Tail Biting Low Density Parity Check Convolutional Codes over Rain Fade channel

A.Wongsriwor¹, V.Imtawil¹, and P.Suthisopapan^{2*}

¹ Department of Electrical Engineering, Khon Kaen University ²Department of Electronics and Telecommunication Engineering, Faculty of Engineering, Rajamangala University of Technology Isan Khon Kaen

E-mail: arisakoe@gmail.com, virasit@kku.ac.th, mr.puripong@gmail.com

Abstract: Rain fade is one of important effects that can not be avoided in Ka-band satellite communications. In order to reduce the effect of rain fade, Tail-Biting low density parity check convolutional codes (TB codes) that have high performance and low complexity is considered. Simulation results show that TB codes with small size of memory can achieve significant coding gain over worst case for rain fade condition. Moreover, the comparison between high performance LDPC and TB codes is also presented. It is found that TB codes with memory of 159 can beat LDPC codes at BER 10⁻⁵.

Keywords-- Ka-band, TB-LDPCCCs, satellite communications

1. Introduction

Ka-band satellite communications is the one type of Satellite communications, that has high frequency range (27GHz-40GHz). Its wavelength is close to the raindrop diameter. As a result, it can easily be disrupted by rainfall. The channel environment of satellite communications is supposed to be an Additive White Gaussian Noise (AWGN) Channel when no rain [1]. However, in the rain fall environment, it can be modeled by rain fade channel [2,3]. Performance of a communication system over a rain fade channel is typically lower than AWGN case. LDPC codes is currently employed over rain fade channel to improve the performance of system [2,3].

LDPC codes were proposed in 1960's by R. G. Gallager and re-discovered in 1996's by D. MacKay and R. Neal. It is well known in literature that LDPC codes can achieve near Shannon limit performance [4]. In 1999's, LDPC convolutional codes (LDPCCCs) were first introduced by A. Jimenez [5]. These codes can transmit data in stream of arbitrary length. LDPCCCs have lower complexity when comparing with LDPC block codes. However, it need *zeros tail* to flush memories, leading to code rate loss. Later, a modified version of LDPCCCs is proposed, called TB, to overcome rate loss problem [6]. In this paper, TB codes is investigated over rain fade environment. In addition, TB codes is compared with a good LDPC codes of girth 6.

The paper is organized as follows. Section 2 introduces the model of rain fade channel. Section 3 describes the structure of LDPC codes and TB codes. The simulation results are shown in section 4. Finally, the conclusion is presented in section 5.

2. System model

Let x(t) be the transmitted signal of the rain fade channel. The received signal is denoted by r(t) and can be expressed as:

$$r(t) = x(t)A\exp(j\varphi) + n(t), \tag{1}$$



Fig. 1 Model of Rain fade channel.

where n(t) is the additive white Gauss noise. The model of Rain fade channel[1] is shown in Fig.1. A and φ represent the envelope and phase, respectively, that are real random variable with probability density function [2,7] as:

$$p(A) = \frac{1}{\sqrt{2\pi\delta'}} \exp\left\{-\frac{(a-m')^2}{2{\delta'}^2}\right\}$$
(2)

$$p(\varphi) = \frac{1}{\sqrt{2\pi\delta''}} \exp\left\{-\frac{(\varphi - m'')^2}{2{\delta''}^2}\right\}$$
(3)

The parameters m' and m'' are mean values of envelope and phase respectively, δ' and δ'' are variance of them. These mean and variances depend on the weather in three conditions; AWGN, light rain and heavy rain, as shown in Table 1.

Table 1 Parameters of rain fade channel [3,6].

weather	m'	δ'	<i>m</i> ″	δ''			
Ideal (AWGN)	1	0	0	0			
Light rain	0.483	0.00003	0.0088	0.00546			
Heavy rain	0.436	0.01386	0.0068	0.00414			

3. Tail-biting LDPC Convolutional codes

The vector $\mathbf{m} = (m_1, m_2, ..., m_K)$ denotes the binary data source that is encoded into $\{c_1, c_2, ..., c_N\}$ codeword by TB encoder of code rate b/c. TB can be generated from (n,k)LDPC codes [8], that has memory (m_s) k-1. TB codes has lower complexity than LDPC codes since the encoder circuits of TB can be easily derived from shift registers [9]. The structure of Parity Check matrix of TB codes is shown in Fig.1.

$$H_{[0,N-1]}^{T} = \begin{bmatrix} H_{0}^{T}(0) & H_{1}^{T}(1) & \cdots & H_{m_{1}}^{T}(m_{s}) & 0 & \cdots & 0 \\ 0 & H_{0}^{T}(1) & \cdots & H_{m_{l}-1}^{T}(m_{s}) & H_{m_{l}}^{T}(m_{s}+1) & 0 & \cdots & 0 \\ \cdot & \ddots & & \ddots & \ddots & \\ H_{m_{l}}^{T}(N) & 0 & \cdots & 0 & H_{0}^{T}(N-m_{s}) & \cdots & H_{m_{l}-1}^{T}(N-1) \\ H_{m_{l}-1}^{T}(N) & H_{m_{l}}^{T}(N+1) & 0 & & \vdots \\ \vdots & & 0 & H_{0}^{T}(N-2) & H_{1}^{T}(N-1) \\ H_{1}^{T}(N) & H_{2}^{T}(N+1) & \cdots & H_{m_{l}}^{T}(N+m_{s}-1) & 0 & \cdots & 0 & H_{0}^{T}(N-1) \end{bmatrix}$$

Fig.1 Structure of Parity Check matrix of TB codes.

