Improved Cavity Perturbation Technique for Accurate Measurement of Complex Permittivity

Chul-Ki Kim
Dept. of Electrical Engineering
KAIST
DaeJeon, Repubilc of Korea
chulki@kaist.ac.kr

Byeong-Yong Park
Dept. of Electrical Engineering
KAIST
DaeJeon, Repubilc of Korea
Bogo30617@kaist.ac.kr

Seong-Ook Park
Dept. of Electrical Engineering
KAIST
DaeJeon, Repubilc of Korea
soparky@kaist.ac.kr

Abstract — **This paper propose the new cavity perturbation technique to calculate the complex permittivity of dielectric material. And then we more expanded the maximum volumes of the sample ratio to cavity resonator than the conventional cavity perturbation technique.**

Keywords—cavity perturbation technique; maximum volume.

I. INTRODUCTION

It is important to know the properties of materials used in microwave engineering applications. Although completely a few technique have been developed for the measurement, one of the most simple and accurate techniques is the cavity perturbation technique (CPT). The conventional cavity perturbation technique can be expressed starting from the Maxwell's equations, and apply the appropriate boundary conditions. These results can be simplified at maximum electric field and magnetic field to know a material's properties in a cavity resonator. As reported, the CPT has some problems that the materials to be measured should be specific shape and limitation of material's volume. Also, we evaluate the depolarizing effect of the sample. Therefore, we should improve the cavity perturbation technique to calculate the properties of materials more exactly, considering the problems.

Improved CPT is proposed by considering depolarizing field within the sample in this paper. Two dielectrics (Al_2O_3, SiC) are employed to verify the proposed method. The maximum volume of the dielectric samples is calculated with consideration of S-band cavity resonator and is compared with previously reported literature [4]. It is increased from 0.17% to 29.05%, using the proposed CPT.

II. IMPROVED CAVITY PERTURBATION METHOD

The variation of introducing a sample into a cavity resonator can be justified in the perturbation equation derived directly from Maxwell's equation [1]:

$$\frac{\Delta\omega}{\omega} = \frac{\int_{V_s}[(\overline{E_s}\cdot\overline{D_0} - \overline{E_0}\cdot\overline{D_s}) - (\overline{H_s}\cdot\overline{B_0} - \overline{H_0}\cdot\overline{B_s})]dV}{\int_{V_c}(\overline{E_0}\cdot\overline{D_0} - \overline{H_0}\cdot\overline{B_0})dV} \quad (1)$$

where $\Delta\omega = \omega_s - \omega_0$ is the complex resonance frequency variation between the complex resonance frequency of the cavity with the sample (ω_s) and without the sample (ω_0). The complex resonance frequency can be separated into real and imaginary parts as (2)

$$\frac{\Delta\omega}{\omega} = \frac{f_s - f_0}{f_s} + \frac{j}{2}\left(\frac{1}{Q_s} - \frac{1}{Q_0}\right). \quad (2)$$

When the sample is inserted where the electric field maximum (magnetic field minimum) in the resonator, we consider the maximum volume and shape at an electrostatic situation. First, we divide the electric field in the interior of the sample into uniform field and non-uniform field due to the depolarization field. Equation (1) can be simplified to (3),

$$\frac{\omega_s - \omega_0}{\omega} = \frac{(\frac{1}{\varepsilon_r^*} - 1)\int_{Vs}\bar{E}_0\cdot(\overline{D}_s^{uni} - \overline{D}_s^{non})dV}{2\varepsilon_0\int_{Vc}|E_0|^2\,dV} \quad (3)$$

where $\overline{D}_s^{uni}$ is uniform field and $\overline{D}_s^{non}$ is the non-uniform field in the sample. To calculate the permittivity more exactly, we concentrate the nonuniform field. Depending on the applied electric field and the sample shape, the sample will be partially polarized like a dipole. Therefore, we consider the nonuniform field. The electric field in the interior of the material sample is the total vector $\overline{E_s}$, representing the sum of the applied field and the depolarizing field.

$$\overline{D}_s^{non} = \varepsilon_r^*N\varepsilon_0(\varepsilon_r^* - 1)\overline{E_s} = \varepsilon_r^*N\varepsilon_0\frac{(\varepsilon_r^* - 1)}{1 + N(\varepsilon_r^* - 1)}\overline{E_0} \quad (4)$$

The depolarizing factor N depends on the geometrical shape of the sample and the applied electric field [2]. Here N^i is the depolarizing factor in i[th] applied axial direction to sample, where $N^a + N^b + N^c = 1$. These relations of the N factor are plotted on fig. 1.

$$N^i = \int_0^\infty \frac{(abc)du}{2(u+i^2)\sqrt{(u+a^2)(u+b^2)(u+c^2)}} \quad (5)$$
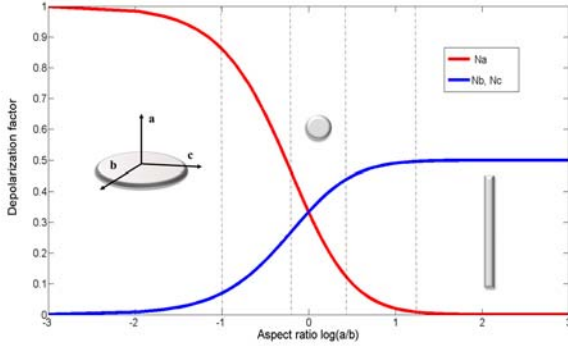
$$i = a,b,c(Field-direction)$$

Fig. 1. Depolarizing factor $N_{a,b,c}$ as function of aspect ratio versus the applied axial-electric field.

