Differential Encoding of 16APSK for BICM-ID

Ruey-Yi Wei, Wen-Yao Hsu
Department of Communication Engineering
National Central University
Taoyuan, Taiwan, R.O.C.
rywei@ncu.edu.tw

James A. Ritcey
Department of Electrical Engineering
University of Washington
Seattle, Washington 98195-2500
{jar7}@uw.edu

***Abstract*—Bit-interleaved coded modulation with iterative decoding (BICM-ID) are suitable for continuous fading channels. In addition, BICM-ID using differential encoding can avoid the rate loss due to pilot symbols. Conventional differential encoding for uncoded modulation was used for BICM-ID. In this paper, we propose new differential encoding of 16APSK (amplitude and phase-shift keying) signals for BICM-ID. We first derive the probability of receiving signals conditioned on the transmission of input bits of general differential encoding. After that, we propose a new algorithm to optimize differential encoding for BICM-ID, and use it to find the differential encoding of 16APSK. Besides, code searches for the proposed differential encoding are performed. Simulation results show that the proposed differential encoding has better error performance than conventional differential encoding, and the searched new codes can further improve error performance.**

***Index Terms*—Differential encoding, noncoherent detection, bit interleaved-coded modulation, APSK, BICM-ID, fading channels**

I. Introduction

The common channel model used for wireless communications is continuous fading channels, and it is generally recognized that bit-interleaved coded modulation (BICM) is the most suitable channel code for such channels [1],[2]. In [3]-[5], BICM with iterative decoding (BICM-ID) which has more coding gain was proposed. However, coherent BICM-ID need pilot symbols for channel estimation. If channels vary fast, the pilot symbols should be transmitted frequently so that the rate loss is large. BICM-ID using differential encoding, called differential BICM-ID hereafter, can avoid the rate loss due to pilot symbols. For high transmission rates, differential BICM-ID using 16-ary amplitude and phase-shift keying (16DAPSK, or called 16-level star QAM) signals was proposed in [6]. However, differential encoding of 16APSK in [6] is used for uncoded modulation originally, so perhaps it is not the best differential encoding for BICM-ID.

In [7] and [8], we proposed a new differential encoding technique called differential encoding by a look-up table, which can be optimized in terms of minimum noncoherent distance. In this paper, we design differential encoding of 16APSK for BICM-ID. The proposed differential encoding is realized by a table also, but is optimized by intra-set distances of set partitioning. To generate soft-output at the noncoherent demodulator, we derive the probability of receiving signals conditioned on the transmission of input bits of general differential encoding. Moreover, convolutional codes

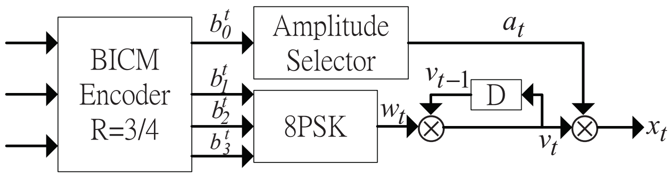

Fig. 1. Block diagrams of the transmitter in [6] (compared to Figure 4).

for the proposed differential encoding are searched. Minimum distance and simulation results both show that the proposed differential encoding and the searched codes outperform those in [6].

II. Review

We first briefly review BICM-ID using 16-DAPSK in [6] whose block diagrams are shown in Fig. 1. Figure 2 illustrates the signal constellation of 16APSK, in which the radiuses of the inner ring and the outer ring are denoted by r_0 and r_1, respectively. The ring ratio r_1/r_0 is 2 in the paper which is the same as [6]. At the transmitter, three data bits are fed into a rate-3/4 BICM encoder. After that, four output bits of the BICM encoder at time t, namely $B_t = b_3^t b_2^t b_1^t b_0^t$, are sent into a 16-DAPSK modulator. Three bits $b_3^t b_2^t b_1^t$ are used for 8DAPSK and the remaining one bit b_0^t is used for 2DASK. In other words, the t-th output symbol of the 16-DAPSK modulator is

$$x_t = a_t v_t \tag{1}$$

where $a_t \in \{r_0, r_1\}$ and v_t is the t-th 8DPSK symbol expressed as $v_t = v_{t-1} w_t$ in which w_t is the t-th 8PSK symbol. The symbol w_t is determined by $b_3^t b_2^t b_1^t$ as $w_t = \mu(b_3^t b_2^t b_1^t)$ where $\mu(.)$ is the 8PSK mapping function, and the amplitude a_t is jointly decided by a_{t-1} and b_0^t. If $b_0^t = 0$, $a_t = a_{t-1}$, while if $b_0^t = 1$, a_t will be switched to the other value.

