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Stable Sparse Channel Estimation Algorithm under Non-Gaussian Noise Environments

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Abstract—**Broadband frequency-selective fading channels usually exhibit the inherent sparse structure distribution in spread time-domain. By exploiting the sparsity, adaptive sparse channel estimation (ASCE) algorithms, e.g., least mean square with reweighted L1-norm constraint (LMS-RL1) algorithm, can bring a considerable performance gain under the assumption of additive white Gaussian noise (AWGN). In the scenarios of real wireless communication systems, however, channel estimation performance is often deteriorated by the unexpected non-Gaussian mixture noises which usually include AWGN and impulsive noises. To design stable communication systems, we propose sign LMS-RL1 (SLMS-RL1) channel estimation algorithm to remove the non-Gaussian noises and to exploit channel sparsity simultaneously. In addition, the regularization parameter (REPA) selection for SLMS-RL1 algorithm is proposed via Monte Carlo method. Simulation results are provided to corroborate our studies.**

Keywords—SLMS-RL1 algorithm; regularization parameter selection; adaptive sparse channel estimation; Gaussian mixture model (GMM).

I. Introduction

Broadband transmission is becoming more and more important in advanced wireless communications systems [1]–[3]. The main impairments in wireless systems are due to multipath propagation as well as harmful additive noises. In such circumstances, accurate channel state information (CSI) is required for stable coherence signal detection [4]. Based on the assumption of Gaussian noise model, second-order statistics based least mean square (LMS) algorithm and its variants have been widely applied in channel estimation due to its simplicity and robustness [5][6]. However, the performance of LMS is usually limited by potential impulsive noises in advanced wireless systems [7][8]. These kinds of impulsive noises are often generated from natural or man-made electromagnetic waves, usually has a long tail distribution and violates the commonly used Gaussian noise assumption [9]. Without loss of generality, Gaussian mixture noise model (GMM) has been used to describe non-Gaussian noise system [8].

To mitigate the harmful GMM noises, it is necessary to develop robust channel estimation algorithms. Based on the assumption of dense finite impulse response (FIR), recently, several effective adaptive channel estimation algorithms have been proposed to achieve the robustness against impulsive interferences [6][10]–[12]. In [6], standard sign least mean absolute (SLMS) is proposed to suppress impulsive noise with using sign LMS algorithm. In [10], an useful standard affine projection sign algorithm (APSA) is proposed to mitigate impulsive noise. In [11], Yoo et. al. propose an improved APSA algorithm by deriving approximate optimal step-size. In [12], Li et. al. propose an effective variable step-size (VSS) sign algorithm for stable channel estimation under Gaussian mixture noise environment. The performance gain is obtained by adjusting the step-size via gradient-based weighted average of the sign algorithm. However, FIR of the real wireless channel is often modeled as sparse or cluster-sparse and hence many of channel coefficients are zero [13]–[17]. Hence, these algorithms may not exploit the sparse structure information. Indeed, some potential performance gain could be obtained if adopting advanced adaptive channel estimation algorithms.

To exploit channel sparsity as well as to remove GMM noises, we proposed a stable sign least mean square algorithm by using reweighted L1-norm constraint (SLMS-RL1), which is one of most effective sparse constraint functions in compressive sensing [18]. It is well known that regularization parameter (REPA) is one of critical parameters to control the performance of SLMS-RL1 algorithm. This paper proposed a Monte Carlo based selection method to choose an appropriate REPA so that SLMS-RL1 algorithm can achieve estimation performance gain as much as possible and also can ensure stable convergence under different GMM noise levels. Simulations results are given to verify the effectiveness of the proposed algorithm.

The rest of the paper is organized as follows. In Section II, we introduce GMM-induced adaptive sparse system model and propose SLMS-RL1 algorithm in Section III. In Section IV, Monte Carlo based computer simulations are given to select REPA. Then, SLMS-RL1 algorithm using the proposed REPA is compared with benchmarking algorithms, i.e. LMS, SLMS and LMS-RL1. Finally, Section V concludes the paper and brings forward the future work.

