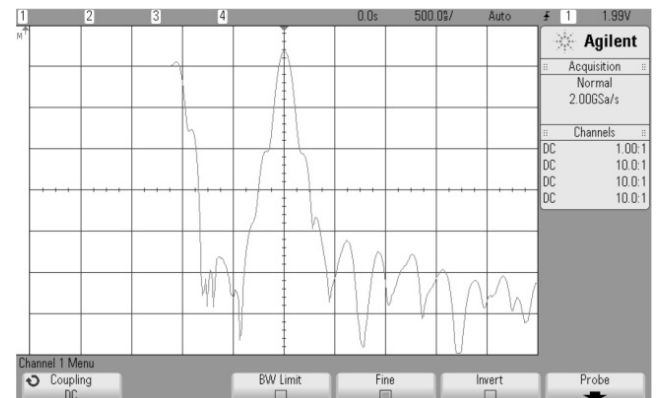
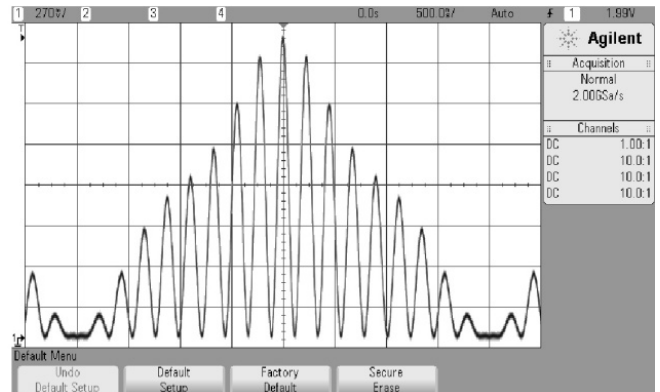


Figure 2. Experimental results of echo patterns.

Figure 1-2 show simulation and experimental results of the echo patterns in time and frequency domain. In other words, it follows that the amplitude frequency characteristic of a system plays no part in determining the symmetry of the pulse. This pulse will be symmetrical about a delayed central time axis, for a symmetrical pulse, if there is no delay distortion. In short, then, “positive” pairs of symmetrical echoes can be used to determine or to vary the amplitude of time and frequency characteristic without affecting the phase.

3. Equalizer Based on Bernstein Polynomials

The n^{th} ($n \geq 1$) Bernstein polynomials is given by [6-10]

$$B_n(f; x) = \sum_{i=0}^n f\left(\frac{i}{n}\right) \binom{n}{i} x^i (1-x)^{n-i} \quad (3.1)$$

Where $f(x)$ is defined in the interval $[0,1]$,

For $i = 0, 1, \dots, n$ where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$

Considering the approximation of a low pass function as shown in Figure 3, we get

$$f\left(\frac{i}{n}\right) = \begin{cases} 1, & 0 \leq i \leq n-K \\ 0, & n-K+1 \leq i \leq n \end{cases} \quad (3.2)$$

Where, K is the number of successive discrete points at the zero values function.

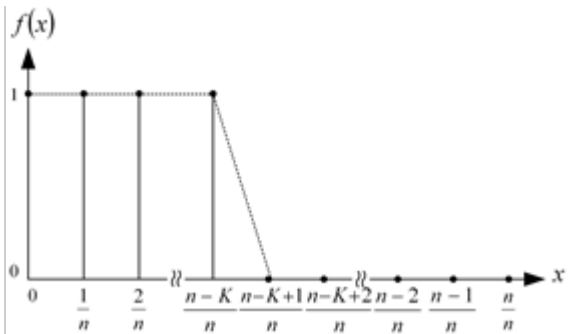


Figure 3. Low-pass function.

Instead of equation (3.2) into equation (3.1) can be.

$$B_{n,K}(f; x) = \sum_{i=0}^{n-K} \binom{n}{i} x^i (1-x)^{n-i} \quad (3.3)$$

Let us consider the interval of x which is defined in the interval $[0,1]$. It must be changed to the interval $[0,\infty]$ for Ω by using the transformation as follows

$$x = \frac{\Omega^2}{1 + \Omega^2} \quad (3.4)$$

Substitution of Eq. (3.4) in to Eq. (3.3) yields

$$B_{n,K}(f; \Omega) = \frac{\sum_{i=0}^{n-K} \binom{n}{i} \Omega^{2i}}{(1 + \Omega^2)^n} \quad (3.5)$$

Using derivation of Herrmann’s polynomials, So Eq. (3.3) can rewrite as

$$B_{n,K}(f; x) = (1-x)^K \sum_{i=0}^{n-K} \binom{n}{i} x^i (1-x)^{n-i} (1-x)^{-K} \quad (3.6)$$

Thus, The equalizer was used with the fourth order bernstein polynomials given by

$$E(\omega\tau) \triangleq \frac{A}{2} [1 + \cos(\omega\tau)] = A \cos^2\left(\frac{\omega\tau}{2}\right) \quad (3.7)$$

Setting

$$y^2 = \alpha^2 \sin^2\left(\frac{\omega\tau}{2}\right) \quad (3.8)$$

Thus

$$\left[1 - \frac{y^2}{\alpha^2}\right] = \cos^2\left(\frac{\omega\tau}{2}\right) \quad (3.9)$$

