An Efficient Amplitude Fluctuation Model for the Wave Propagation in Solar Corona Based on Rytov's First Iteration Method

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***Abstract*—In this paper, a new numerical model based on Rytov's first iteration method is implemented to analyze the amplitude fluctuation when the Electromagnetic (EM) waves pass through the solar corona during the superior solar conjunction. Integrated with the geometrical model of the deep space communication, the proposed amplitude fluctuation model incorporates both the solar wind density model and its irregularities spectrum model. In addition, the scintillation index model is further derived from the proposed model, which is normally used to characterize the amplitude fluctuation. Simulation results indicate that the proposed model can quantitatively evaluate the amplitude fluctuation under various scenarios. Besides, the derived scintillation index model achieves better accuracy compared with other models and the measurement data collected by the deep space probe. The good performance in terms of both efficiency and accuracy makes the proposed model is possible applied in forecast the amplitude fluctuation in the deep space communication.**

Keywords—EM wave propagation; solar corona; solar wind density; amplitude fluctuation; scintillation index

I. INTRODUCTION

With the development of state-of-the-art interstellar exploration, deep space communication, which is the only way that enables the data to be commuted between the Earth and the probe, has attracted intensive research attention over the past decades [1-4]. Apart from the long distance path loss, the radio link will be affect by the complex Interplanetary Space (IPS) media, such as the solar wind plasma that emits from the Sun [3]. When the EM wave encounters the solar wind scintillation, its amplitude, phase and angle-of-arrival will be fluctuated. This is more severely during the superior solar conjunction (the probe moves in the other side of the Sun relative to the Earth) since the wave will propagate through the solar corona with high density. In order to enhance the communication performance, some strategies (such as, increasing the EM wave frequency, antenna aperture, or even using the relay communication and optical communication technologies [5-7]) have been proposed. However, an available and precise prediction of the amplitude fluctuation is critical for the link margin design when we minimize solar wind scintillation impact [2].

Since the solar wind density irregularities play a key role in amplitude fluctuation, it is necessary to utilize the comprehensive density model and irregularities model. Based on long time observation, some solar wind density models have been proposed [4]. Nevertheless, the exactly three dimensions evolution of the solar wind density can hardly be obtained due to the spatial and temporal characteristics of the solar wind. In [8], an approximated calculation approach was proposed. In general, it can be availably used in improving the prediction of amplitude fluctuation. However, the complicated numerical integration leads to a lack of flexibility. Even though a flexible model was proposed in [9], its accuracy is difficult to be guaranteed. All of the mentioned drawbacks make these models immature for the truly practical applications. The scintillation index derived from amplitude fluctuation is also used to investigate the EM wave propagating through the solar corona. A simplified scintillation index model was given in [1], which only considers the effect of heliocentric distance. A modified model was proposed in [3]. The author in [10] used the Fresnel propagation filter to describe the scintillation index model, and further demonstrated the scintillation index has the relation with solar wind irregularities [11]. All of these models are roughly in calculation, however, that need further study.

Motivated by the above description, the solar wind fluctuation along the full EM link and the outer scale of the turbulence were taken into consideration to derive an efficient amplitude fluctuation model. The rest of this paper is organized as follows. Section II outlines the background model. In section III, we solve the EM wave equation with Rytov's first iteration method. Based on some assumptions, the new analytical formulations of amplitude fluctuation and scintillation index model are derived. The performance of the proposed model is evaluated and show better accuracy in section IV. Section V draws the conclusion.

II. BACKGROUND MODEL

A. Geometrical Model for the Deep Space Communication

Figure 1 shows the geometry of the radio wave passing through the solar corona between the Earth and the probe during the superior solar conjunction. The solar radius is R_s. The distance of each side of triangle between the Sun, Earth, and probe is defined as L, L_{se} and L_{sp}. r is specified as the heliocentric distance. We further define the SEP (Sun-Earth-Probe) angle and SPE (Sun-Probe-Earth) angle as α and β, respectively. Hence, the distance of communication link can be explicitly written as

$$L=L_{se}\cos[\arcsin(r/L_{se})]+L_{sp}\cos[\arcsin(r/L_{sp})] \quad (1)$$

B. Solar Wind Density Model

Since the solar corona is filled with the charged particles that streamed from the Sun, it is necessary to know the plasma density distribution. As the velocity of the solar wind is much slower than the EM wave propagation speed, the solar wind density is normally assumed not change remarkably in a short time, which is also called as Taylor's frozen theory. In this paper, we used a widely accepted Guhathakurta's empirical formula to describe the electron density $N_e(r,\theta)$ [12].

$$N_e(r,\theta)=N_p(r)+[N_{cs}(r)-N_p(r)]\cdot\exp[-\theta^2/w^2(r)]\times 10^6 \quad (2)$$

where, $N_p(r)$ and $N_{cs}(r)$ correspond to the electron density at the poles and the current sheet, respectively. θ means the angular distance from the current sheet and the half-angular width of the current sheet is denoted by $w(r)$.