II. System Model

Consider an additive noise interference channel, which is modeled by the unknown N-length finite impulse response (FIR) vector $\boldsymbol{w}=[w_0,w_1,\cdots,w_{N-1}]^T$ at discrete time index n. Hence, the received signal can be expressed as

$$d(n)=\boldsymbol{x}^T(n)\boldsymbol{w}+z(n), \tag{1}$$

where $\boldsymbol{x}(n)=[x(n),x(n-1),\cdots,x(n-N+1)]^T$ is the input signal vector of the N most recent input samples; $\boldsymbol{w}$ is an N-dimensional column vector of the unknown system that we wish to estimate, and $z(n)$ is non-Gaussian noise which can be described by Gaussian mixture model (GMM) [8] as

$$p\left(z(n)\right)=\left(1-\phi\right)\cdot\mathcal{CN}\left(0,\sigma_n^2\right)+\phi\cdot\mathcal{CN}\left(0,T\sigma_n^2\right), \tag{2}$$

where $T\gg 1$ denotes the impulsive noise strength and $\mathcal{CN}(0,\sigma_n^2)$ denotes the Gaussian distributions with zero mean and variance σ_n^2, and ϕ is the mixture parameter, which can decide the level of impulsive noise in GMM. According to (2), one can find that stronger impulsive noises could be described by larger T as well as larger ϕ. Hence, variance of GMM noise $z(n)$ can be denoted as

$$\sigma_z^2=E\left(z^2(n)\right)=(1-\phi)\sigma_v^2+\phi T\sigma_v^2. \tag{3}$$

Note that $z(n)$ reduces to Gaussian noise if $\phi=0$. The objective of the adaptive channel estimation is to perform adaptive estimate of $\boldsymbol{w}(n)$ with limited complexity and memory given sequential observation $\{d(n),\boldsymbol{x}(n)\}$ in the presence of additive GMM noise $z(n)$. According to (1), instantaneous estimation error $e(n)$ can be written as

$$e(n)=d(n)-\boldsymbol{w}^T(n)\boldsymbol{x}(n), \tag{4}$$

where $\boldsymbol{w}(n)$ is the estimator of $\boldsymbol{w}$ at iteration n and $\boldsymbol{v}(n)=\boldsymbol{w}(n)-\boldsymbol{w}$.

III. Proposed SLMS-RL1 Algorithm

To estimate $\boldsymbol{w}(n)$ under GMM noise environments, a stable SLMS-RL1 channel estimation algorithm is proposed. Firstly, cost function of the algorithm is written as

$$G(n)=\underbrace{\left\|e(n)\right\|_1}_{\text{update error}}+\lambda\underbrace{\left\|\boldsymbol{f}(n)\boldsymbol{w}(n)\right\|_1}_{\text{sparse constraint}}, \tag{5}$$

where λ denotes a positive REPA which can balance the update error term and the sparse constraint term in (5), and the vector $\boldsymbol{f}(n)$ are defined as

$$[\boldsymbol{f}(n)]_i=\frac{1}{\delta_r+\left|[\boldsymbol{w}(n-1)]_i\right|},\ i=0,1,\cdots,N-1, \tag{6}$$

where δ_r being some positive number and hence $[\boldsymbol{f}(n)]_i>0$ for $i=0,1,\cdots,N-1$. The update equation can be derived by differentiating (5) with respect to $\boldsymbol{w}(n)$. Then, the resulting update equation is:

$$\begin{aligned}\boldsymbol{w}(n+1)&=\boldsymbol{w}(n)-\mu\frac{\partial G(n)}{\partial\boldsymbol{w}(n)}\\&=\boldsymbol{w}(n)+\mu\boldsymbol{x}(n)\operatorname{sgn}\left(e(n)\right)-\frac{\rho\operatorname{sgn}\left(\boldsymbol{w}(n)\right)}{\delta_r+\left|\boldsymbol{w}(n-1)\right|},\end{aligned} \tag{7}$$

