THP Based on Cholesky Factorization with Unitary Transformation for Multiuser MIMO

Satoshi Denno* and Tatsuya Itakura*
* Graduate School of Natural Science and Technology, Okayama University,
3-1-1, Tsushima-Naka, Kita-ku, Okayama 700-8530, Japan
Email: denno@okayama-u.ac.jp

***Abstract*—This paper proposes two types of Tomlinson-Harashima precoding (THP) based on Cholesky factorization for multiuser MIMO systems where the receiver has at least two antennas. In the system with the THP, a linear filter is introduced for the receiver. It is shown by computer simulation that the BER performance of the THP is much superior to that of the conventional THP.**

I. Introduction

Multiuser MIMO (MU-MIMO) has been considered for high speed wireless communication in the IEEE wireless local area networks and the cellular systems, since MU-MIMO can enhance system throughput without additional spectrum [1]. The block diagonalization technique, a representative of linear MU-MIMO techniques, has been intensively investigated [2]. On the other hand, non-linear MU-MIMO techniques have been proposed, for instance, the vector perturbation [4] and the Tomlinson-Harashima precoding (THP) [5], [6] because non-linear MU-MIMO has been shown to achieve much better performance than linear MU-MIMO [3]. Especially, because THP achieves superior performance with relatively lower computational load, techniques to improve THP have been proposed [7]. However, the signal to noise ratio of the signal received at the user is different from each other in most THP. Even though the THP achieves superior performance in terms of system throughput, the THP can not provide users with signals having equal quality.

This paper proposes two types of THP based on Cholesky factorization that equalize the SNR of the received signals at all the users in the MU-MIMO system where the user terminal has at least two antennas.

II. Tomlinson-Harashima Precoding based on Cholesky Factorization for Multiuser MIMO

We assume that an MU-MIMO where N_{T} antennas and N_{R} antennas are installed at a transmitter on an access point and a receiver on a user terminal, and M terminals surround the access point. We set $N_{\mathrm{T}} = MN_{\mathrm{R}}$ for simplifying the following notation. Let $\mathbf{Y}_k \in \mathbb{C}^{N_{\mathrm{R}}\times 1}$ and $\mathbf{N}_k \in \mathbb{C}^{N_{\mathrm{R}}\times 1}$ denote a received signal vector and a noise vector at the kth receiver, the received signal vector containing all the signals received at the user terminals, $\mathbf{Y} \in \mathbb{C}^{MN_{\mathrm{R}}\times 1}$, and the noise vector containing all these noise vectors, $\mathbf{N} \in \mathbb{C}^{MN_{\mathrm{R}}\times 1}$, are defined as $\mathbf{Y} = \left(\mathbf{Y}_1{}^{\mathrm{T}} \cdots \mathbf{Y}_M{}^{\mathrm{T}}\right)^{\mathrm{T}}, \mathbf{N} = \left(N_1{}^{\mathrm{T}} \cdots N_M{}^{\mathrm{T}}\right)^{\mathrm{T}}$, where superscript T represents transpose of a vector. As is well-known, the MU-MIMO system can be modeled as,

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}. \tag{1}$$

In (1), $\mathbf{X} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ and $\mathbf{H} \in \mathbb{C}^{MN_{\mathrm{R}}\times N_{\mathrm{T}}}$ represent a transmission signal vector and a channel matrix, which is defined as $\mathbf{H} = \left(\mathbf{H}_1{}^{\mathrm{T}} \cdots \mathbf{H}_M{}^{\mathrm{T}}\right)^{\mathrm{T}}$, where $\mathbf{H}_k \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{T}}}$ represents a sub-channel matrix to the kth receiver. When the THP is applied, the transmission signal vector $\mathbf{X}$ is expressed with a precoding matrix $\mathbf{F} \in \mathbb{C}^{N_{\mathrm{T}}\times MN_{\mathrm{R}}}$ as follows.

$$\mathbf{X} = \beta\mathbf{F}\mathbf{V} \tag{2}$$

$\mathbf{V} \in \mathbb{C}^{MN_{\mathrm{R}}\times 1}$ and β in (2) denote a precoder input vector and a normalization coefficient defined as,

$$\beta = \sqrt{\frac{E_{\mathrm{s}}}{\sum_{i=1}^{MN_{\mathrm{R}}} \sigma_{V_i}^2 |\mathbf{F}_i|^2}}. \tag{3}$$