At the receiver, the t-th received symbol is

$$y_t = h_t x_t + n_t \tag{2}$$

where h_t is an unknown channel coefficient and n_t is zero-mean complex AWGN (additive white Gaussian noise) with a variance of $N_0/2$ per dimension. With the assumption of $h_t = h_{t-1}$, (2) can be rewritten as

$$y_t = y_{t-1} w_t \left(\frac{a_t}{a_{t-1}}\right) + n'_t \tag{3}$$

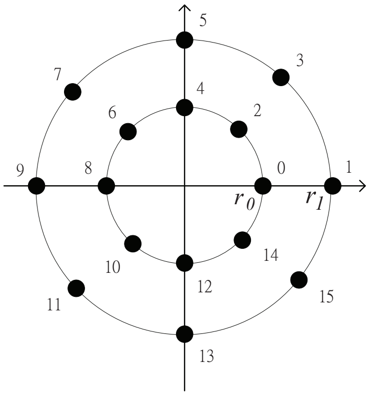

Fig. 2. The signal constellation of 16APSK.

where $n'_t = n_t - n_{t-1}w_t(\frac{a_t}{a_{t-1}})$. The noise variance of n'_t is

$$N'_0 = N_0(1 + (\frac{a_t}{a_{t-1}})^2) \tag{4}$$

where $N'_0 = 2N_0$ if $b^t_0 = 0$, and $N'_0 = N_0(1 + R^2_1)$ or $N'_0 = N_0(1 + R^2_2)$ if $b^t_0 = 1$, in which $R_1 = \frac{r^2_1}{r^2_0}$ and $R_2 = \frac{r^2_0}{r^2_1}$. Based on (3), the probability of receiving y_t conditioned on the transmission of B_t is

$$P(y_t|b^t_3 b^t_2 b^t_1, b^t_0 = 0) = \frac{1}{2\pi N_0} e^{\frac{-|y_t - y_{t-1}\mu(b^t_3 b^t_2 b^t_1)|^2}{2N_0}} \tag{5}$$

and

$$P(y_t|b^t_3 b^t_2 b^t_1, b^t_0 = 1) = \frac{1}{\pi N_0(1 + R^2_1)} e^{\frac{-|y_t - y_{t-1}R_1\mu(b^t_3 b^t_2 b^t_1)|^2}{N_0(1+R^2_1)}} + \frac{1}{\pi N_0(1 + R^2_2)} e^{\frac{-|y_t - y_{t-1}R_2\mu(b^t_3 b^t_2 b^t_1)|^2}{N_0(1+R^2_2)}}. \tag{6}$$

With (5) and (6), soft output using log-MAP or max-log-MAP can be generated. Simulation results in [6] showed that the bit error rate (BER) is the best when $\mu(.)$ is the set partitioning labeling of 8PSK in [3].

Then differential encoding in [7] and [8] are introduced as follows. Differential encoding is defined as a function which maps B_t to $x_t = E(x_{t-1}, B_t)$ such that for $B_t \neq B'_t$, $x_t = E(x_{t-1}, B_t) \neq x'_t = E(x_{t-1}, B'_t)$. Conventionally, $x_t = E(x_{t-1}, B_t)$ is represented by a formula or a rule. For DPSK, $x_t = E(x_{t-1}, B_t) = x_{t-1}w_t$ where $w_t = \mu(B_t)$. In [7], we proposed that $x_t = E(x_{t-1}, B_t)$ can be realized by a look-up table. For differential QAM (quadrature amplitude modulation), it was shown that $x_t = E(x_{t-1}, B_t)$ realized by an optimized table outperforms other differential encoding using rules.