where $\rho=\mu\lambda$. In Eq. (7), since $\operatorname{sgn}\left(\boldsymbol{f}(n)\right)=\boldsymbol{1}_{1\times N}$, hence one can get $\operatorname{sgn}\left(\boldsymbol{f}(n)\boldsymbol{w}(n)\right)=\operatorname{sgn}\left(\boldsymbol{w}(n)\right)$. Note that although the weight vector $\boldsymbol{w}(n)$ changes in every stage of this sparsity-aware SLMS-RL1 algorithm, it does not depend on $\boldsymbol{w}(n)$, and the cost function $G(n)$ is convex. In (7), we can find that REPA selection is an important step for designing SLMS-RL1 channel estimation algorithm. In other words, suitable REPA can ensure SLMS-RL1 algorithm to exploit channel sparity efficiently, and vice versa.

Tab. 1. Simulation parameters.

Parameters	Values
Training signal	Pseudo-random Binary sequences
Channel length	$N=80$
No. of nonzero coefficients	$K\in\{4,8,16\}$
Distribution of nonzero coefficient	Random Gaussian $\mathcal{CN}(0,1)$
Received SNR for channel estimation	$SNR=10\text{dB}\}$
GMM noise distribution	$\alpha_1=\alpha_2=0,\ \sigma_1^2=10^{(-SNR/10)}$ $\sigma_2^2=T\sigma_1^2, T\in\{200,400,600\}$
Step-size	$\mu=0.01$
Threshold of the (S)LMS-RL1	$\delta_r=0.05$

IV. Monte-Carlo bsaed REPA Selection Method and Numerical Simulations

In this section, the proposed SLMS-RL1 algorithm is evaluated in different scenarios: SNR, impulsive-noise strength T , mixture parameters ϕ as well as channel sparsity K . For achieving average performance, $M=1000$ independent Monte-Carlo runs are adopted. The simulation setup is configured according to the typical broadband wireless communication system [3]. The signal bandwidth is 50MHz located at the central radio frequency of 2.1GHz. The maximum delay spread of $0.8\mu s$. Hence, the maximum length of channel vector $\boldsymbol{w}$ is $N=80$ and its number of dominant taps is set to $K\in\{2,4,8,16\}$. To validate the effectiveness of the proposed algorithms, average mean square error (MSE) standard is adopted. Channel estimators are evaluated by average MSE which is defined by

$$\text{MSE}\left\{\boldsymbol{w}(n)\right\}=10\log_{10}\left\{E\left\|\boldsymbol{w}(n)-\boldsymbol{w}\right\|_2^2\right\}, \tag{8}$$

where $\boldsymbol{w}$ and $\boldsymbol{w}(n)$ are the actual signal vector and reconstruction vector, respectively. The results are averaged over $M=1000$ independent Monte-Carlo runs. Each dominant channel tap follows random Gaussian distribution as $\mathcal{CN}(0,\sigma_w^2)$ which is subject to $E\{\|\boldsymbol{w}\|_2^2\}=1$ and their positions are randomly decided within the $\boldsymbol{w}$. The received SNR is defined as P_0/σ_z^2, where P_0 is the received power of the pseudo-random (PN) binary sequence for training signal. In addition, threshold parameter of SLMS-RL1 is set as $\delta_r=0.05$ [19]. Detailed parameters for computer simulation are listed in Tab. 1.