The parity check matrix of TB codes can be derived from parity check matrix of LDPC codes that can be easily explained by example.

Example Derive TB codes with rate = $\frac{1}{2}$, b=1, c=2, N = 20, and $m_s = 4$ from (10, 5)LDPC code.



Fig.2 Cutting pattern of TB code.

Let A is a parity check matrix of LDPC code. Firstly, cut A into A_0 and A_1 by repeatedly move c = 2 units to the right and c-b = 1 units down that is shown in Fig.2.



Fig. 3 Composition of B from A₀ and A₁.

Then, compose matrix B by paste A_0 and A_1 as shown in Fig.3. Then cut k-1 = 4 last row of B. Finally, paste it to the right corner as displayed in Fig. 4.

1	1	0	0	0	0	0	0	0	0										
1	0	0	0	0	0	0	0	0	0										
0	0	1	1	1	0	0	0	0	0										
0	0	0	0	0	0	1	1	0	0										
0	1	1	0	1	1	0	1	0	0										
0	0	0	1	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	1	0	0
					_					0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0

Fig. 4 Cutting of *k*-1 last row of B.

	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1
0	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	1	0	0

Fig.5 Parity check matrix of TB code.

4. Simulation results

All simulations are performed using Matlab software. The (5040,2520) LDPC codes with girth 6 and TB codes with $m_s = 59$ and 159 (deriving from (120,60) and (320,160) LDPC codes respectively) are selected.

The performance of TB codes with maximum iteration of TB decoder is 5,15 and 25 are compared to find the suitable iteration for primary study. Figure 6 shows the performance of TB codes with maximum iteration of 5,15 and 25. It is seen that the performance of TB codes with maximum iteration of 15 and 25 have a similar performance that is better than decoding iteration = 5 about 1 dB at BER= 10^{-5} . From this result the maximum iteration of 15 is selected for all simulations.

Figure 7 shows the performances of uncode and TB codes with minimum m_s = 59 that can be derived for rate 1/2. It can be seen that the performance of TB codes can obtain about 8 dB over uncode at BER= 10⁻⁵, proves that the TB codes is an efficient code to resist rain fade.

Figure 8 shows the performances of TB codes with $m_s = 59$, 74, 119 and 159. As can be seen, TB codes with high m_s have higer performance than low m_s such as $m_s = 159$ is better than $m_s = 59$ about 1 dB at 10⁻⁵. Which shows that the performance of TB codes depend on the size of memory.

In order to confirm that the TB codes have a high performance. The comparison of the performance between TB codes and good LDPC are conducted. Figure 9 shows the performances of LDPC with block length of 1200 and TB codes with m_s =119 and 159. It can be seen that the performance of the LDPC codes similar to the TB code for

 $m_s = 119$. However, when increasing the memory m_s to 159, the performance of TB codes is superior to LDPC.

From all results, show that TB codes is a good way to resist rain fade that have a high performance and low complexity.



Fig.6 Performance of TB codes with maximum iteration=5,15 and 25.



Fig. 7 Performance of TB codes with m_s =59 and uncode.



Fig.8 Performance of TB codes with m_s =59, 74, 119 and 159.



Fig.9 Performance of LDPC codes and TB-LDPCCCs.

5. Conclusion

In this paper, the applications of TB codes over Ka-Band satellite communications are investigated. As discussed above, the results prove that coding gain can be enhanced by adopting TB codes in rain fade environment. TB codes provide lower complexity, compared with LDPC block codes, while maintaining good bit error rate performance. The performance of TB codes can be further improved by increasing the memory order m_s .

Acknowledgements

This work is financially supported by Faculty of Engineering, Rajamangala University of Technology Isan, Khonkaen Campus and Department of Electrical Engineering, Faculty of Engineering, KhonKaen University.

References

[1] C.I. Oh, S.H. Choi, D.I. Chang and D.G. Oh, "Analysis of the Rain fading channel and the system applying MIMO", *IEEE International Symposium on Communications and Information Technologies*, September 2006.

[2] X. Da, Y. Wang and T. Xie ,"Performance of LDPC Codes for Satellite Communication in Ka Band",*IEEE*

International Conference on Wireless Communications, Networking and Mobile Computing, September 2009.

[3] F.X. Zhang and S.W. Xu, "Performance simulation of using non-binary LDPC codes to against rain fade in Kaband", *IEEE International Conference on Information and Automation*. August 2015.

[4] D.J.C. MacKay and R.M. Neal, "Near Shannon limit performance of low density parity check codes", *Electronics Letters*, vol.33, pp. 457 - 458, March 1997.

[5] A. Jimenez and K.S. Zigangirov, "Periodic time-varying convolutional codes with low-density parity-check matrices", *IEEE International Symposium on Information Theory*, August 1998.

[6] M. B. S. Tavares, K.S. Zigangirov and G. P. Fettweis, "Tail-Biting LDPC Convolutional Codes Based on

Protographs", *IEEE 66th Vehicular Technology Conference*, September 2007.

[7] Chun Loo, "Impairment of digital transmission through a Ka band satellite channel due to weather Conditions", *International Journal of Satellite Communications*, May 1998.

[8] A. E. Pusane, R. Smarandache, P.O Vontobel, D.J.Costello, "Deriving Good LDPC Convolutional Codes from LDPC Block Codes", *IEEE International Symposium on Information Theory*, June 2007.

[9] M. B. S. Tavares, " On Low-Density Parity-Check Convolutional Codes: Constructions, Analysis and VLSI Implementation", 2010.