

# Relay/Antenna Selection Using Adaptive Discrete Stochastic Approximation in MIMO Relaying Channels

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**Abstract —** In the paper, a computationally-efficient layered relay-and-antenna selection (LRAS) is proposed by using an adaptive discrete stochastic approximation (A-DSA) technique for two-way multiple-input multiple-output (MIMO) amplify-and-forward (AF) relaying systems in correlated channels. This LRAS algorithm is studied based on the achievable sum-rate (ASR) maximization under an equal power allocation. Notably, the A-DSA LRAS enables to reduce the complexity burden significantly compared to other existing selection strategies for MIMO two-way multiple-AF-relay systems.

**Keywords —** amplify-and-forward; MIMO relay; relay/antenna selection; two-way

## I. INTRODUCTION

A promising spectral-efficient two-way relaying protocol in conjunction with the multiple-input multiple-output (MIMO) technology has drawn considerable interest in recent years [1]. The amplify-and-forward (AF) relaying protocol [2] is more appealing owing to a simpler relaying strategy and hardware implementation. Fortunately, the implementation complexity of MIMO relaying networks with multiple relays is alleviated significantly by means of the utilization of antenna selection (AS) techniques. Most of AS schemes used to non-regenerative relaying networks deal with single-antenna relay models [3]. A cross-entropy (CE) relay subset selection [4] is addressed to reduce the computational load while still maximizing the achievable sum-rate (ASR) for a two-way MIMO AF relaying network. The authors [5] investigate the outage probability of relay selection in an underlay cognitive radio (CR) system with a secondary multi-relay network operating in the AF mode in the presence of primary user (PU) interference. In [6], an aggressive discrete stochastic approximation (ADSA) based relay antenna selection algorithm is investigated to maximize the achievable sum-rate (ASR) by taking both the antenna spatial correlation and channel estimation error into consideration in a three-node MIMO AF two-way relaying system. The distributed relay selection problem is studied based on the postprocessing signal-to-noise ratio (SNR) in spectral efficient broadcasting networks employing virtual MIMO in [7]. The asymptotic performance of the spectral and energy efficiencies of the multi-pair two-way relaying system is provided in [8] when each user has a single antenna and the relay is equipped with very large number of antennas. The authors in [9] evaluate the maximization of energy efficiency

in MIMO AF relay systems in combination with relay antenna selection mechanism.

In this paper, a computationally-efficient layered relay-and-antenna selection (LRAS) strategy is developed based on an adaptive DSA scheme [10] to maximize the channel capacity of the two-way MIMO AF multi-relay system. The A-DSA RAS selection strategy operates in a two-stage mode to determine the transmit-and-receive antenna subset from the pre-selected single relay. However, this exhaustive search strategy exhibits an exponential increase in computational complexity. Fortunately, by means of a two-stage RAS algorithm, the overall complexity burden is reduced effectively. Moreover, simulation results depict the superior sum-rate performance of the proposed A-DSA LRAS method over other existing selection ones. *Notations:* Symbols for matrices (vectors) are denoted by boldface upper (lower) case letters.  $(\cdot)^*$ ,  $(\cdot)^T$ ,  $(\cdot)^H$ , and  $(\cdot)^{-1}$  define the complex conjugation, transpose, Hermitian transpose, and matrix inversion, respectively.  $E\{\cdot\}$  and  $\det(\cdot)$  indicate the expected-value and determinant operators, respectively.  $C^{M \times M}$  denotes space of  $M \times M$  matrices with complex entries.  $(\mathbf{A})_{i,j}$  represents the  $(i,j)$ th entry of matrix  $\mathbf{A}$ .  $\mathbf{B}^{1/2}$  is the square root of a nonnegative definite matrix  $\mathbf{B}$ .