At time t, a differential detector at the receiver determines B_t based on two consecutive symbols r_{t-1} and r_t. Therefore, (x_{t-1}, x_t) forms a super-symbol of interest. The noncoherent distance between two super-symbols $\mathbf{c} = (x_{t-1}, x_t)$ and $\mathbf{c}' = (x'_{t-1}, x'_t)$ is denoted by $d_{nc}(\mathbf{c}, \mathbf{c}')$, and the minimum noncoherent distance of a differential encoder is defined by

$$d_{\min} = \min_{B_t \neq B'_t} d_{nc}((x_{t-1}, x_t), (x'_{t-1}, x'_t)) \tag{7}$$

where $x_t = E(x_{t-1}, B_t)$ and $x'_t = E(x'_{t-1}, B'_t)$. It is necessary that a differential encoder has $d_{\min} > 0$.

Let M denote the number of signal points in the used constellation, and all (x_{t-1}, x_t) codewords which correspond to the same B_t form a group. Consequently, there are totally M groups, denoted by G_0, G_1, $\cdots$, G_{M-1}, and each group contains M codewords. If two codewords $\mathbf{c}$ and $\mathbf{c}'$ have small $d_{nc}(\mathbf{c}, \mathbf{c}')$, one should put them into the same group if possible; Otherwise, they should be put into two different groups which differ smallest number of bits in B_t. The algorithm to optimize differential encoding is listed as follows.

- Step 1 Sort all possible codeword pairs $\mathbf{c} = (x_{t-1}, x_t)$ and $\mathbf{c}' = (x'_{t-1}, x'_t)$ according to $d_{nc}(\mathbf{c}, \mathbf{c}')$, from small values to large values.
- Step 2 Arrange G_0, G_1, $\cdots$, G_{M-1}: Take codeword pairs one by one from the sorted codeword pairs in order. Put two codewords of a pair into the same group if possible; Otherwise, put them into two different groups.
- Step 3 Assign B_t to G_0, G_1, $\cdots$, G_{M-1}: Randomly assign patterns of B_t to groups $G_0, G_1, \cdots, G_{M-1}$, and then tries all possible switches one by one. Find the best assignment which minimizes the number of different bits in B_t between two groups with small distances.

III. CONDITIONAL PROBABILITY FOR GENERATING SOFT-OUTPUT

The set partitioning labeling for 8PSK is resulted from the set partitioning in [9]-[11] which has minimum intra-set squared Euclidean distances $\Delta^2_0 = 0.586$, $\Delta^2_1 = 2$ and $\Delta^2_2 = 4$. However, it is not shown that differential encoding of 16APSK in Fig. 1 is suitable for BICM-DE. The set partitioning resulted from this differential encoding is shown in Fig. 3 where b^t_0 and a_{t-1} jointly determine which subset (B0 or B1) is used. This set partitioning is not reasonable with respect to Euclidean distances because two nearest points of the inner circle are still in the same subset B0. Therefore, we propose general differential encoding for BICM-ID instead of conventional differential encoding, as illustrated in Fig. 4. The output symbol of the differential encoding is determined by x_{t-1} and all four bits B_t jointly, i.e., $x_t = E(x_{t-1}, B_t)$. Obviously the differential encoding in [6] is a special case of the general differential encoding. We propose to realize differential encoding by a look-up table and optimize the table in the next section.

To generate soft-output for general differential encoding, the probability of (5) and (6) should be re-derived as follows. First modify (3) as

$$y_t = y_{t-1}(\frac{x_t}{x_{t-1}}) + n'_t. \tag{8}$$

Based on (8) with $x_t = E(x_{t-1}, B_t)$, the conditional probability of receiving y_t is

$$P(y_t|x_{t-1}, B_t) = \frac{1}{\pi \tilde{N}_0} e^{-\frac{1}{\tilde{N}_0}|y_t - y_{t-1}\frac{E(x_{t-1}, B_t)}{x_{t-1}}|^2} \tag{9}$$

