A Realization of Determinant Criterion for STTC Design

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Abstract: In a MIMO (Multiple-Input Multiple-Output) system, space-time trellis coding (STTC) is one kind of space-time coding technique which can obtain both of high diversity and coding gain. In this paper, we propose a high-speed algorithm to solve the determinant calculations in determinant criterion of STTC. Some experimental results indicate that this algorithm is very available for developing an effective signaling scheme of space-time trellis codes.

1. Introduction

Multiple-input multiple-output (MIMO) systems have better performance under fading channels and can provide high data rates and high diversity for satisfying the modern wireless communications demands. A MIMO system where multiple transmit antennas N_T and receive antennas N_R are used, has been adopted as next generation (4G) technology in wireless communications. Because a MIMO system emerges a lot of new technologies such as space-time coding and decoding so that it makes wireless communications an exciting and challenging field. The main features of MIMO systems are broadband and diversity techniques, the former provides high data transmission rate and the latter realizes the better signal qualities in fading channel environments. Also, the low power consumptions are strictly required.

The space-time (ST) coding can make good use of the transmit diversity to alleviate the effect of fading and achieve high date transmission rate without adding extra bandwidth and power consumption. Hence, to develop good ST codes is very important. The first ST code with a normalized rate of 1 symbol per channel use (pcu) was proposed by Alamouti over two transmit antennas and two time periods in [1]. In [2], Tarokh gave the design criteria, which is a tradeoff between constellation size, data rate, complexity, and diversity advantage for ST code, and he presented STTC. By expanding the Alamouti code, a lot of researches for the new type ST codes called ST block code (STBC) were presented. STBC can achieve the maximum possible diversity advantage with a simple decoding algorithm. However, no coding gain can be provided by STBC. However, STTC is able to combat the effect of fading, which can simultaneously offer a substantial coding gain, spectral efficiency, and diversity improvement on flat fading channels.

In order to develop good STTC, the rank criterion and the determinant criterion are proposed [2], [3]. The rank criterion guarantees to achieve maximum possible diversity $N_T N_R$ if matrix $\mathbf{A(c, e)}$ (defined by Eq.(5)) has full rank for all the codewords, and the determinant criterion can achieve maximum coding gain if the minimum determinant of $\mathbf{A(c, e)}$ should be maximized over all codewords.

However, how to calculate $\det(\mathbf{A(c, e)})$ has not been satisfied in the determinant criterion. Generally, the calculation of $\det(\mathbf{A(c, e)})$ is realized by the combinations of all $\mathbf{c}$ and $\mathbf{e}$. Thus, to find an effective method to calculate $\det(\mathbf{A(c, e)})$ is very important. In this paper, we present an high-speed algorithm to calculate $\det(\mathbf{A(c, e)})$ so that the determinant criterion can be realized easily.

2. MIMO Systems and STTC Structures

We consider a MIMO system with N_T transmit antennas and N_R receive antennas shown as Fig.1, where the STC is a space-time coder and the STD is a space-time decoder. In the STC, the serial binary inputs are changed into the parallel symbol vector $\boldsymbol{x}$, then encoded to a ST codeword $\mathbf{X}_t = (x_t^1, x_t^2, \cdots, x_t^{N_T})^T$ and the received signal matrix $\boldsymbol{Y}_t$ can then be written as

$$\mathbf{Y}_t = \mathbf{HX}_t + \mathbf{N}_t \tag{1}$$

where $\boldsymbol{H}$ is a $N_R \times N_T$ channel matrix whose entries are independent, identically distributed (i.i.d.) complex Gaussian random variables with variance 0.5 in each dimension, i.e., $h_{i,j} \sim CN(0,1)$; $\mathbf{N}_t = (n_t^1, n_t^2, \cdots, n_t^{N_R})^T$ is a noise matrix with i.i.d. complex Gaussian entries. In this paper, we assume the channels to be quasi-static and flat Rayleigh fading.
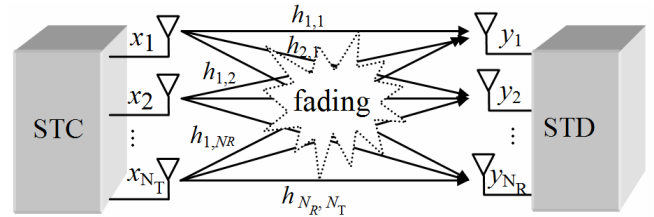

