Verification of Iterative Algorithms for Electrical-Thermal Analysis of Power Delivery Network

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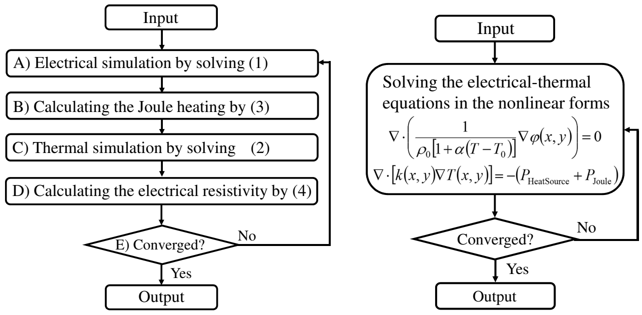
Abstract: In this paper, electrical-thermal co-simulations of a power delivery network (PDN) are performed by two types of iterative algorithms: Newton's method and an iterative electrical-thermal co-simulation (IETC) method. The conventional Newton's method directly solves the nonlinear problem of the co-simulation by a well-known iterative algorithm. On the other hand, the IETC method transforms the original problem to linear systems of equations and solves them by an iterative procedure. We show some numerical results of an example PDN to compare the efficiency of the IETC method and that of the conventional Newton's method.

Keywords— **electrical-thermal co-simulation, iterative algorithm, iterative electrical-thermal co-simulation (IETC) method, Newton's method, power delivery network (PDN)**



(a) (b)

Figure 1. The procedures of the electrical-thermal co-simulation based on (a) the IETC method and (b) Newton's method.

1. Introduction

The electrical behavior of an integrated circuit (IC) is affected by heating effects, and the temperature distribution on a conductor plane of a power delivery network (PDN) depends on the electrical power dissipated on the plane. These electrical-thermal relationships indicate that co-simulation techniques considering both electrical and thermal effects are required to verify the power and thermal integrity of the PDN. In general, for such a co-simulation, we have to solve a nonlinear system of equations. Nonlinear equations can be solved by using the well-known Newton's method, which uses the iterative algorithm of quadratic convergence. Recently, the iterative electrical-thermal co-simulation (IETC) method has been proposed to verify hybrid electrical-thermal phenomena [1]. The IETC method is the iterative method which is as accurate as Newton's method. However, the superiority of the IETC method over the conventional Newton's method has not been discussed well, especially from the viewpoint of the computational efficiency. In this paper, we compare these two iterative methods in terms of the number of the iterations and the CPU time in the electrical-thermal co-simulation.

2. Electrical-thermal co-simulation

The governing equation for the steady-state voltage distribution is expressed as

$$\nabla \cdot \left(\frac{1}{\rho(x,y,T)} \nabla \phi(x,y)\right) = 0 \tag{1}$$

where $\rho(x,y,T)$ is the temperature dependent electrical resistivity, and $\phi(x,y)$ is voltage distribution.

On the other hand, for steady state temperature distribution, the governing heat equation is written as

$$\nabla \cdot (k(x,y) \nabla T(x,y)) = -P(x,y) \tag{2}$$

where $k(x,y,T)$ is the thermal conductivity of solid medium, $T(x,y)$ is the temperature distribution, and $P(x,y)$ is the heat source including the external heat and the Joule heating generated by the Ohmic loss in conductors.

Additionally, the electrical resistivity ρ is dependent on a temperature, and the Joule heating $P_{\mathrm{Joule}}(x,y)$ depends on the electric field $\vec{E}$ and the current density $\vec{J}$ as

$$P_{\mathrm{Joule}}(x,y) = \vec{J}(x,y) \cdot \vec{E}(x,y) = \frac{1}{\rho}(\nabla \phi(x,y))^2 \tag{3}$$

$$\rho = \rho_0[1 + \alpha(T - T_0)] \tag{4}$$

where T_0 is the reference temperature on the conductors, ρ_0 is the resistivity at T_0, and α is the temperature coefficient of a conductor. It can be seen from (4) that the electrical resistivity increases when the temperature rises.

If (4) is substituted into (1), T appears in the denominator of the coefficient of (1). Additionally, if (3) is substituted into (2), T again appears in the denominator, and (2) includes the square of the gradient of ϕ. Therefore, assuming that T and ϕ are the variables, (1) and (2) have to be solved together and form a nonlinear system of equations. To solve this problem, we adopt two methods: One is Newton's method, which is used for a general nonlinear problem. The other is the IETC method proposed in [1].