As the dominant mode in the rectangular cavity resonator is TE$_{10n}$ mode, the electric field E_0 can be written as,

$$E_0 = E_{max} \sin\frac{\pi x}{a} \ \sin\frac{n\pi z}{d} \qquad (6)$$

where 'a' is the longer side of rectangular section, 'd' is the length of cavity, ε_r^* is the complex permittivity ($\varepsilon_r' - i\varepsilon_r''$) and 'n' is integer at TE$_{10n}$ mode. (n=1,2,3,…)

To evaluate the volume integral of the sample, we also define the effective sample volume,

$$V_s' = \mathrm{h}t'w'. \qquad (7)$$

The effective thickness t' and w' are given by

$$t' = \left(\frac{t}{2} + \frac{a}{2\pi}\sin\left(\frac{\pi t}{a}\right)\right) \text{ and } w' = \left(\frac{w}{2} + \frac{L}{2\pi n}\sin\left(\frac{n\pi w}{L}\right)\right)$$

We substitute (2), (4) and (7) into (3), and then evaluate the right side equation, (3) can be rewritten as (8),

$$\frac{f_0 - f_s}{f_0} + \frac{j}{2}\left(\frac{1}{Q_s} - \frac{1}{Q_0}\right)$$
$$= 2(\varepsilon_r^* - 1)\frac{V_s'}{V_c} - \frac{2(\varepsilon_r^* - 1)^2 N}{1 + N(\varepsilon_r^* - 1)}\frac{V_s'}{V_c}. \quad (8)$$

Finally, equation (8) is separated and then we equal the real and imaginary part of it, becomes (10)

$$\frac{f_0 - f_s}{f_0}$$
$$= (\varepsilon_r' - 1)2\frac{V_s'}{V_c}$$
$$- \frac{2N[((\varepsilon_r' - 1)^2 - \varepsilon_r''^2)(N\varepsilon_r' - N + 1) + 2(\varepsilon_r' - 1)\varepsilon_r''^2 N]}{(N\varepsilon_r' - N + 1)^2 + \varepsilon_r''^2 N^2}\frac{V_s'}{V_c}.$$
$$(9a)$$

$$\frac{1}{Q_s} - \frac{1}{Q_0}$$
$$= 4\varepsilon_r''\frac{V_s'}{V_c}$$
$$- 4\frac{V_s'}{V_c}\frac{N[((\varepsilon_r' - 1)^2 - \varepsilon_r''^2)N\varepsilon_r'' - 2(\varepsilon_r' - 1)\varepsilon_r''(N\varepsilon_r' - N + 1)]}{(N\varepsilon_r' - N + 1)^2 + \varepsilon_r''^2 N^2}.$$
$$(9b)$$

We added the new part generated by polarization to the real part (9a) and imaginary part (9b) in order to minimize the range of error. Evaluating the two equations and then we calculate the complex permittivity and analysis the effect of depolarization more exactly than the original CPT.
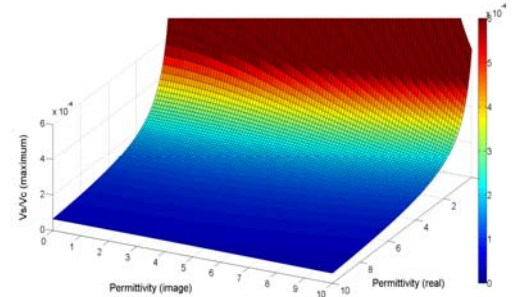
III. MAXIMUM SAMPLE VOLUME

The Maximum range of the sample volume with the complex permittivity was evaluated in [4]. By equating the real parts and considering the frequency shift range condition $[(f_0 - f_s)/f_0 \leq 10^{-3}]$ in [4], the paper evaluate the following:
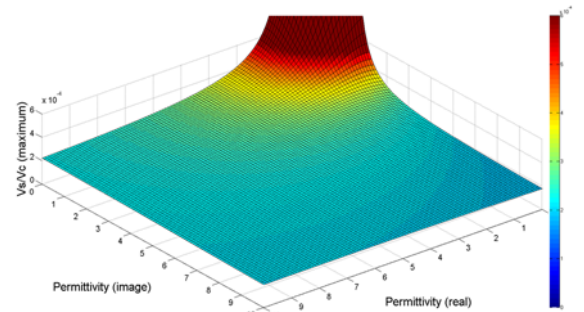
$$\frac{f_0 - f_s}{f_0} = 2 \cdot \frac{V_s}{V_c} \cdot \frac{N(\varepsilon_r'^2 + \varepsilon_r''^2) + (1 - 2N)\varepsilon_r'^2 + N - 1}{(N\varepsilon_r' - N + 1)^2 + (N\varepsilon_r'')^2}$$
$$\leq 10^{-3} \qquad (10)$$