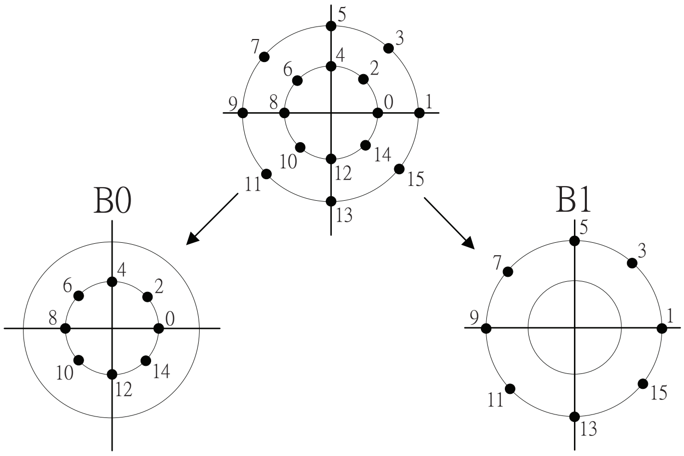

Fig. 3. The first set partitioning of 16APSK resulted from conventional differential encoding.

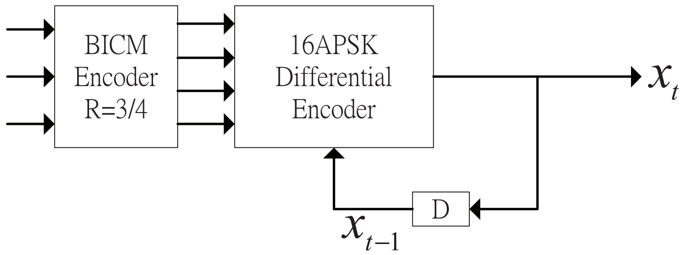

Fig. 4. Block diagrams of the proposed transmitter.

where $\tilde{N}_0$ is defined as

$$\tilde{N}_0 = \begin{cases} 2N_0 & \text{if } \frac{a_t}{a_{t-1}} = 1 \\ N_0(1+R_1^2) & \text{if } \frac{a_t}{a_{t-1}} = R_1 \\ N_0(1+R_2^2) & \text{if } \frac{a_t}{a_{t-1}} = R_2 \end{cases} \quad (10)$$

which can be compared with (4). The probability of (9) needs the information of x_t, so we simplify it in the following. Because (x_{t-1}, x_t) and $(x_{t-1}e^{j\theta}, x_t e^{j\theta})$ always have the same probability of (9), they are noncoherently indistinguishable and must correspond to the same input bits B_t. In other words, for any (x_{t-1}, x_t) and B_t satisfying $x_t = E(x_{t-1}, B_t)$, we have $x_t e^{j\theta} = E(x_{t-1}e^{j\theta}, B_t)$ where $\theta \in \{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \cdots, \frac{7\pi}{4}\}$. Consequently, $a_t v_t = E(a_{t-1}v_{t-1}, B_t)$ means $a_t v_t v_{t-1}^* = E(a_{t-1}, B_t)$. Therefore, (8) can be expressed as

$$\begin{aligned} y_t &= y_{t-1}\left(\frac{a_t v_t v_{t-1}^*}{a_{t-1}}\right) + n_t' \\ &= y_{t-1}\left(\frac{E(a_{t-1}, B_t)}{a_{t-1}}\right) + n_t'. \end{aligned} \quad (11)$$

Based on (11), the conditional probability of (9) is rewritten as

$$P(y_t|a_{t-1}, B_t) = \frac{1}{\pi\tilde{N}_0} e^{-\frac{1}{\tilde{N}_0}|y_t - y_{t-1}\frac{E(a_{t-1},B_t)}{a_{t-1}}|^2}. \quad (12)$$

The probability of (12) only needs the amplitude of x_t, and use $x_{t-1} = a_{t-1}$ to obtain x_t.

In case of the conventional differential encoding of 16APSK, i.e., $w_t = v_t v_{t-1}^* = \mu(b_3^t b_2^t b_1^t)$ and the amplitude a_t is jointly decided by a_{t-1} and b_0^t, (12) becomes

$$P(y_t|a_{t-1}, B_t) = \frac{1}{\pi\tilde{N}_0} e^{-\frac{1}{\tilde{N}_0}|y_t - y_{t-1}\mu(b_3^t b_2^t b_1^t)\frac{a_t}{a_{t-1}}|^2} \quad (13)$$