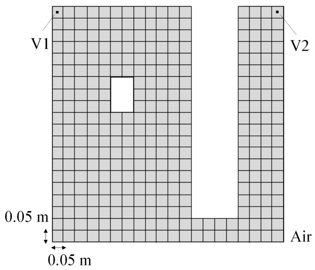
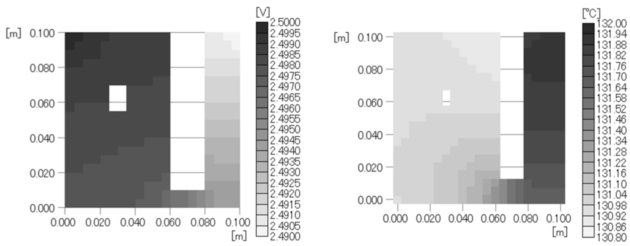


Figure 2. The square meshes for the example plane of the PDN.

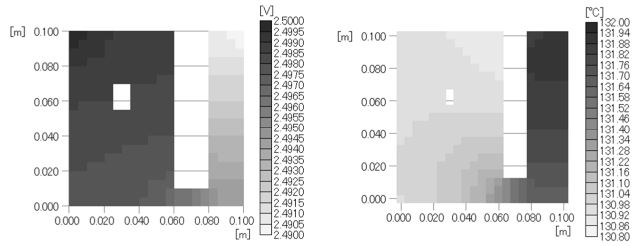


(a) (b)

Figure 3. The results of the IETC method. (a) Voltage distribution. (b) Temperature distribution.

In the case of Newton's method, we solve the nonlinear system of equations composed of (1) and (2) along with (3) and (4). The procedure of the electrical-thermal co-simulation based on Newton's method is shown in Figure 1(b). In this procedure, given the initial values, the iterative variables of the voltages and temperatures are calculated by solving (1) and (2) by means of the conventional Newton's method.

On the other hand, the IETC method follows the procedure in Figure 1(a). One of the important points of this procedure is that it transforms the original nonlinear problem into a set of linear equations. The procedure of the IETC method proceeds as follows: In Step A), the electrical simulation is performed to obtain the voltage distribution by solving (1). In this step, the temperature is assumed to be constant, and therefore, the nonlinear equation (1) becomes a linear one: if the temperature in the denominator in (1) is constant, there is no temperature variable and nonlinear term in (1). Furthermore, due to this fact, (1) can be solved separately from (2). In Step B), the heat source including the Joule heating is calculated by using the voltage distribution and (3). In Step C), the thermal simulation is performed with the previously-calculated heat sources to obtain the temperature distribution by assuming the voltage is constant. This assumption transforms (2) into a linear one as with (1) in Step A). In Step D), the electrical resistivity is updated by using the temperature distribution and (4). Step E) checks the convergence of the iterative solutions of the voltages and temperatures, and if converged, the iterative calculation is terminated; otherwise, the above steps are repeated.



(a) (b)

Figure 4. The results of Newton's method. (a) Voltage distribution. (b) Temperature distribution.

Table 1. Number of iterations and CPU time.

Methods	# of iterations	CPU time (ms)
IETC method	28	54
Newton's method	61	205

3. Numerical Results

To compare the two methods, we calculate the steady-state voltage and thermal distribution on the copper plane of the example PDN shown in Figure 2. We apply 2.5 V to V1 and 2.49 V to V2 in Figure 2. The initial temperature of the plane and the ambient temperature are 20 °C. In addition, the resistivity ρ_0 of the plane is 1.68×10^{-8} Ω·m, the temperature coefficient α is 4.4×10^{-3}, and the thermal conductivity k is 400 W/(m·K). The heat transfer coefficient of 5 W/($\mathrm{m}^2 \cdot \mathrm{K}$) is used for the convection boundary. The plane is discretized by using the 0.05 m square meshes as illustrated in Figure 2. In the simulation, we assume that the voltage variable is placed at the center of a square cell, and the temperature variable is at each corner of the cell. In other words, the voltage and temperature variables are staggered on the plane. The iterations of both methods are terminated if the norm of the variable vector is less than 10^{-9}. The numbers of the voltage and temperature variables are 320 and 385.

The voltage and temperature distributions obtained by the two methods are shown in Figures 3 and 4. It is confirmed that both voltage and temperature distributions of the IETC method are almost the same as those of Newton's method. The CPU times and the numbers of the iterations are listed in Table 1. Table 1 indicates that the IETC method is about 4 times faster than Newton's method in the electrical-thermal co-simulation of the example PDN.

4. Conclusion

The two types of electrical-thermal co-simulation methods, the IETC method and Newton's method, have been performed and compared with each other. The numerical results showed that the IETC method was more efficient than Newton's method in the example co-simulation.

It should be noted that the above conclusion makes sense only for the simple example in this paper. Therefore, we can not assert that Newton's method is always less efficient for the PDN simulation. We are planning to provide theoritical discussions about the both co-simulation methods in the future.

References

[1] J. Xie and M. Swaminathan, “Electrical-thermal co-simulation of 3D integrated systems with micro-fluidic cooling and Joule heating effects,” *IEEE Trans. Compon., Packag. Manuf. Technol.*, vol. 1, no. 2, pp.234-246, Feb. 2011.