Then

$$B_4(f; x^2) = \sum_{i=0}^4 \left(1 - \frac{i^2}{n^2 \alpha^2}\right)^{-1} \binom{4}{i} x^{2i} (1-x^2)^{4-i} \quad (3.10)$$

where $\alpha = 1.1$

The transfer function of equalizer base on bernstein polynomials is defined by

$$T(x^2) = \frac{1}{2.22x^8 + 0.94x^6 + 0.88x^4 + 0.21x^2 + 1} \quad (4)$$

substitution $x^2 = \alpha^2 \left(\frac{\Omega^2}{1 + \Omega^2}\right)$ in to Eq.4., we obtain the new equation as

$$T(\Omega^2) = \frac{\Omega^8 + 4\Omega^6 + 6\Omega^4 + 4\Omega^2 + 1}{9.55\Omega^8 + 9.25\Omega^6 + 8.13\Omega^4 + 4.26\Omega^2 + 1} \quad (5)$$

where $\Omega = j\omega$

Figure 4 shows characteristic of equalizer based on bernstein polynomials by using the MATLAB simulation. The advantage of this design is to get smooth magnitude response and zero phase response. Therefore, the proposed equalizer in this paper selected the bernstrin polynomials for proving the efficiency in equalizing the amplitude distortion.

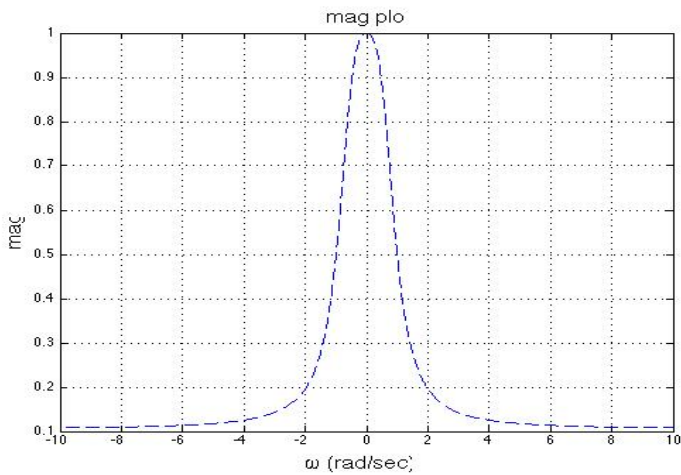
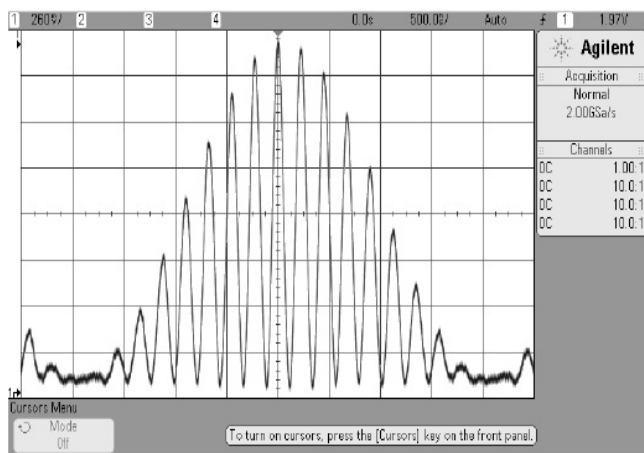
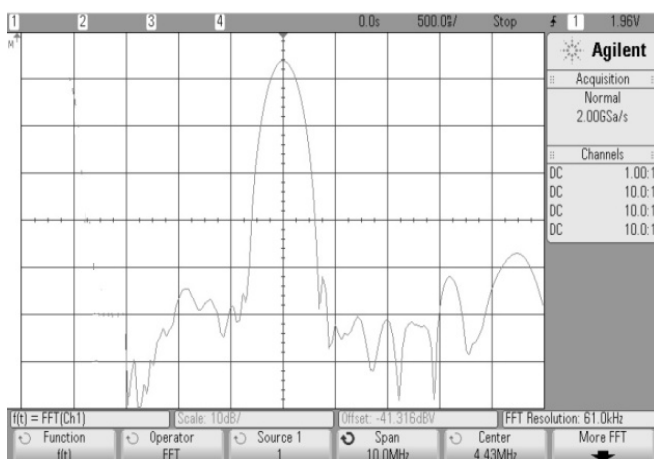


Figure 4. magnitude response of equalizer base on bernstein polynomials.



(a) Time domain



(b) frequency domain

Figure 5. experimental results of echo patterns.

Figure 5 demonstrates the TV signal from equalizer. It shows experimental results without echo pair and amplitude distortion.

4. Conclusions

This paper introduces an echo paired suppression technique from effect of amplitude distortion. The echo suppression technique is designed by equalizer based on Bernstein polynomial. It has a substantial contribution to the correction of the echo distortion in the broadcast color TV transmission system. The advantage of this design is to get smooth magnitude response. Therefore, the proposed equalizer was selected for proving the efficiency in equalizing the amplitude distortion. The output signal from equalizer shows experimental results without echo pair and amplitude distortion.

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