If $b_0^t = 1$, we have $\frac{a_t}{a_{t-1}} = R_1$ or R_2, so

$$\begin{aligned} P(y_t|b_3^t b_2^t b_1^t, b_0^t = 1) &= P(y_t|b_3^t b_2^t b_1^t, b_0^t = 1, a_{t-1} = r_0) \\ &\quad + P(y_k|b_3^t b_2^t b_1^t, b_0^t = 1, a_{t-1} = r_1) \\ &= \frac{1}{\pi N_0(1+R_1^2)} e^{-\frac{1}{\tilde{N}_0}|y_t - y_{t-1}R_1\mu(b_3^t b_2^t b_1^t)|^2} \\ &\quad + \frac{1}{\pi N_0(1+R_2^2)} e^{-\frac{1}{\tilde{N}_0}|y_t - y_{t-1}R_2\mu(b_3^t b_2^t b_1^t)|^2} \end{aligned} \quad (14)$$

whose right-hand side is the same as the right-hand side of (6); if $b_0^t = 0$, we have $a_t = a_{t-1}$, so

$$\begin{aligned} P(y_t|b_3^t b_2^t b_1, b_0^t = 0) &= P(y_t|b_3^t b_2^t b_1, b_0^t = 0, a_{t-1} = r_0) + \\ &\quad P(y_t|b_3^t b_2^t b_1, b_0^t = 0, a_{t-1} = r_1) \\ &= \frac{2}{2\pi N_0} e^{-\frac{1}{\tilde{N}_0}|y_t - y_{t-1}\mu(b_3^t b_2^t b_1^t)|^2}. \end{aligned} \quad (15)$$

whose right-hand side is twice the right-hand side of (5). This means that (5) has a little bug which should be corrected by multiplying it by two.

To verify the correctness of (15), simulations of a rate-3/4 convolutional code from [6] and a rate-1/2 convolutional code from [4] are performed. The correlated fading channel with $f_D T_s = 0.01$ in [12] is used, and each frame contains 1200 symbols. At the receiver, max-log-MAP is used for soft-outputs of the differential detector and the BICM decoder. Extrinsic information between the differential detector and the BICM decoder is exchanged. The iteration number of this BICM-ID is three because more iterations do not obtain obvious gains. Figure 5 compares (15) with the original probability (5). The modified probability (15) outperforms the original probability (5) slightly because their difference is small.

IV. The Proposed Differential Encoding for BICM-ID

To maximize minimum intra-set distances of differential encoding by a table, we need to modify Step 3 of constructing tables. The proposed Step 3 is

Step 3 Assign B_t to G_0, G_1, $\cdots$, G_{M-1}: Divide the set of M groups into two subsets. Try all $\frac{1}{2}\binom{M}{M/2}$ possibilities of set partitioning and choose the one that maximizes the minimum intra-set distance Δ_1. Input bits $b_0^t = 0$ and 1 is assigned to two subsets. Then for $i \in \{1, 2, \cdots, \log_2 M - 1\}$, successively divide each subset which contains $M/2^i$ groups into two subsets containing $M/2^{i+1}$ groups. For each partitioning, try all possibilities of set partitioning and choose the one that maximizes the minimum intra-set distance Δ_{i+1}, and assign $b_i^t = 0$ and 1 to two subsets.

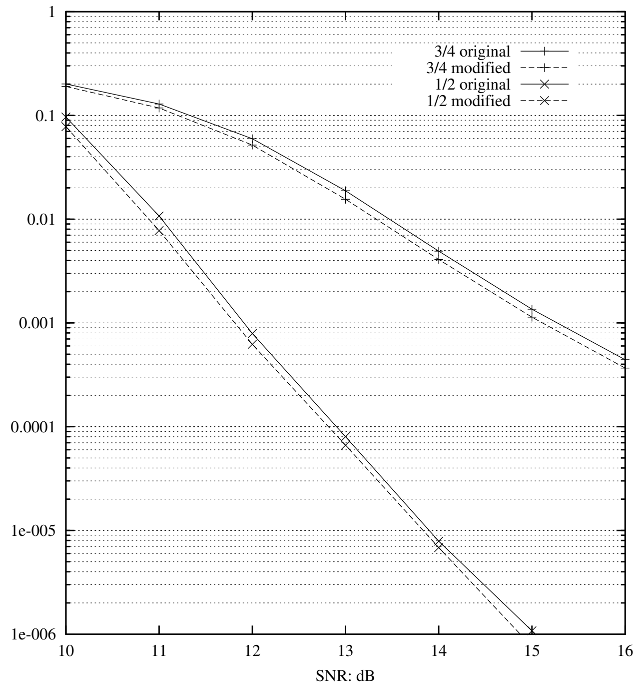

Fig. 5. The comparison between (5) and (15).