C. Solar Wind Irregularities and Its Spectrum Model

In this study, we introduce the solar wind density relative fluctuation factor $\mu=\delta N_e/N_e$. Considering the effect of outer scales l_o and inter scales l_i, many space explorations have measured the solar wind irregularities spectrum as [9], $\Phi_N(\kappa)=C_n^2\kappa^{-p}$, $\kappa_o\leq\kappa\leq\kappa_i$, where, κ is the wavenumber, $\kappa_o=2\pi/l_o$ and $\kappa_i=2\pi/l_i$. C_n^2 is the function characterizing the strength of solar wind fluctuation. With a generalized Kolmogorov spectrum model (p=11/3) [9], the spatial spectrum of the solar wind irregularities can be simplified as

$$\Phi_N(\kappa)=\frac{2\Gamma(11/6)\kappa_o^{2/3}}{3(2\pi)^{3/2}\Gamma(4/3)}\left\langle\delta N_e^2\right\rangle\kappa^{-11/3} \quad (3)$$

where, $\Gamma(\,)$ is the gamma function, δN_e^2 denotes the variance of the solar wind density fluctuation.

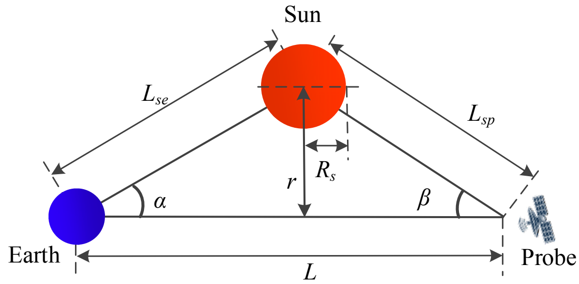

Figure 1. Geometric diagram of the deep space communication during the superior solar conjunction

III. AMPLITUDE FLUCTUATION MODEL

According to the description above, when the EM waves propagate in the variable solar corona media, the amplitude will be severely distorted and even result in losing the signal at the receiver. In this paper, we assume that the diffract angle is very small, thus, the downlink is close to the line-of-sight propagation. The EM waves propagate through the solar corona media can be determined by the following scalar wave equation

$$\nabla^2 E+k^2\left[1+\delta\varepsilon(s,t)E\right]=0 \quad (4)$$

where, $k=2\pi/\lambda$ is the wavenumber, λ is the wavelength. The dielectric constant of the media is $\delta\varepsilon(s,t)$. For the sake of simplicity, we omit the time t in the following expressions. With the assumption of weak scattering, the wave equation can be solved by the Rytov's first iteration approximation.

$$E(S)=E_0(S)\exp\left(-k^2\iiint d^3r\,\delta\varepsilon(s)A(s,s')\right) \quad (5)$$

where, E_0 is the un-turbulent electric field strength in position s. $A(s,s')$ is constituted by Green function and the ratio of electric field $E_0(s)/E_0(s')$. Since the log-amplitude fluctuation $\chi=\ln(E/E_0)$ is normally described as the amplitude fluctuation, we can use its variance to describe the variability of EM wave amplitude fluctuation,

$$\begin{aligned}\left\langle\chi^2\right\rangle=&\frac{k^4}{16\pi^2}\iiint d^3s\iiint d^3s'\cos\left(ks^2/2z\right)\\&\cdot\cos\left(ks'^2/2z'\right)\left\langle\delta\varepsilon(s)\delta\varepsilon(s')\right\rangle\end{aligned} \quad (6)$$

where, <> means the ensemble average. z denotes distance in line-of-sight direction. Since the dielectric constant fluctuation has the relationship with the fluctuation of solar wind, $\delta\varepsilon=r_e\lambda^2\delta N_e/\pi$, we obtain the amplitude fluctuation variance as,

$$\left\langle\chi^2\right\rangle=\frac{2^{1/3}\pi^{10/3}\Gamma(11/6)\left\langle\delta N_e^2\right\rangle r_e^2\lambda^{17/6}}{12\Gamma(4/3)\Gamma(17/6)\cos(\pi/12)}L^{\frac{11}{6}}l_o^{-\frac{2}{3}} \quad (7)$$

