Plasmonic Ship-Wake on Graphene

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***Abstract*—Kelvin predicted that the semi-angle of the V-shaped wedge behind a ship moving in deep water region is $19.5°$, independent of the ship's velocity. The predication has been challenged recently that the semi-angle would transit from the Kelvin angle to the Mach angle as the ship's velocity increases. In this paper, we show the graphene plasmons excited by a swift charged particle would have the similar phenomenon. When the velocity of the charged particle is relatively slow, the graphene plasmons excited would accumulate along the caustic boundary of the graphene plasmons pattern, forming the plasmonic Kelvin angle. At large velocity, however, no graphene plasmons would accumulate along any boundary, thereby the caustics disappear and the effective semi-angle of the graphene plasmons approaches the Mach angle.**

Keywords—graphene plasmons; Kelvin angle;

I. INTRODUCTION

Lord Kelvin predicted that the wave pattern generated behind a boat on the water's surface would form a V-shape with a fixed semi-angle of 19.5° that is independent of the ship's velocity [1, 2]. However, the argument was recently challenged that the semi-angle of the wave pattern would transit from the Kelvin angle to the Mach angle as the ship's velocity increases [3, 4]. This phenomenon is not unique in deep water waves, and we find its counterpart in graphene plasmons, one kind of two-dimensional (2D) plasmons. The fact that 2D plasmons exhibit a dispersion similar to that of deep-water waves in the long wavelength limit has already been known for more than fifty years [5]. This implies that many deep-water-wave phenomena can find counterparts in graphene plasmons.

The caustic wave theory is adopted to study the wave focus effect along the boundary. Caustics are a boundary, across which there is a jump-wise variation of the number of rays reaching each point [6]. We find that at a relatively small velocity ($\sim 0.1c$ or smaller, where c is the velocity of light in vacuum) of the swift charged particle, the stimulated graphene plasmons are confined within the caustic boundaries with a semi-angle of 19.5°, i.e., the Kelvin angle. Each point within the caustic boundaries is covered twice by rays, whereas outside there is no ray. As the velocity of the charged particle increases, the graphene-plasmonic rays reaching the caustic boundaries become weak, which blurs and eventually eliminates the caustics. The calculation shows the effective semi-angle of the plasmonic ship-wake approaches the Mach angle, being similar to the recent studies of ship waves in fluid mechanics.

II. THEORETICAL MODEL

The calculation model is shown in Fig. 1, where a particle with charge q moves along $\hat{z}$ direction with a uniform velocity v parallel to an isolated graphene sheet at $y = d$. The current density that this charged particle produces is

$$\mathbf{J}(\mathbf{r},t) = \hat{z}qv\delta(x)\delta(y)\delta(z-vt) \tag{1}$$

The evanescent fields from the swift charged particle can excite graphene plasmons on graphene. In the calculation we set $d = 1\mu\text{m}$. The isolated graphene is assumed to have chemical potential $\mu_c = 0.15\text{eV}$, and scattering rate $\Gamma = 0.11\text{meV}$ at the room temperature $T = 300\text{K}$ [7]. The frequency dependent complex conductivity $\sigma(\omega)$ of the isolated graphene is computed from Kubo formula [7, 8]. With the prescribed parameters, the intraband conductivity dominates.

We first make Fourier transform of (1) to get the current density at each frequency as

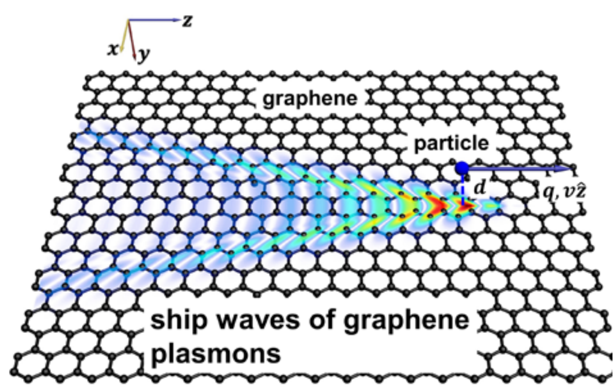

Fig.1 The plasmonic ship-wake on graphene excited by a charged particle moving along the z-direction with a constant velocity v.The distance between the graphene and the particle is d.