To utilize the proposed algorithm, a definition of additive distance for fading channels is needed. We propose to define the squared noncoherent distance between $\mathbf{c}$ and $\mathbf{c}'$ by

$$d_{nc}^2(\mathbf{c}, \mathbf{c}') = \log \max \{\Pr(\mathbf{c} \to \mathbf{c}'), \Pr(\mathbf{c}' \to \mathbf{c})\} \qquad (16)$$

where $\Pr(\mathbf{c} \to \mathbf{c}')$ is the pairwise error probability [13, eqn.(5)] of deciding $\mathbf{c}'$ when $\mathbf{c}$ is transmitted.

For conventional differential encoding, the minimum intra-set distances are $\Delta_0^2 = 2.345$, $\Delta_1^2 = 2.461$, $\Delta_2^2 = 3.688$ and $\Delta_3^2 = 4.382$. The table for 16APSK obtained by the proposed algorithm is shown in Table I. For convenience of presentation, we use $s_t \in \{0, 1, 2, \cdots, 15\}$ to denote a signal point of the constellation in Fig. 2. According to s_{t-1} and B_t, s_t can be obtained by this table. After the first set partitioning, the first eight groups belong to the subset of $b_0^t = 0$ and the last eight groups belong to the subset of $b_0^t = 1$. The first set partitioning of this differential encoding is illustrated in Fig. 6. Note that $b_0^t \oplus (s_{t-1} \& 1) = 0$ chooses B0 while $b_0^t \oplus (s_{t-1} \& 1) = 1$ selects B1 where $\oplus$ denotes the modulo-two addition and & represent bitwise-and operation. Then by the second set partitioning, the subset of $b_0^t = 0$ is divided into two subsets; One is the first four groups and the other is the next four groups. The partitioning for the subset of $b_0^t = 1$ is similar. And so on. The new differential encoding has $\Delta_1^2 = 3.120$ while Δ_0, Δ_2 and Δ_3 are the same.

For two code sequences of the BICM encoder, the squared noncoherent distance is $\sum_{i=0}^{3} d_i \Delta_i^2$ where d_i denotes the Hamming distance in level i. For the minimum squared noncoherent distance of the 8-state rate-3/4 convolutional code in [6], the set partitioning labeling used in [6] is 8.49, and the proposed differential encoding in Table I is 9.15.

TABLE I
THE PROPOSED ENCODING TABLE FOR 16APSK WHERE $k = 0, 2, 4, 6, 8, 10, 12$ OR 14.

$b_3^t b_2^t b_1^t b_0^t$	$s_{t-1} = 0 + k$	$s_{t-1} = 1 + k$
0000	0+k mod 16	1+k mod 16
1000	8+k mod 16	9+k mod 16
0100	4+k mod 16	5+k mod 16
1100	12+k mod 16	13+k mod 16
0010	3+k mod 16	2+k mod 16
1010	11+k mod 16	10+k mod 16
0110	7+k mod 16	6+k mod 16
1110	15+k mod 16	14+k mod 16
0001	1+k mod 16	0+k mod 16
1001	9+k mod 16	8+k mod 16
0101	5+k mod 16	4+k mod 16
1101	13+k mod 16	12+k mod 16
0011	2+k mod 16	3+k mod 16
1011	10+k mod 16	11+k mod 16
0111	6+k mod 16	7+k mod 16
1111	14+k mod 16	15+k mod 16