## II. SYSTEM AND CHANNEL MODEL

The information-bearing signals are exchanged between two  $N$ -antenna terminals A and B assisted by  $K$  half-duplex  $R$ -antenna relays ( $R_k, k \in \Omega_R = \{1, 2, \dots, K\}$ ) in a two-way MIMO AF relaying system under correlated fading channels. The direct links between two terminals are assumed to be absent due to large path loss. In addition, a total of  $L$  RF chains are available for reception and transmission at the pre-selected single relay. Due to time slots devoted to reception and transmission independently, the receive antenna set is not necessarily the same as the transmit antenna set. Let  $\Gamma$  denote the source index, *i.e.*,  $\Gamma = A$  and B. Elements of all channel links of the terminal-to- $k$ th relay ( $\Gamma$ -R) channels  $\mathbf{H}_{\Gamma R,k} \in C^{R \times N}$  and the  $k$ th relay-to-terminal (R- $\Gamma$ ) channels  $\mathbf{H}_{R \Gamma,k} \in C^{N \times R}$  for all  $k \in \Omega_R$  are modeled based on the Kronecker correlation channel model [11], given by

$$\mathbf{H}_{\Gamma R, k} = \mathbf{R}_{\Gamma R, k}^{1/2} \mathbf{H}_{\Gamma R, k}^W \mathbf{T}_{\Gamma R, k}^{1/2}, \quad (1)$$

$$\mathbf{H}_{R \underline{\Gamma}, k} = \mathbf{R}_{R \underline{\Gamma}, k}^{1/2} \mathbf{H}_{R \underline{\Gamma}, k}^W \mathbf{T}_{R \underline{\Gamma}, k}^{1/2}, \quad (2)$$

where  $\mathbf{H}_{\Gamma R, k}^W \in C^{R \times N}$  and  $\mathbf{H}_{R \underline{\Gamma}, k}^W \in C^{N \times R}$  are, respectively, the  $\Gamma$ -R and the R- $\underline{\Gamma}$  spatially white complex Gaussian random matrices in which elements are independent identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit covariance.  $\mathbf{R}_{\Gamma R, k} \in C^{R \times R}$  and  $\mathbf{R}_{R \underline{\Gamma}, k} \in C^{N \times N}$  are the receive-side spatial correlation matrices and  $\mathbf{T}_{\Gamma R, k} \in C^{N \times N}$  and  $\mathbf{T}_{R \underline{\Gamma}, k} \in C^{R \times R}$  are the transmit-side correlation matrices. The perfect synchronization is assumed at all wireless links. The received signal vector at the  $i$ th relay during the first time slots is expressed as

$$\mathbf{y}_k = \mathbf{H}_{\Gamma R, k} \mathbf{s}_\Gamma + \mathbf{H}_{R \underline{\Gamma}, k} \mathbf{s}_{\underline{\Gamma}} + \mathbf{n}_k, \quad k \in \Omega_R, \quad (3)$$

where  $\mathbf{s}_\Gamma \in C^{N \times 1}$ ,  $\Gamma = A, B$ , are the source signal vectors with  $E\{\mathbf{s}_\Gamma \mathbf{s}_\Gamma^H\} = \frac{P_\Gamma}{N} \mathbf{I}_N$  and  $\mathbf{n}_k \in C^{R \times 1}$  stands for the ZMCSCG noise vector with the variance matrix  $E\{\mathbf{n}_k \mathbf{n}_k^H\} = \sigma_R^2 \mathbf{I}_R$  at relay. Here,  $P_\Gamma$ ,  $\Gamma = A, B$ , indicate the source transmission power. The received signals at both terminals in the second time slot are given as

$$\mathbf{y}_\Gamma = \sum_{k=1}^K (\rho_k \mathbf{H}_{R \Gamma, k} \mathbf{H}_{\Gamma R, k} \mathbf{s}_\Gamma + \rho_k \mathbf{H}_{R \Gamma, k} \mathbf{n}_k) + \mathbf{n}_\Gamma, \quad \Gamma = A, B, \quad (4)$$

where  $\mathbf{n}_\Gamma$  is the noise vector with the variance matrix  $E\{\mathbf{n}_\Gamma \mathbf{n}_\Gamma^H\} = \sigma_\Gamma^2 \mathbf{I}_N$  at terminal  $\Gamma$ . The power normalization coefficient of the  $k$ th relay  $\rho_k$  is given by

$$\rho_k = \sqrt{\frac{P_k}{\text{tr}\left\{\frac{R_k}{N} \mathbf{H}_{\Gamma R, k} \mathbf{H}_{\Gamma R, k}^H + \frac{R_k}{N} \mathbf{H}_{R \underline{\Gamma}, k} \mathbf{H}_{R \underline{\Gamma}, k}^H + \sigma_R^2 \mathbf{I}_R\right\}}}, \quad k \in \Omega_R, \quad (5)$$

where  $P_k$  denotes the transmit power for the  $k$ th relay node.