$\mathbf{F}_i$, $\sigma_{V_i}^2$ and E_{s} in (3) represent the ith column of the matrix $\mathbf{F}$, the power of ith entry in the vector $\mathbf{V}$ and the desired transmission power. Let $\mathbf{S} \in \mathbb{C}^{MN_{\mathrm{R}}\times 1}$ denote a vector containing modulation signals, the vector $\mathbf{V}$ is defined as,

$$\mathbf{V} = \mathbf{P}\mathbf{S} - \mathbf{B}\mathbf{V}. \tag{4}$$

In (4), $\mathbf{B} \in \mathbb{C}^{MN_{\mathrm{R}}\times MN_{\mathrm{R}}}$ and $\mathbf{P} \in \mathbb{C}^{MN_{\mathrm{R}}\times MN_{\mathrm{R}}}$ are a feedback filter and a permutation matrix defined below. In this paper, the desired transmission power is defined as $E_{\mathrm{s}} = \mathrm{E}[\mathbf{S}^{\mathrm{H}}\mathbf{S}]$ where $\mathrm{E}[a]$ denotes an ensemble average of a. The precoding matrix $\mathbf{F}$, the permutation matrix $\mathbf{P}$ and the feedback filter $\mathbf{B}$ are obtained in the following techniques.

III. Proposed THP

A. SVD for the Sub-Channel Matrix

The sub-channel matrix to kth receiver $\mathbf{H}_k$ is decomposed with the singular value decomposition (SVD).

$$\mathbf{H}_k = \mathbf{W}_k^{\mathrm{H}}\mathbf{\Gamma}_k\mathbf{Q}_k \tag{5}$$

Superscript H, $\mathbf{W}_k \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{R}}}$, $\mathbf{Q}_k \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{T}}}$, and $\mathbf{\Gamma}_k \in \mathbb{R}^{N_{\mathrm{R}}\times N_{\mathrm{R}}}$ denote Herimitian transpose, unitary matrices, and a diagonal matrix. Let $\mathbf{W} \in \mathbb{C}^{MN_{\mathrm{R}}\times MN_{\mathrm{R}}}$ denote a unitary matrix defined as $\mathbf{W} = \mathrm{diag}[\mathbf{W}_0, \cdots \mathbf{W}_{M-1}]$ that has the vectors $\mathbf{W}_k$ in the diagonal positions, a matrix $\mathbf{\Psi}_{\mathrm{u}} \in \mathbb{C}^{MN_{\mathrm{R}}\times MN_{\mathrm{R}}}$ is defined as,

$$\mathbf{\Psi}_{\mathrm{u}} = \mathbf{W}\mathbf{H}(\mathbf{W}\mathbf{H})^{\mathrm{H}} + \gamma^{-1}\mathbf{I}, \tag{6}$$

where $\gamma = \frac{E_{\mathrm{s}}}{\mathrm{E}[\mathbf{N}^{\mathrm{H}}\mathbf{N}]}$. The matrix $\mathbf{\Psi}_{\mathrm{u}}$ is Cholesky-factorized as follows.

$$\mathbf{P}\mathbf{\Psi}_{\mathrm{u}}^{-1}\mathbf{P}^{\mathrm{T}} = \mathbf{L}_{\mathrm{u}}^{\mathrm{H}}\mathbf{D}_{\mathrm{u}}\mathbf{L}_{\mathrm{u}} \tag{7}$$

$(\bullet)^{-1}$, $\mathbf{D}_{\mathrm{u}} \in \mathbb{R}^{MN_{\mathrm{E}} \times MN_{\mathrm{E}}}$, and $\mathbf{L}_{\mathrm{u}} \in \mathbb{C}^{MN_{\mathrm{R}} \times MN_{\mathrm{R}}}$ in (7) represent inverse, a diagonal matrix and a lower triangular matrix, respectively. However, all the diagonal entries are 1 in the lower triangular matrix $\mathbf{L}_{\mathrm{u}}$. In addition, the permutation matrix is determined that minimizes the trace of the diagonal matrix $\mathbf{D}_{\mathrm{u}}$. The precoding matrix $\mathbf{F}$ is defined with the matrices introduced above.

$$\mathbf{F} = \mathbf{H}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{P}^{\mathrm{T}}\mathbf{L}_{\mathrm{u}}^{\mathrm{H}}\mathbf{D}_{\mathrm{u}} \tag{8}$$

In addition, the feedback filter $\mathbf{B}$ is defined as,

$$\mathbf{B} = \mathbf{L}_{\mathrm{u}}^{-1} - \mathbf{I}, \tag{9}$$

where $\mathbf{I} \in \mathbb{R}^{MN_{\mathrm{E}} \times MN_{\mathrm{R}}}$ represents the identity matrix. An estimated transmission signal vector $\bar{\mathbf{S}}_k$ is obtained at the kth receiver as $\bar{\mathbf{S}}_k = \mathrm{M}\left[\beta^{-1}\mathbf{W}_k^{\mathrm{H}}\mathbf{Y}_k\right]$ where $\mathrm{M}\left[\bullet\right]$ represents the modulo operation [5], [6].