$$\mathbf{J}(\mathbf{r},\omega)=\hat{z}\frac{q}{4\pi^2\rho}e^{i\omega z/v}\delta(\rho), \quad (2)$$

where $\rho=\sqrt{x^2+y^2}$. Each frequency component of the graphene plasmons can be exactly derived by taking the residue of Sommerfeld pole [9, 10]. The vertical component of electrical field E_y is used to represent the transverse-magnetic (TM) graphene plasmons

$$E_y(\omega)=\frac{-iq\varepsilon_0\omega^2}{\pi v\sigma^2}\frac{e^{ik_y d+ik_x x+ik_z z}}{k_x}, \quad (3)$$

where $k_y=-2\omega\varepsilon_0/\sigma(\omega)$ indicates the confinement of graphene plasmons, $k_z=\omega/v$ and $k_x=\sqrt{\omega^2\varepsilon_0\mu_0-k_y^2-k_z^2}$ are the wave vectors of the graphene plasmons, and ε_0 and μ_0 are the constitutive parameters of vacuum. The field distribution at time t is the Fourier integral of

$$\begin{aligned}E_y(y=d,t)&=\int\frac{-iq\varepsilon_0\omega^2}{\pi v\sigma^2}\frac{e^{ik_y d}e^{ik_x x+ik_z z-i\omega t}}{k_x}d\omega\\&=\int e_y(\omega)e^{ik_y d}e^{i\psi(\omega)t}d\omega\end{aligned}, \quad (4)$$

where $\psi(\omega)=k_x\frac{x}{t}+k_z\frac{z}{t}-\omega$, and $e_y(\omega)=\frac{-iq\varepsilon_0\omega^2}{\pi v\sigma^2}\frac{1}{k_x}$.

We numerically carry out the integration in (4) to get the wave patterns. At first, we set the velocity of the particle to be $v=0.1c$. The results are shown in Fig. 2a. The top part is the absolute value of the electrical field $|E_y(r,t)|$ and the bottom part is the absolute value of the total electrical field $|E_{tot}(r,t)|$. The arrow indicates the position of the charged particle. The semi-angle of 19.5°, i.e., the Kelvin angle, is clearly seen. A plane-like wave is inside the wave pattern.

We can get an intuitive picture by combining the ray theory and Kelvin's model for Kelvin wedge [1, 2]. As shown in Fig. 2b. When the particle moves from point A to point B with velocity v. The ship waves of graphene plasmons excited at point A will propagate in all directions with different frequencies. In the propagation direction of angle θ(as indicated in Fig. 2b), where $\cos\theta=k_z/\sqrt{k_x^2+k_z^2}$, the phase velocity v_p of graphene plasmons must satisfy the stationary condition $v_p=v\cos\theta(\omega)$ [1, 2]. When the particle arrives at point B, the dashed blue circle represents the loci of all the arrived phases of graphene plasmon waves. However, the group velocity v_g is only about half of the phase velocity v_p, thus the loci of the energy of arrived waves form the solid blue circle with the diameter only half of the blue dashed one. The waves propagating in the direction $\theta=35°$ form the Kelvin caustic boundary with angle $\alpha=\sin^{-1}(1/3)=19.5°$, as shown by the solid black lines in Figs. 2b and c.