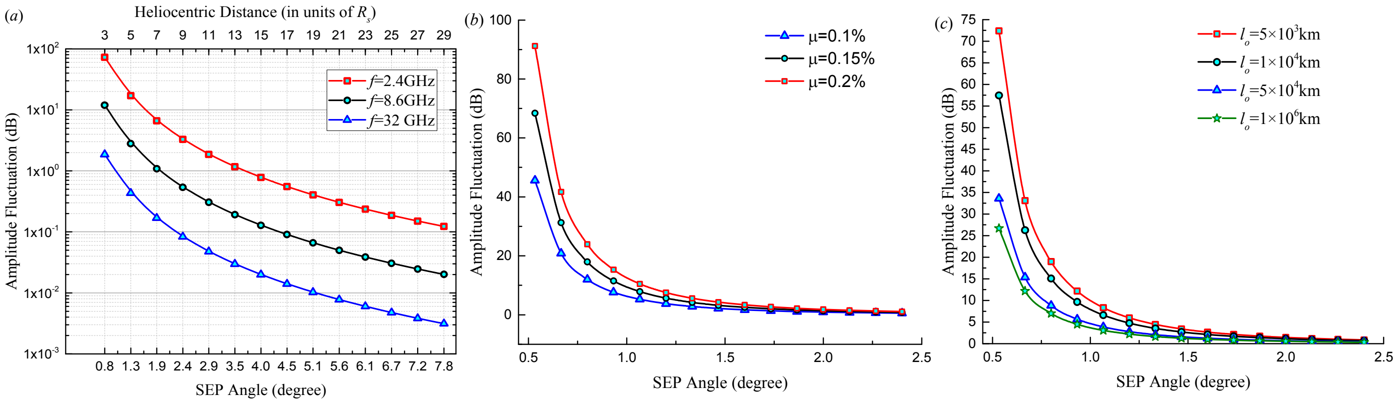

Figure 2. Results of amplitude fluctuation with different SEP angle under (a) various frequencies, (b) various solar wind density relative fluctuation factors, (c) various outer scales.

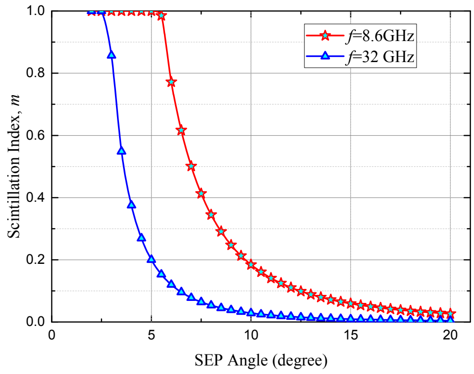

Figure 3. The variation of the scintillation index.

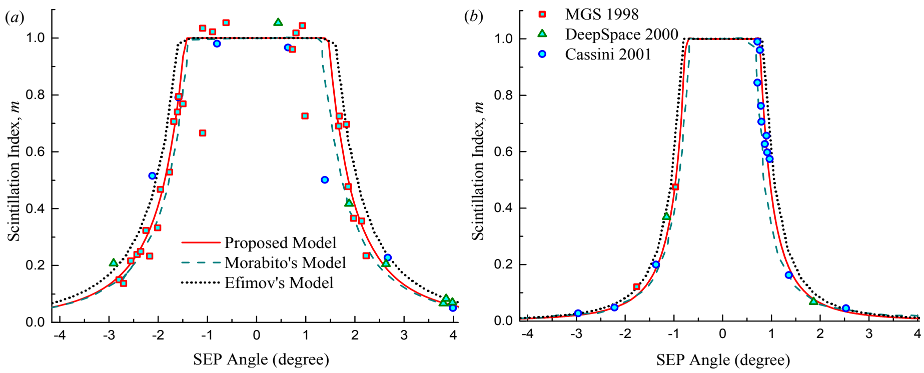

Figure 4. The performance of the models. (*a*) *f*=8.4GHz (*b*) *f*=32GHz.