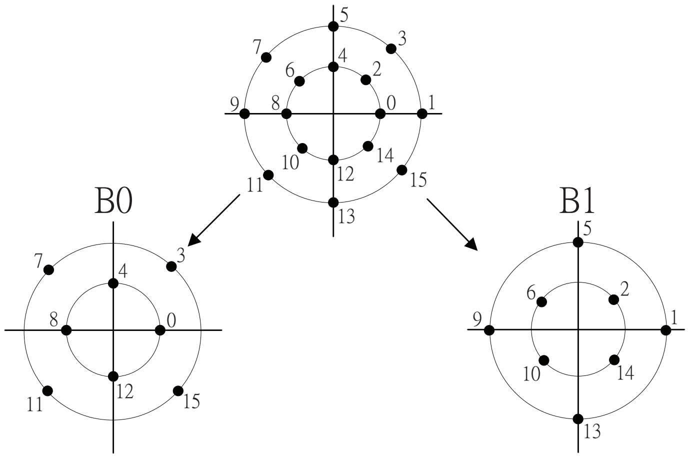

Fig. 6. The first set partitioning of 16APSK resulted from conventional differential encoding..

To find better codes for the proposed differential encoding, rate-3/4 convolutional codes are randomly searched. The first criterion is maximizing the minimum Hamming distance for diversity, and the second criterion is maximizing the minimum noncoherent distance. The minimum Hamming distance of the original 8-state rate-3/4 convolutional code in [6] is 3. The obtained 8-state, 16-state and 32-state convolutional codes have the minimum Hamming distance 4, and minimum squared noncoherent distance 11.49, 13.27 and 13.53, respectively. The code polynomials for 8-state, 16-state and 32-state codes are [4444;0642;2051], [4444;2501;7720] and [8888;6,0,14,10;15,2,4,15] (octal), respectively. These codes are simulated with simulation parameters being the same as those for Fig. 5. The simulation results in Fig. 7 show the improvement of the proposed differential encoding on the differential encoding in [6], which is approximately 0.85 dB gain at bit error rate of 10^{-6}. Besides, for the proposed differential encoding, the searched 8-state code offers approximately 0.6 dB gain over the original code in [6] at bit error rate of 10^{-6}.

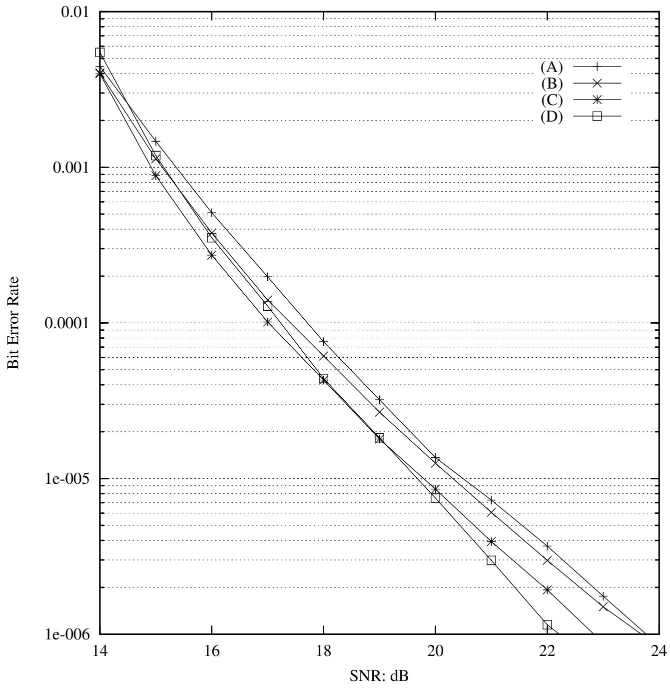

Fig. 7. Simulation results of the differential encoding in [6] and the proposed new differential encoding. (A) differential encoding in [6] with mixed labeling for $\mu(.)$, using the convolutional code in [6]. (B) differential encoding in [6] with set partitioning labeling for $\mu(.)$, using the convolutional code in [6]. (C) the proposed differential encoding, using the 8-state convolutional code in [6]. (D) the proposed differential encoding, using the searched 8-state convolutional code.