Figure 1: A MIMO system

The STTC is one of signal processing techniques and can improve the transmit performance of a MIMO system. Let (a_t, b_t) be binary inputs at time t and make $\mathbf{u}_t = (a_t, b_t, a_{t-1}, b_{t-1}, \ldots, a_{t-s/2}, b_{t-s/2})$ where s denotes the number of memory elements in the encoder, and $\mathbf{G}$ be a generation matrix with N_T columns and $m+s$ rows where m is the number of binary inputs at time t. The STTC with M-PSK modulation is given by

$$\mathbf{c} = (c_t^1, c_t^2) = \mathbf{u}_t \mathbf{G} \ (\text{mod } M). \tag{2}$$

The transmitted signals can be obtained by $\mathbf{X}_t = (x_t^1, x_t^2)^T = (\exp(j\pi c_t^1/2), \exp(j\pi c_t^2/2))^T$.

For example, the generation matrix $\mathbf{G}$ introduced by Tarokh in [2] is

$$\mathbf{G}^T = \begin{bmatrix} 0 & 0 & 2 & 1 \\ 2 & 1 & 0 & 0 \end{bmatrix} \qquad (3)$$

and the STTC structure can be represented by a trellis diagram shown as in Fig.2, which indicates the principle of the encoder of STTC. Fig.2 shows the four-state STTC structure where the nodes of $q_{0,0}$, $q_{0,1}$, $q_{1,0}$, and $q_{1,1}$ represent the states, the edges represent state transitions, and the sequences of the number represent the codeword transmitted from antennas. When the binary input 10 are inputted at state of $q_{0,1}$, the transition edge is 10, the output is (1, 2), and the next state will be $q_{1,0}$. It is not difficult to make sure the output **c** of STTC are (0, 0), (0, 3), (3, 1), (1, 2), (2, 0) with the initial state $q_{0,0}$ when binary input sequence is 00→11→01→ 10→00 in Fig.2. It should be noted when we create the transmitted symbol, both of the first state and the last state should be $q_{0,0}$. We can use the trellis diagram to create and detect the codewords.
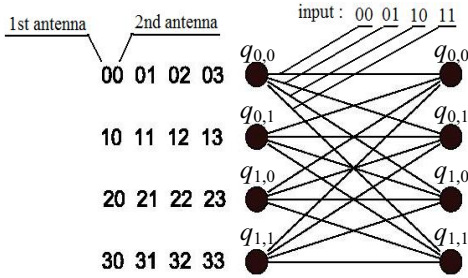

Figure 2:   Trellis diagram for a generation matrix by Tarokh

3. The Proposed Algorithm

Traditionally, the design of STTC is treated as a problem of minimizing the probability of error for a fixed rate and SNR.

3. 1 Pair-wise Error Probability

In the quasi-static fading model, the Chernoff bound of the pair-wise error probability [3] of deciding in favor of an erroneous symbol vector **e** when transmitting **c** is as follows

$$P_{c \to e} \leq \det\left(\mathbf{I}_{N_T} + \rho\mathbf{A(c,e)}\right)^{-N_R} \qquad (4)$$

where $\mathbf{I}_{N_T}$ is an $N_T \times N_T$ identity matrix; ρ denotes the average SNR at each receive antenna; $\mathbf{A(c, e)}$ is the distance matrix given by

$$\mathbf{A(c, e)} = \mathbf{B(c, e)} \, \mathbf{B^*(c, e)} \qquad (5)$$

where

$$\mathbf{B(c,e)} = \begin{bmatrix} c_1^1 - e_1^1 & c_2^1 - e_2^1 & \cdots & c_L^1 - e_L^1 \\ c_1^2 - e_1^2 & c_2^2 - e_2^2 & \cdots & c_L^2 - e_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{N_T} - e_1^{N_T} & c_2^{N_T} - e_2^{N_T} & \cdots & c_L^{N_T} - e_L^{N_T} \end{bmatrix} \qquad (6)$$

and c_t^i and e_t^i denote the transmitted symbol and the erroneous detected symbol from antenna i at time t, the L and N_T denote the length of frae and the number of transmit antennas. At high SNR, Eq.(4) leads to the rank criterion and the determinant criterion for STTC designs where the rank and the determinant of $\mathbf{A(c, e)}$ must be calculated.