Apart from the variance of the amplitude fluctuation, the scintillation index is also used to characterize the effect of solar wind on EM wave propagation. It is usually defined as the intensity standard deviation normalized to the average received intensity. Since the scintillation index has a relation with the amplitude fluctuation, $S_4^2 = 4\langle\chi^2\rangle$, we have

$$S_4 = \sqrt{\frac{2^{7/3}\pi^{10/3}\Gamma(11/6)\langle\delta N_e^2\rangle r_e^2\lambda^{17/6}}{12\Gamma(4/3)\Gamma(17/6)\cos(\pi/12)} L^{\frac{11}{6}} l_o^{-\frac{2}{3}}} \tag{8}$$

IV. Numerical Results And Discussions

In this section, the impact of solar wind scintillation on EM wave amplitude fluctuation has been analyzed via simulation. Besides, its performance is also compared with some existing solutions.

A. Simulation Settings

The simulation is built under the assumption that L_{sp}=1.5AU (AU=1.5×10^{11}m), which mimics the case of Mars ellipse orbit during the superior solar conjunction. The heliographic latitude and longitude ϕ is 0° and 50°, respectively. The title angle of the dipole axis with respect to rotation axis is 15°, the angle between the heliomagnetic and heliographic equators γ_0 is 0°. In order to evaluate the amplitude fluctuation with different parameters, the following parameters have been considered. The EM wave frequencies f are [2.4, 8.4, 32] GHz, the turbulence outer scale l_o are [1×10^6, 5×10^6, 1×10^7, 5×10^7] m and the solar wind density relative fluctuation factor μ are [0.1%, 0.15%, 0.2%]. These parameters are compromised with the result assumed by [12].

B. Amplitude Fluctuation Dependencies on Parameters

In this part, we focus on the performance of the amplitude fluctuation under different parameters, such as frequencies, relative fluctuation factors and various turbulent eddy outer scales. The results are demonstrated in Figure 2. As we can see in Figure 2 (a), the amplitude fluctuation decreases when the SEP angle and the heliocentric distance (in the bottom and top horizontal axes, respectively) increase. This is reasonable since the solar wind density will be diluted in the radial direction which induces the amplitude fluctuation decreases simultaneously. Besides, the amplitude fluctuation suffers by an average of 73% less amplitude fluctuation than *f*=8.4GHz and even 91% less than *f*=2.4GHz, as the statistical results shown in (a). Similar results can also be obtained from (b) and (c), when the solar wind density relative fluctuation factors decrease from 0.2% to 0.1% and outer scale of the turbulent eddies increase from $1\times10^3\ km$ to $5\times10^4\ km$.

C. Scintillation Index Performance

Figure 3 illustrates the scintillation index by varying the SEP angle under various EM wave frequencies. Intuitively, the scintillation index will be reduced at high frequency. The EM signal f=32GHz reached strong scintillation (S_4>0.3) at $4R_s$ and saturated at $3R_s$, which is smaller than f=8.4GHz. The effectiveness of the proposed scintillation index model is also compared with other models and the measurement data that were collected by the various probes. As we can see in Figure 4(a), the proposed model has an average of 18.2% deviation, while the deviation of Morabito's model and Efimov's model are 28.5% and 50.3%, compared with the data collected by with the data collected by the MGS 1998, Deep Space 2000, and Cassini 2001. The similar results are also demonstrated in Figure 4(b). Note that, the positive SEP angle means the egress to superior solar conjunction and negative for ingress. Since the effect of the outer scale and the solar wind fluctuation along the EM wave link have been taken into consideration, the proposed model achieves better performance than the other models, which means that it can be effectively used in predicting the scintillation index in the future deep space exploration.

V. CONCLUSIONS

Since the amplitude fluctuation caused by the solar wind plasma gives rise to severe effect on the radio signal, a precise amplitude fluctuation model is urgent needed in the deep space communication, especially during the superior solar conjunction. With the introduction of the geometric model of the deep space communication, solar wind density and its irregularities model, an efficient amplitude fluctuation model and its derived scintillation model are proposed by taking advantage of the Rytov's first iteration method. The amplitude fluctuation under various parameters is further analyzed by the derived model. The extensive simulation results demonstrated that the proposed model can be effectively used to predict the amplitude fluctuation and scintillation index with high accuracy.

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