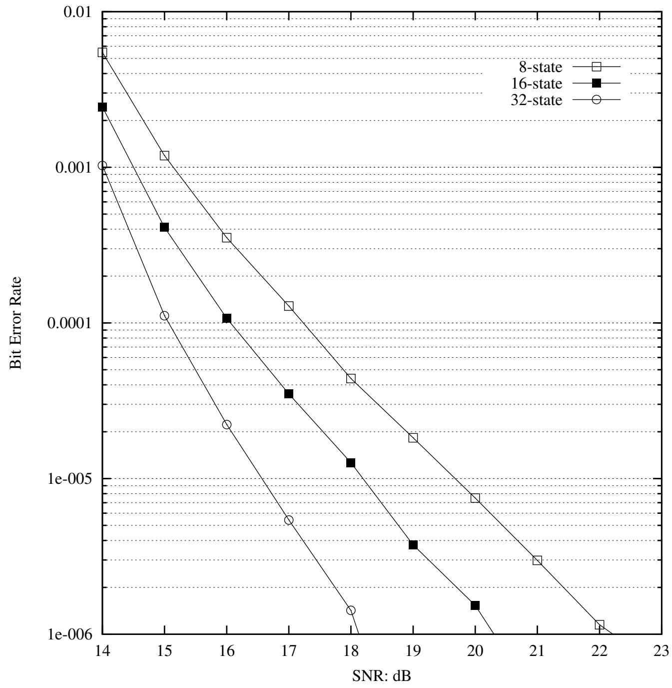

Fig. 8. Simulation results of the proposed differential encoding using the searched convolutional code.

In Fig. 8, it is observed that codes with more states can further enhance the error performance significantly.

V. CONCLUSIONS

In this paper, we consider differential BICM-ID using 16APSK. We derive the probability of receiving signals conditioned on the transmission of input bits of general differential encoding, which also indicates a little error in the equation in [6]. We also propose new differential encoding of 16APSK for BICM-ID by optimizing intra-set distances. Besides, code searches are performed. The superiority of the proposed differential encoding and the searched codes is verified by simulations .

REFERENCES

[1] E. Zehavi, "8-PSK trellis codes for a Rayleigh channel," *IEEE Trans. Commun.,* vol. 40, no. 5, pp. 873-884, May 1992.

[2] G. Caire, G. Taricco and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inform. Theory,* vol. 44, no. 3, pp. 927-945, May 1998.

[3] X. Li and J. A. Ritcey, "Bit-interleaved coded modulation with iterative decoding," *IEEE Commun. Lett.,* vol. 1, no. 6, pp 169-171, Jun. 1997.

[4] X. Li and J. A. Ritcey, "Trellis-coded modulation with bit interleaving and iterative decoding," *IEEE Journal Selec. Areas in Commun.,* vol. 17, no. 4, pp. 715-724, Apr. 1999.

[5] X. Li, A. Chindapol and J. A. Ritcey, "Bit-interleaved coded modulation with iterative decoding and 8PSK signaling," *IEEE Trans. Commun.,* vol. 50, no. 8, pp. 1250-1257, Aug. 2002.

[6] D. Liang, S. X. Ng and L. Hanzo, "Soft-decision star-QAM aided BICM-ID" *IEEE Signal Proc. Lett.,* vol. 18, no. 3, pp. 169-172 Mar. 2011.

[7] R. Y. Wei, "Differential encoding by a look-up table for quadrature-amplitude modulation," *IEEE Trans. Commun.,* vol. 59, pp. 84-94, Jan. 2011.

[8] R. Y. Wei and L. T. Chen, "Further results on differential encoding by a table," *IEEE Trans. Commun.,* vol. 60, pp. 2580-2590, Sep. 2012.

[9] G. Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. Inform. Theory,* vol. 28, pp. 55-67, Jan. 1982.

[10] G. Ungerboeck, "Trellis-coded modulation with redundant signal sets-Part I: Introduction," *IEEE Commun. Mag.,* vo1. 25, no. 2, pp. 5-11, Feb. 1987.

[11] G. Ungerboeck, "Trellis-coded modulation with redundant signal sets-Part II: State of the art," *IEEE Commun. Mag.,* vo1. 25, no. 2, pp. 12-21, Feb. 1987.

[12] C. Xiao, "A generalized simulation model for Rayleigh fading channels with accurate second-order statistics," in *Proc. IEEE VTC2002-Spring*, Birmingham, Al, May 6-9, 2002.

[13] Y. M. Chen and Y. L. Ueng "Noncoherent amplitude/phase modulated transmission schemes for Rayleigh block fading channels," *IEEE Trans. Commun.,* vol. 61, pp. 217-227, Jan. 2013.