Here, we consider how to calculate $\det(\mathbf{A(c, e)})$ by an effective method. By Eq.(5), we know that the $\mathbf{A(c, e)}$ can be represented by

$$\mathbf{A(c,e)} = \sum_{t=1}^{L} \mathbf{b}_t \mathbf{b}_t^* \qquad (7)$$

$$\mathbf{b}_t = \begin{bmatrix} c_t^1 - e_t^1 & c_t^2 - e_t^2 & \cdots & c_t^{N_T} - e_t^{N_T} \end{bmatrix}^T. \qquad (8)$$

The question is how to obtain the minimum $\det(\mathbf{A(c, e)})$ with all combination of the codewords **c** and **e**. From Eq.(7), we know that all of $\det(\mathbf{A(c, e)})$ can be obtained by $\det\left(\sum_{t=1}^{L} \mathbf{b}_t \mathbf{b}_t^*\right)$. It is evident that the calculation of $\det(\mathbf{A(c, e)})$ is based on the combinations of all **c** and **e**.

3. 2 State Diagram Representation

By Eq.(7), we know that the determinant can be calculated by means of adding $\mathbf{b}_t \mathbf{b}_t^*$ which consist of   *c* and *e* generated by each state transition of the trellis diagram.

For example, we consider the calculation of the minimum determinant of **G** represented by Eq.(3). Without loss of generality, we assume that *c* is the all zero codeword and we can get the state diagram represented by Fig.3 [2].
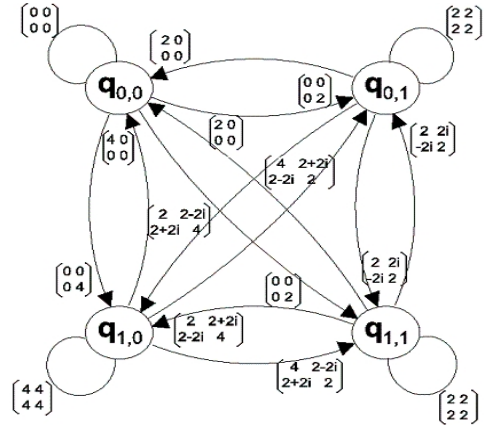

Figure 3:   State diagram for a generation matrix by Tarokh

In Fig.3, the nodes represent the states, the edges represent the state transitions, and the labels on the edge represent $\mathbf{b}_t \mathbf{b}_t^*$. For a simple case, we can obtain the minimum determinant by the observation method in Fig.3, that is, we find a loop that the first state and end state are $q_{0,0}$ such as $q_{0,0} \to q_{0,1} \to q_{0,0}$ and $q_{0,0} \to q_{1,1} \to q_{0,0}$, the determinant of the sum of $\mathbf{b}_t \mathbf{b}_t^*$ is 4. We can obtain the minimum determinant with this state diagram by searching a path whose starting state and final state are $q_{0,0}$ and which contains at least one state different from $q_{0,0}$. But it is impossible to calculate the minimum determinant by observations with a many-state diagram even if we only consider that *c* is all zero codeword, so we need to develop an algorithm to compute the minimum determinant.

3. 3 Tree Structure Representation

The calculation of the minimum determinant can be represented by a tree structure. As an example, consider the generation matrix expressed by Eq.(3) again. We should consider all of the *c* and *e* when calculating the minimum

determinant. No generality is lost by taking c to be all zero codeword at $q_{0,0}$ because of its geometrically uniform[2]. By this assumption, we only need to consider all e, in other words, we should consider all trellis states when detecting the codeword e. We can represent its detecting process by a tree structure as Fig.4.
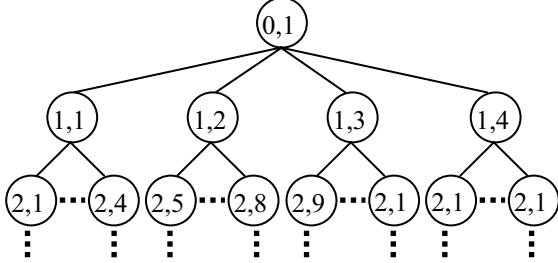