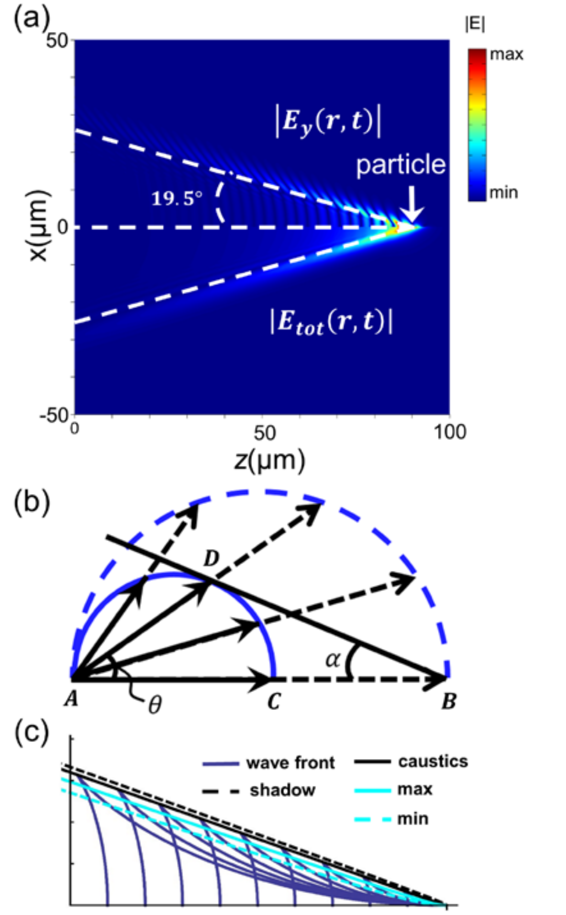

Fig. 2. The plasmonic ship-wake excited by a swift charged particle with velocity $v=0.1c$. (a) The absolute value of electric field $|E_y(r,t)|$ (top) and the absolute value of the total electrical field $|E_{tot}(r,t)|$ (bottom) of the wave patterns. (b) The mathematical model to determine the Kelvin angle. (c) The field distribution along in the neighborhood of caustic.

The integral in (4) can be evaluated asymptotically with the stationary phase methods [2]. The stationary value $\omega=\omega_s$ is calculated with $\frac{d\psi(\omega)}{d\omega}=\frac{dk_x}{d\omega}\frac{x}{t}+\frac{dk_z}{d\omega}\frac{z}{t}-1=0$ .The integral in (4) can be approximated as

$$E_y(t)=2\,\mathrm{Re}\left[e_y(\omega_s)\frac{\sqrt{2\pi}}{\sqrt{\psi''(\omega_s)t}}e^{ik_y(\omega_s)d+i\psi(\omega_s)t+i\pi/4}\right]. \quad (5)$$

Equation (5) diverges when $\psi''(\omega)=0$. In this case the path of steepest descent has to be chosen differently. It can be shown that for the frequency component $\omega=\omega_c$, where

$$\frac{d^2\psi(\omega)}{d\omega^2}=\frac{d^2k_x}{d\omega^2}=0, \quad (6)$$

the rays run together near such points, and the loci of such points form caustics or a caustic boundary, which separates a region without rays from another region covered twice by rays. The black solid line in Fig. 2 c shows the caustic boundary. When $v = 0.1c$, the exact angle of the caustics is $\alpha = 19.5°$.

With the caustic wave theory, the asymptotic form of $E_y(y,t)$ can be calculated with Airy integral,

$$\frac{E_y(t)}{4\pi} = \mathrm{Re}\left\{ e_y(\omega_c)\left[\frac{t}{2}\psi'''(\omega_c)\right]^{-\frac{1}{3}} e^{ik_y(\omega_c)d + i\psi(\omega_c)t}\mathrm{Ai}(\mathrm{X})\right\}, \quad (7)$$

where Ai(X) is the Airy integral function and

$$\mathrm{X} = \frac{t\psi'(\omega_c)}{\left(t\psi'''(\omega_c)/2\right)^{1/3}}. \quad (8)$$

When $\mathrm{X} = 0$, (8) represents the caustic boundary. When $\mathrm{X} > 0$, (8) represents the caustic shadow region, where, because of the Airy function, the field decays exponentially. When $\mathrm{X} = 0.66$, Airy function reaches $1/e$ of the its maximum value. We set $\mathrm{X} = 0.66$ to represent the boundary of the caustic shadow, as shown by the dashed black line in Fig. 2c. The Airy function reaches maximum at $\mathrm{X} = -1.02$. The region between $\mathrm{X} = 0$ and $\mathrm{X} = -1.02$ corresponds to the caustic zone. The field in the caustic zone is relatively strong in the neighborhood of caustics as the wave-field focusing effect on caustics [6]. The field focusing in the caustic zone can be observed in Fig. 2a near the caustic boundary. The Airy function vanishes at $\mathrm{X} = -2.34$. It means the field distribution has a minimum value, as shown by the dashed cyan line in Fig. 2c.