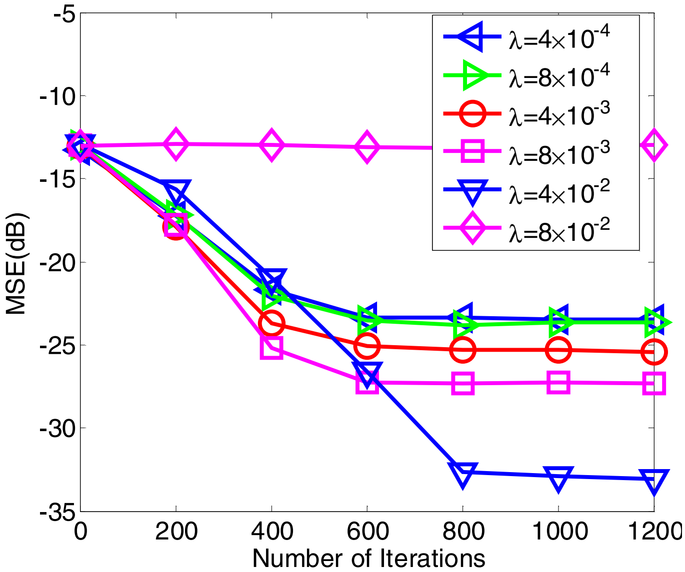

Fig. 1. Monte Carlo simulations averaging over 1000 runs for channl sparsity $K = 4$, GMM with mixture parameter $\phi = 0.1$ and impulsive-noise strength $T = 400$ in different reguarlization parameter λ.

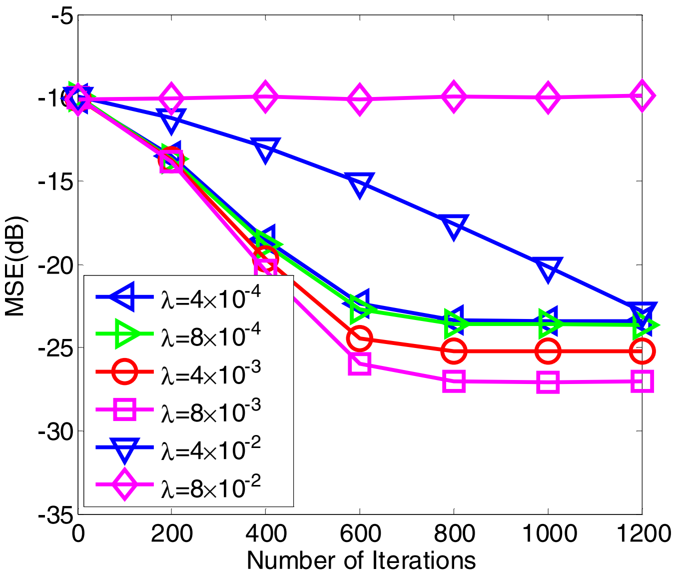

Fig. 2. Monte Carlo simulations averaging over 1000 runs for channl sparsity $K = 8$, GMM with mixture parameter $\phi = 0.1$ and impulsive-noise strength $T = 400$ in different reguarlization parameter λ.

In the first example, average MSE curves of the proposed algorithm are depicted under different channel sparsity, i.e., $K \in \{4,8,16\}$ as shown in Figs. 1~3. Three figures show that MSE curves depend highly on regularization parameter λ. Under the simulation environment as listed in Tab. I, Fig. 1 shows that $\lambda = 4 \times 10^{-2}$ is feasible parameter for channel sparsity $K = 4$ while Figs. 2~3 demonstrate that $\lambda = 8 \times 10^{-2}$ is suggested parameter for $K \in \{8,16\}$. In practical system scenarios, channel sparsity (K) is often changed randomly. Hence, stability of channel estimation algorithm is the most important for selecting regularization parameter empirically. Considering the three representative cases $K \in \{4,8,16\}$ as shown in Figs. 1~3, $\lambda = 8 \times 10^{-3}$ is selected as for the SLMS-RL1 which can ensure stable convergence while without sacrificing significant MSE performance.