Figure 4: The detecting process represented by a tree structure

The nodes represent the states, and the nodes at the depth t represent the state at time t. All of the nodes are labeled by number like "i,j". The node labeled "i,j" means that the j-th node at the depth i, and we indicate the node as $N_{i,j}$ from here. The root node represents the initial state. Each node has four child nodes because there should be four cases of detecting. Each node has the matrix $\mathbf{D}$ which can be obtained as follows. For a node $N_{i,j}$ whose parents has the matrix $\mathbf{D}_{parent}$, the matrix $\mathbf{D}_{child}$ of $N_{i,j}$ is calculated by $\mathbf{D}_{parent} + \mathbf{b}_t\mathbf{b}_t^*$, where the $\mathbf{b}_t\mathbf{b}_t^*$ is the matrix represented on the edge which connects the parent node of $N_{i,j}$ to $N_{i,j}$ in the state diagram. Note that we assume the root node matrix $\mathbf{D}_{root}$ is a zero matrix. The tree structure has the following property.

Property 1: The distance matrices as Eq.(7) for all $\mathbf{c}$ and $\mathbf{e}$ are identical with the matrices of the nodes at the depth L.

From *Property* 1, the matrix having the minimum determinant exists in all matrices $\mathbf{D}$ of the nodes at the depth L. On the other hand, if a node $N_{k,j}$ whose state is $q_{0,0}$ has $\mathbf{D}_{k,j}$, there is a matrix $\mathbf{D}_{L,m}$ which is identical to $\mathbf{D}_{k,j}$, because there is a path between $N_{k,j}$ and $N_{L,m}$ surely whose node states are all of $q_{0,0}$ and all edges have zero matrices. It means that we can obtain the minimum determinant of the distance matrix by (1) searching nodes whose states are $q_{0,0}$; (2) finding a minimum determinant in all determinants of such nodes obtained by (1).

Property 2: Blum proved the following Eq.(9) [8].

for any integer $k \geq 1$,

$$\det(\sum_{t=1}^{k} \mathbf{b}_t\mathbf{b}_t^*) \leq \det(\sum_{t=1}^{k+1} \mathbf{b}_t\mathbf{b}_t^*) \qquad (9)$$

Eq.(9) means that the determinant value doesn't get smaller when adding the matrix $\mathbf{b}_t\mathbf{b}_t^*$, so we have the following *Reducing Rules*. By these rules, a lot of the redundancy nodes can be eliminated so that we can calculate the minimum determinant quickly. In fact, these rules enable the calculation to be finished at most at the depth as the trellis states number in the tree structure.

Reducing Rule 1: When we obtained a minimum determinant as a candidate, we can eliminate all nodes whose determinants are not less than the candidate.

Reducing Rule 2: Let a node N_2 be the descendant node of node N_1. If both N_1 and N_2 have the same states, we can eliminate all descendant nodes of N_2.

3. 4 High-Speed Algorithm

In section 3.3, we know the calculation of the minimum determinant can be realized by making tree structure. Here, we propose a high-speed algorithm for calculating the minimum determinant of the distance matrix based on the tree structure. The outline of the algorithm is as follows.

First, we assume an initial value of the minimum determinant "*candidate*" to be infinity and the parent node set $\mathbf{N}_p$ to contain the root node. Next, we make the child nodes of the node(s) in $\mathbf{N}_p$ and make a child node set $\mathbf{N}_c$ contain these child nodes. Then, we calculate the determinants of the nodes in the $\mathbf{N}_c$. Second, we choose the one node denoted by N_{act} by ascending order of the determinant value from $\mathbf{N}_c$. We deal with N_{act} by one of the following processing (1) – (4) in numerical order.

(1) By *Reducing Rule* 2, we eliminate the N_{act} if the state of N_{act} has already existed in the path from the root node to N_{act}.

(2) By *Reducing Rule* 1, we eliminate the N_{act} and the nodes in $\mathbf{N}_c$ which have not chosen yet if the determinant of N_{act} is not less than *candidate*.

(3) *By Property* 1, we renew *candidate* with the determinant value of N_{act} if the state of N_{act} is the same as $q_{0,0}$, and we eliminate the N_{act} and the nodes in $\mathbf{N}_c$ which have not chosen yet because of *Reducing Rule* 1.