### III. LAYERED RAS STRATEGIES

#### A. Relay Selection

The lower bound ASR of the one-way wireless link from  $\underline{\Gamma}$  to  $\Gamma$  with the aid of the  $R_k$  relay node is given by [12]

$$C_{\underline{\Gamma} \Rightarrow \Gamma, k} = \frac{1}{2} E \left\{ \log_2 \det \left( \mathbf{I}_N + \frac{\rho_k^2 P_\Gamma}{N} \mathbf{H}_{R \Gamma, k}^H \mathbf{H}_{\Gamma R, k} \mathbf{H}_{\Gamma R, k}^H \mathbf{H}_{R \Gamma, k} \mathbf{R}_{z_{\Gamma, k}}^{-1} \right) \right\}, \quad (6)$$

where  $\mathbf{z}_{\Gamma, k} \triangleq \rho_k \mathbf{H}_{R \Gamma, k} \mathbf{n}_k + \mathbf{n}_\Gamma$  in (6) introduces an equivalent noise vector at terminal  $\Gamma$  with the corresponding covariance matrix  $\mathbf{R}_{z_\Gamma} = E\{\mathbf{z}_\Gamma \mathbf{z}_\Gamma^H\} = \rho_k^2 \sigma_R^2 \mathbf{H}_{R \Gamma, k} \mathbf{H}_{R \Gamma, k}^H + \sigma_\Gamma^2 \mathbf{I}_N$  and the factor 1/2 accounts for the half-duplex mode. Thus, the lower bound ASR of the two-way channel link between  $\underline{\Gamma}$  and  $\Gamma$  with the use of the  $R_k$  relay is described as

$$C_{\underline{\Gamma} \Rightarrow \Gamma, k} = C_{\underline{\Gamma} \Rightarrow \Gamma, k} + C_{\Gamma \Rightarrow \underline{\Gamma}, k}. \quad (7)$$

Note that  $C_{\underline{\Gamma} \Rightarrow \Gamma, k}$  is a function of channel matrices  $\mathbf{H}_{\Gamma R, k}$  and  $\mathbf{H}_{R \underline{\Gamma}, k}$ ,  $\Gamma = A, B$ . In order to maximize the ASR lower bound, the adaptive discrete stochastic approximation (A-DSA) is developed. Denote the set of all  $K$  possible candidate relays as  $I_R = \{R_1, R_2, \dots, R_K\}$ , the A-DSA RS scheme can be formulated as follows:

$$R_{k^*} = \arg \max_{R_k \in I_R} \xi_k, \quad (8)$$

where  $\xi_k \triangleq \frac{1}{2} \{I(\mathbf{y}_\Gamma; \mathbf{s}_\Gamma) + I(\mathbf{y}_\Gamma; \mathbf{s}_{\underline{\Gamma}})\} = \sum_{\Gamma=A}^B C_{\underline{\Gamma} \Rightarrow \Gamma, k} (\mathbf{H}_{R \Gamma, k}, \mathbf{H}_{R \Gamma, k})$ .