The wave pattern of graphene plasmons excited by the swift charged particle with velocities $0.5c$ and $0.7c$ are shown in Figs. 3a and b, respectively. The outmost perceivable fields at large velocities of the particle are contributed by waves with large propagation angle θ, corresponding to group velocity v_g'. It can be shown that as the particle's velocity v increases, v_g' tends to be independent of v. Therefore, $\alpha \approx v_g'/\mathrm{v} \propto 1/v$ and the effective semi-angle of the wave pattern behaves like Mack angle. In Fig. 2a, we find that in the horizontal line $z = 0$, the absolute of the total electrical field at the caustic point is 0.6 relative to its maximum value along the line. We use this as a

criterion to determine the angle of the ship waves at different velocities of the charged particle, and the results are shown in Fig. 3c. We plot the Mach angle line $\alpha = 4.5/v$ with the coefficient 4.5 adopted from the wave pattern when the particle's velocity is $0.9c$. The results show clearly the transition from the Kelvin angle of 19.5° at small velocities of

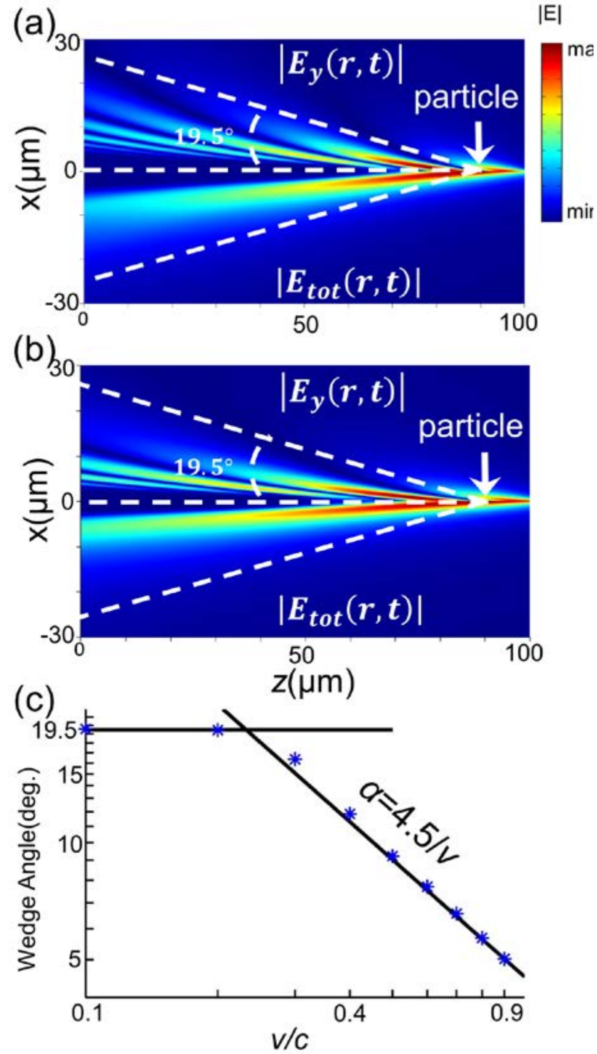

Fig. 3 The plasmonic ship-wake excited by a swift charged particle with velocity $v = (a)\ 0.5c$ and $(b)\ 0.7c$ .The absolute value of electric field $|E_y(r,t)|$ (top) and the absolute value of the total electrical field $|E_{tot}(r,t)|$ (bottom) of the wave patterns. (c) The semi-angle change from Kelvin angle to Mach angle as the particle's velocity increases.

the charged particle to the Mach angle at large velocities, being similar to the recent studies in fluid mechanics [3, 4].

III. CONCLUSION

In conclusion, we incorporate the recent development in fluid mechanics and the caustic wave theory to study of graphene plasmon excitation, and reveal a novel wave phenomenon of graphene plasmons. We find that graphene plasmons excited by a swift charged particle moving above graphene can form a caustic wave pattern with the semi-angle equal to the Kelvin angle, when the velocity of the charged particle is slow. At large velocities, the effective semi-angle of graphene plasmons approaches the Mach angle.

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