(4) Otherwise, we do nothing.

After finishing the processing (1)-(4), we chose the next node as N_{act} in $\mathbf{N}_c$ and repeat (1)-(4) again until all of the nodes in $\mathbf{N}_c$ have been chosen. If there are some nodes in $\mathbf{N}_c$, we change $\mathbf{N}_c$ into $\mathbf{N}_p$ and we do the same processing.

By above processing, we can make the tree structure. It can be seen that the depth of tree does not exceed the number of its trellis states.

According to the above explanations, we propose a high-speed algorithm for calculating the minimum determinant.

Algorithm for calculating $\det(\mathbf{A}(\mathbf{c}, \mathbf{e}))$:

Step 1: Let *candidate* be infinity, and put the root node in the set $\mathbf{N}_p$.

Step 2: Make a child node set $\mathbf{N}_c$ of the nodes in $\mathbf{N}_p$.

Step 3: Choose one node N_{act} in ascending order from $\mathbf{N}_c$.

Step 4: If the state of N_{act} has already existed in the path from the root node to N_{act}, eliminate the N_{act}, and go to Step 7.

Step 5: If the determinant of N_{act} is not less than the minimum candidate, eliminate the N_{act} and the nodes in $\mathbf{N}_c$ which have not chosen yet, and go to Step 7.

Step 6: If the state of N_{act} is $q_{0,0}$, renew the minimum candidate with the determinant of N_{act}, and eliminate the N_{act} and the nodes in $\mathbf{N}_c$ which have not been chosen yet.

Step 7: Choose the next node in $\mathbf{N}_c$ and go to Step 4.

Step 8: If there's no node in N_c, ouput the minimum candidate and finish the calculation.

Step 9: Change the N_c into N_p and go to Step 2.

The proposed algorithm is in the assumption for fixing c as zero codeword. When c is not fixed, this algorithm also can be used by the following extensions. (1) Each node in the tree structure represents a pair of both states of detecting and coding. (2) In Step 6, check if the states of a pair indicated by N_{act} are equal to each other.

4. Performance of the Algorithm

To examine our algorithm's performance, we made experiments with the 20000 generation matrices with 4 rows and 2 columns, and 20000 ones with 5 rows and 2 columns. We examined how deeply the tree is when the calculation finished. The depth of finishing calculation is three if the all of the nodes in N_c can eliminated at the depth three. The results which are the ratio of the number of the generation matrices finishing calculation at each depth are shown in Table 1 where you can find that a lot of the calculation can finish in the depth of three or four and the maximum depth is the same number as the trellis states. It follows that this algorithm can calculate the minimum determinant for almost all of the generation matrices.

Table 1: The depth finishing the calculation

The depth finishing calculation	The size of generation matrix	
	4 x 2	5 x 2
2	60.295 %	17.35 %
3	32.96 %	42.255 %
4	6.745 %	26.72 %
5		9.52 %
6		2.435 %
7		1.715 %
8		0.05 %

For verifying this algorithm, we calculated the minimum determinants of the distance matrices for the generation matrices proposed up to now. The calculating time using MATLAB are shown in Table 2 where the generation matrix proposed by Meixia [7] is calculated by the extended algorithm which c is not fixed. It can be seen that all of the calculations finished in less than 1.5 ms.

Table 2: The time to calculate the minimum determinants

Generation matrix	Proposer	Time[ms]
$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 2 & 1 & 0 & 0 \end{bmatrix}$	Tarokh, et al.[2]	1.3281
$\begin{bmatrix} 0 & 2 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$	Meixia, et al.[7]	1.4531
$\begin{bmatrix} 0 & 2 & 1 & 2 \\ 2 & 3 & 2 & 0 \end{bmatrix}$	Bahador, et al.[9]	1.3750
$\begin{bmatrix} 2 & 0 & 1 & 3 \\ 2 & 2 & 0 & 1 \end{bmatrix}$	S. Baro, et al.[6]	1.3438
$\begin{bmatrix} 3 & 2 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$	T. INOUE, et al.[10]	1.3281

Generally, when we develop a new STTC based on the determinant criterion, the minimum determinants of all generation matrices must be calculated. This high-speed algorithm is very useful for STTC design.

Acknowledgment

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