B. Triangulation for the Sub-Channel Matrix

A matrix $\mathbf{\Psi}_{\mathrm{t}} \in \mathbb{C}^{MN_{\mathrm{R}} \times MN_{\mathrm{R}}}$ is defined and is Cholesky-factorized as,

$$\mathbf{\Psi}_{\mathrm{t}} = \mathbf{H}\mathbf{H}^{\mathrm{H}} + \gamma^{-1}\mathbf{I} = \mathbf{L}_{\mathrm{t}}^{\mathrm{H}}\mathbf{D}_{\mathrm{t}}\mathbf{L}_{\mathrm{t}}. \tag{10}$$

$\mathbf{D}_{\mathrm{t}} \in \mathbb{R}^{MN_{\mathrm{R}} \times MN_{\mathrm{R}}}$ and $\mathbf{L}_{\mathrm{t}} \in \mathbb{C}^{MN_{\mathrm{R}} \times MN_{\mathrm{R}}}$ in (10) are defined in the similar manners as $\mathbf{D}_{\mathrm{u}}$ and $\mathbf{L}_{\mathrm{u}}$. The lower triangular matrix $\mathbf{L}_{\mathrm{t}}$ is further decomposed as,

$$\mathbf{L}_{\mathrm{t}} = \tilde{\mathbf{L}}_{\mathrm{t}}\dot{\mathbf{L}}_{\mathrm{t}}. \tag{11}$$

$\dot{\mathbf{L}}_{\mathrm{t}} = \mathrm{diag}[\dot{\mathbf{L}}_{\mathrm{t},1} \cdots \dot{\mathbf{L}}_{\mathrm{t},M}]$ where $\dot{\mathbf{L}}_{\mathrm{t},k} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{R}}}$ denotes a lower triangular matrix which is a transformed sub-channel matrix of the kth user. Cholesky factorization is applied as,

$$\mathbf{P}\left(\tilde{\mathbf{L}}_{\mathrm{t}}^{\mathrm{H}}\mathbf{D}_{\mathrm{t}}\tilde{\mathbf{L}}_{\mathrm{t}}\right)^{-1}\mathbf{P}^{\mathrm{T}} = \ddot{\mathbf{L}}_{\mathrm{t}}^{\mathrm{H}}\ddot{\mathbf{D}}_{\mathrm{t}}\ddot{\mathbf{L}}_{\mathrm{t}}. \tag{12}$$

The optimum permutation matrix $\mathbf{P}$ is also determined so as to minimize the trace of $\ddot{\mathbf{D}}_{\mathrm{t}}$. With the matrices introduced above, the precoding matrix is obtained as follows.

$$\mathbf{F} = \mathbf{H}^{\mathrm{H}}\dot{\mathbf{L}}_{\mathrm{t}}^{-1}\mathbf{P}^{\mathrm{T}}\ddot{\mathbf{L}}_{\mathrm{t}}^{\mathrm{H}}\ddot{\mathbf{D}}_{\mathrm{t}} \tag{13}$$

The feed back filter $\mathbf{B}$ is obtained by substituting $\ddot{\mathbf{L}}_{\mathrm{t}}$ for $\mathbf{L}_{\mathrm{u}}$ in (9). While the THP proposed in the section prevents inter-user interference, the precoder only transforms the sub-channel matrix to the lower triangular matrix $\dot{\mathbf{L}}_{\mathrm{t},k}$. Therefore, the V-BLAST is applied for all the receivers.

IV. Computer Simulation

4 antennas and 2 antennas are placed at the transmitter and the receives, i.e., $N_{\mathrm{T}} = 4$ and $N_{\mathrm{R}} = 2$, respectively. We assume that 2 users with the receiver exist within a coverage area of the transmitter, i.e, $M = 2$. However, an average channel gain between the transmitter and the receivers is set equal. Rayleigh fading based on the Jake's model is applied for each path. Fig.1 shows the BER performance of the proposed THP in comparison with the conventional THP. Because the SNR of all the received signals is equalized by the proposed THP, the proposed THP achieves much better performance than the conventional THP. The performance of the THP with the SVD (UC-THP) is a little bit superior to that of the THP with the triangulation (BC-THP).

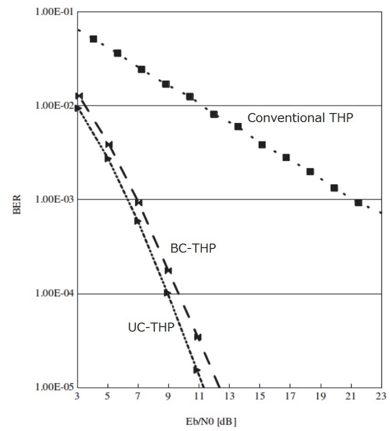

Fig. 1. BER performance comparison of THP

V. Conclusion

This paper proposes two types of THP based on Cholesky factorization for MU-MIMO systems where the terminal has at least two antennas. Actually, a THP with the SVD is proposed as well as that with the triangulation. The BER performance of the proposed THP is evaluated by computer simulation. As a result, the THP with the SVD achieves much better BER performance than the conventional THP.

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