Here,  $I(x; y)$  defines the mutual information between  $x$  and  $y$ . Let  $\psi_R = \{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_K\}$  express the set of all  $K$  possible relay, where  $\mathbf{e}_k$  denotes a unit vector of length  $K$  with a one in the  $k$ th position and others are zero. For each iteration  $k$ , the DSA algorithm updates the  $K \times 1$  state occupation probability vector  $\pi[k] = [\pi[k, R_1], \pi[k, R_2], \dots, \pi[k, R_K]]^T$  with  $\sum_{k=1}^K \pi[k, R_k] = 1$  and the element  $\pi[k, R_k] \in [0, 1]$ . Let  $R^{(k)}$  denote the selected relay at iteration  $k$  and the sequence of relays  $\{R^{(k)}\}$  match to the sequence  $\{\mathbf{D}[k]\} \in \psi_R$  of unit vectors with  $\mathbf{D}[k] = \mathbf{e}_k$  if  $R^{(k)} = R_k$ ,  $k \in \Omega_R$ . In each iteration  $k$ , the A-DSA LRAS compares the sum rates between the currently elected relay  $R^{(k)}$  and the other relay  $\hat{R}^{(k)}$  chosen uniformly from the remaining ( $K-1$ ) possible candidates then updating  $\pi[k]$  if the condition of  $\xi_{\hat{R}^{(k)}} > \xi_{R^{(k)}}$  is met. The iteration process will repeat until the iteration times  $k=10K$  is reached. Finally, the A-DSA algorithm outputs the optimal relay node  $R_{k^*}$  associated with the location index with the largest element in  $\pi[k]$ .

#### B. Antenna Selection

In the sequel, LRAS selection schemes is applied once again to explore the  $L$ -antenna subset from the selected relay  $R_{k^*}$ . With the utilization of AS techniques,  $L$  out of  $R$  antennas are selected from the best relay  $R_{k^*}$  for reception and transmission. Notice that the receive antenna set is not necessarily the same as the transmit antenna set at relay  $R_{k^*}$ . Denote the set of all  $M = C_L^R \times C_L^R$  possible receive-and-transmit antenna-pair choices as  $I_A = \{\vartheta_1, \vartheta_2, \dots, \vartheta_M\}$ . Thus, the A-DSA AS strategy can be formulated as

$$\vartheta_{m^*} = \arg \max_{\vartheta_m \in I_A} \xi_{\vartheta_m}, \quad (9)$$

where  $\xi_{\vartheta_m} = \sum_{\Gamma=A}^B C_{\underline{\Gamma} \Rightarrow \Gamma, k} (\mathbf{H}_{R \Gamma, k}^{(\vartheta_m)}, \mathbf{H}_{R \Gamma, k}^{(\vartheta_m)})$ . The procedures for the A-DSA AS scheme are similar to the DSA RS scheme. The proposed A-DSA RAS scheme is able to asymptotically converge to the optimal solutions of the relay and the antenna subset.

### IV. NUMERICAL RESULTS

An AF-MIMO two-way multi-relay system equipped with multiple antennas at each node is considered over correlated fading channels. In realistic communication scenario of “ $l=1$ ” is evaluated to exploit the feasibility on practical applications in wireless sensor networks. The SNR is defined as  $P_\Gamma / \sigma_n^2$ . Parameters of  $P_\Gamma = P_\underline{\Gamma} = P_R$  and  $P_k = P_R / K$  are used for  $\Gamma = A, B$ , and all  $k \in \Omega_R$ , and the noise variance at

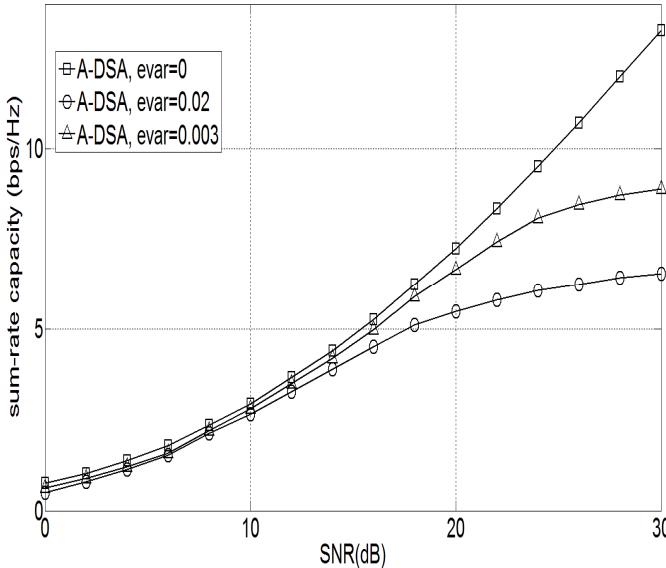


Fig. 1. Sum-rate capacity of an A-DSA RAS technique for an  $[N, R, K, I, L] = [2, 4, 10, 1, 2]$  two-way MIMO AF relaying system with spatial correlation coefficients  $\rho = 0.6$  in terms of SNR.