We can compare their results and ours. From equation (9), we have the maximum volume ratio of sample to the cavity resonator $(V_s/V_c)_{max}$. By substituting the N factor according to the shapes of the sample in (11), Fig. 2 shows the maximum range of the sample volume as a function of the complex permittivity, when the sample is a dielectric rod/bar (h=a), sphere/cube and strip/disk. It is also found that the maximum volume of the sample that can be used in CPT depends on the shape of the sample. These results show that a dielectric strip and disk may take more advantages of CPT than the other types.

$$(\varepsilon_r' - 1)2\frac{V_s'}{V_c}$$
$$- \frac{2N[((\varepsilon_r' - 1)^2 - \varepsilon_r''^2)(N\varepsilon_r' - N + 1) + 2(\varepsilon_r' - 1)\varepsilon_r''^2 N]}{(N\varepsilon_r' - N + 1)^2 + \varepsilon_r''^2 N^2}\frac{V_s'}{V_c}$$
$$\leq 10^{-3}. \qquad (11)$$
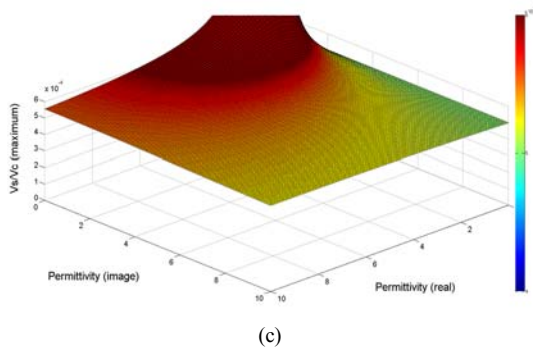

(a)

(b)

(c)

Fig. 2. The maximum range of the sample volume $(V_s/V_c)_{max}$ (a) a dielectric rod/bar, (b) a dielectric sphere/cube and (c) a dielectric strip/disk.

It is apparent that (11) try to expand the maximum volume depended not only on the shape but also the complex permittivity of the sample. To tell the advanced results, we compare two reference materials in [4]. One is low-loss Al_2O_3 ($\varepsilon_r^* = 8.9 - 0.004i$) and another is high-loss SiC ($\varepsilon_r^* = 26.66 - 27.99i$) at 2.45GHz. In table. 1, we demonstrate that $(V_s/V_c)_{max}$ are expanded from 0.17% to 29.05%. Fig. 3 is the graph showing the table visually.

TABLE I.　　EXPANDING THE MAXIMUM VOLUME OF THE SAMPLE

The shape of the sample	Material			
	Al_2O_3		SiC	
	Original	Improve	Original	Improved
Rod / Bar	0.0000633	0.0000719 (13.57% Up)	0.000019	0.000023 (21% Up)
Strip / Disk	0.000563	0.000564 (0.17% Up)	0.000509	0.000510 (0.2% Up)
Sphere / Cube	0.000179	0.000231 (29.05% Up)	0.000167	0.000177 (6% Up)
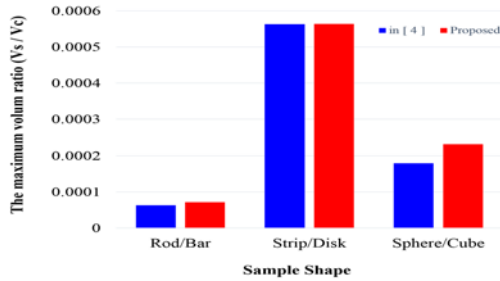
(a)
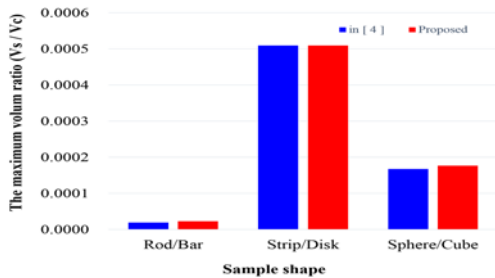
(b)

Fig. 3. The maximum range of the sample volume $(V_s/V_c)_{max}$
(a) Al₂O₃, (b) SiC.

IV. CONCLUSION

In this paper, we investigate the complex permittivity of the dielectric material more exactly and overcome the limit of the sample geometry using the proposed CPT. Based on the proposed technique, we evaluated the non-uniform field in the CPT and then calculated the complex permittivity. We also expanded the maximum volume of the sample to the cavity resonator, comparing the paper and our results.

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