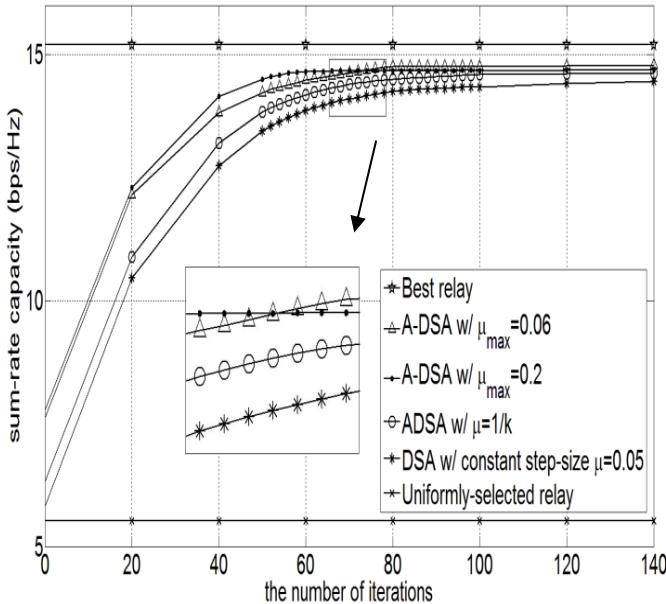


Fig. 2. Sum-rate convergence of the DSA RAS using adaptive step sizes for an  $[N, R, K, I, L] = [2, 4, 10, 1, 4]$  two-way MIMO AF relaying system in terms of iterations at SNR=30dB.

each single node is set to be  $\sigma_n^2 = \sigma_{n_r}^2 = \sigma_{n_k}^2 = 1$  for  $\Gamma = A, B$ . In addition, the 4 quadrature-amplitude-modulation (QAM) scheme is adopted in computer simulations. The  $(i,j)$ -th elements of both transmit and receive correlation matrices are  $\mathbf{R}_{i,j} = \mathbf{T}_{i,j} = \rho^{|i-j|}$  in accordance with the exponential correlation channel model [10]. The maximum numbers of iterations employed for both the A-DSA RS and AS algorithms are  $10K$  and  $10M$ , respectively. Parameters of  $\eta_R = \eta_A$ ,  $\mu_{\max}$ , and  $\mu_{\min}$  are, respectively, given as 0.0005, 0.06, and 0.03.

In Fig. 1, the sun-rate capacity of the A-DSA LRAS with

$[N, R, K, I, L, \rho] = [4, 4, 8, 3, 4, 0]$  is simulated.  $\sigma_e^2$  denotes the channel estimation-error variance for each node. In this simulation scenario, the channel estimation models are defined as

$$\begin{aligned}\mathbf{H}_{TR_i}^w &= \bar{\mathbf{H}}_{TR_i}^w + \Delta\mathbf{H}_{TR_i}^w \\ \mathbf{H}_{R_i\Gamma}^w &= \bar{\mathbf{H}}_{R_i\Gamma}^w + \Delta\mathbf{H}_{R_i\Gamma}^w, \quad \Gamma = A, B, i = 1, 2, \dots, K,\end{aligned}\quad (10)$$

where  $\bar{\mathbf{H}}_{TR_i}^w$  and  $\bar{\mathbf{H}}_{R_i\Gamma}^w$  are the estimation CSIs while  $\Delta\mathbf{H}_{TR_i}^w$  and  $\Delta\mathbf{H}_{R_i\Gamma}^w$  are the associated CSI estimation errors whose elements are modeled as i.i.d. ZMCSCG random variables each with covariance  $\sigma_{e,TR_i}^2$  and  $\sigma_{e,R_i\Gamma}^2$ , respectively. The ASR decreases in Fig. 1 as the channel estimation error increases. Thus, it is essential to take the antenna correlation and channel estimation error into account.

In Fig. 2, the convergence behavior and steady-state ASR comparisons of an adaptive DSA LRAS selection are shown in terms of the number of iterations with spatial correlation coefficient  $\rho = 0.6$  and  $\text{SNR} = 30\text{dB}$ . Simulation results demonstrate that an A-DSA LRAS algorithm with using a time-varying step size is able to provide a rapid convergence speed and a uniformly better steady-state ASR capacity than that of the DSA LRAS scheme with the use of a pre-selected step size. It is also noted that an A-DSA LRAS with the aid of a time-varying step size comes near to the best steady-state ASR status around 60 iteration rounds as compared with 100 and 140 iterations required by the ADSA LRAS counterpart with a time-varying step size  $\mu = 1/k$  and the DSA LRAS with a pre-determined step size  $\mu = 0.05$ . It is worth noting that an adaptive DSA RS with  $\mu_{\max} = 0.2$  produces a faster convergence speed but a worse steady-state ASR capacity than that of the same one with the use of  $\mu_{\max} = 0.06$ . Therefore, the choice of  $\mu_{\max}$  serves as a trade-off between the convergence rate and the steady-state ASR performance.

## V. CONCLUSIONS

In this paper, a layered A-DSA RAS strategy is proposed to enhance the ASR for a two-way MIMO multiple-AF-relay system over correlated fading channels. Simulation results illustrate that the A-DSA LRAS algorithm with the use of a time-varying step size is capable of providing a rapid convergence rate and a uniformly better steady-state ASR capacity than other DSA-based LRAS techniques with either a time-varying or constant step size.

## REFERENCES

- [1] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379-389, Feb. 2007.
- [2] S. W. Peters and R. W. Heath, Jr., "Nonregenerative MIMO relaying with optimal transmit antenna selection," *IEEE Signal Process. Lett.*, vol. 15, pp. 421-424, 2008.
- [3] H. Eghbali, S. Muhamadat, S. A. Hejazi, and Y. Ding, "Relay selection strategies for single-carrier frequency-domain equalization multi-relay cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2034-2045, May 2013.
- [4] J.-C. Chen and C.-K. Wen, "Near-optimal relay subset selection for two-way amplify-and-forward MIMO relaying systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, pp. 37-42, Jan. 2011.

- [5] M. Seyfi, S. Muhandat, and J. Liang, "Relay selection in cognitive radio networks with interference constraints," *IET Commun.*, vol. 7, no. 10, pp. 922-930, Mar. 2013.
- [6] G. Liu, H. Ji, Y. Li, and X. Zhang, "Sum rate maximization antenna selection via discrete stochastic approximation in MIMO two-way AF relay with imperfect CSI," in *Proc. IEEE Global Commun. Conf.*, Anaheim, CA, Dec. 2012, pp. 2487-2492.
- [7] S.-J. Lu, R. Y. Chang, and W.-H. Chung, "Distributed relay selection for virtual MIMO in spectral efficient broadcasting networks," in *Proc. IEEE Wireless Commun. and Networking Conf.*, Shanghai, China, Apr. 2013, pp. 3688-3692.
- [8] H. Cui, L. Song, and B. Jiao, "Multi-pair two-way amplify-and-forward relaying with very large number of relay antennas," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2636-2645, May 2014.
- [9] X. Zhou, B. Bai, W. Chen, and Y. Han, "On energy efficiency maximization of AF MIMO relay systems with antenna selection," in *Proc. IEEE Global Conf. on Signal and Inform. Process. (GlobalSIP)*, Atlanta, Georgia, Dec. 2014, pp. 88-92.
- [10] I. Berenguer, X. Wang, and V. Krishnamurthy, "Adaptive MIMO antenna selection via discrete stochastic optimization," *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4315-4329, Nov. 2005.
- [11] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502-513, Mar. 2000.
- [12] B. Hassibi and B. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Trans. Inform. Theory*, vol. 49, no. 4, pp. 951-963, Apr. 2003.