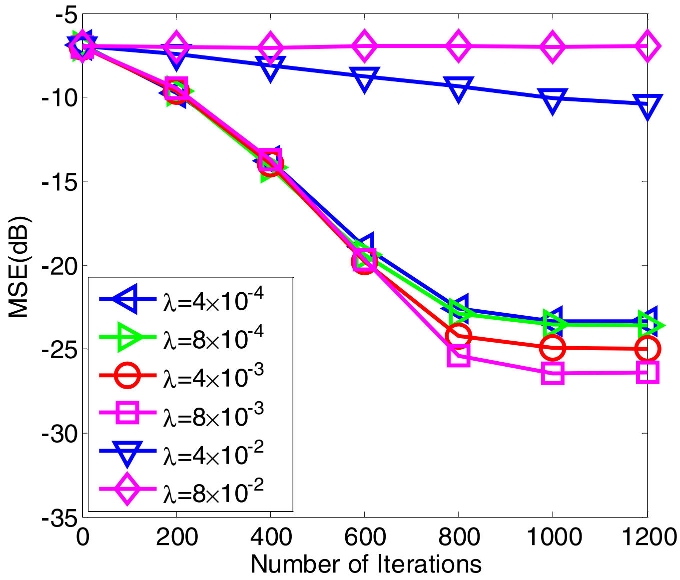

Fig. 3. Monte Carlo simulations averaging over 1000 runs for channl sparsity $K = 16$, GMM with mixture parameter $\phi = 0.1$ and impulsive-noise strength $T = 400$ in different reguarlization parameter λ.

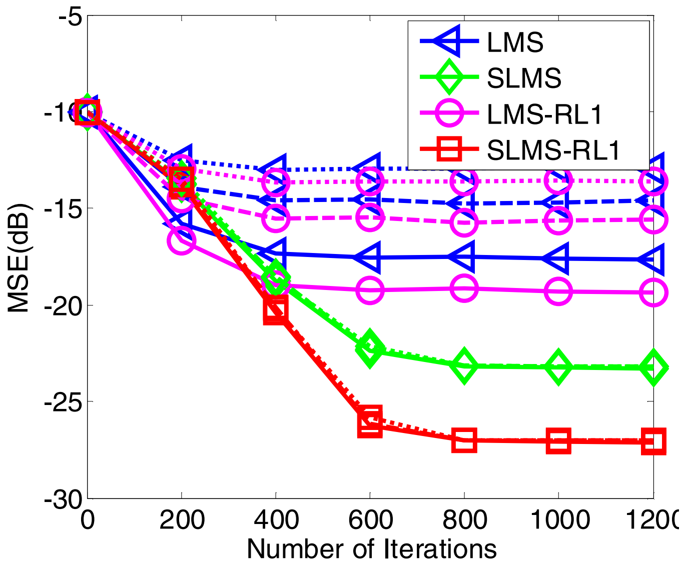

Fig. 4. Monte Carlo simulations averaging over 1000 runs for with mixture parameter $\phi = 0.1$, regularization paramter $\lambda = 8 \times 10^{-3}$, channl sparsity $K = 8$, $SNR = 10\text{dB}$ in $T \in \{200, 400, 600\}$. Case 1 ($T = 200$): solid curves. Case 2 ($T = 400$): dashed curves. Case 3 ($T = 600$): dotted curves.

In the second example, average MSE curves of the proposed algorithm with regularization parameter $\lambda = 8 \times 10^{-3}$ are depicted under GMM impulsive-noise parameters, i.e., $T \in \{200,400,600\}$ as shown in Fig. 4. Under the certain circumstance, e.g., $SNR = 10\text{dB}$, GMM noise mixture parameter $\phi = 0.1$ as well as channel sparisty $K = 8$, one can find that proposed SLMS-RL1 is better than the state-of-the-art three algorithms under GMM noise with positive mixture parameters (ϕ). In Fig. 4, MSE curves of LMS-types algorithms are decided by the different parameters (T). In other words, LMS-type algorithms are sensitive to T. In turn, SLMS-type algorithms are stable to different impulsive-noise parameters, i.e., $\in\{200,400,600\}$. The main reason of SLMS-type algorithms is utilized the sign function which is stable impulsive noise. Hence, the proposed algorithm is also stable for different GMM mixture parameters (ϕ).

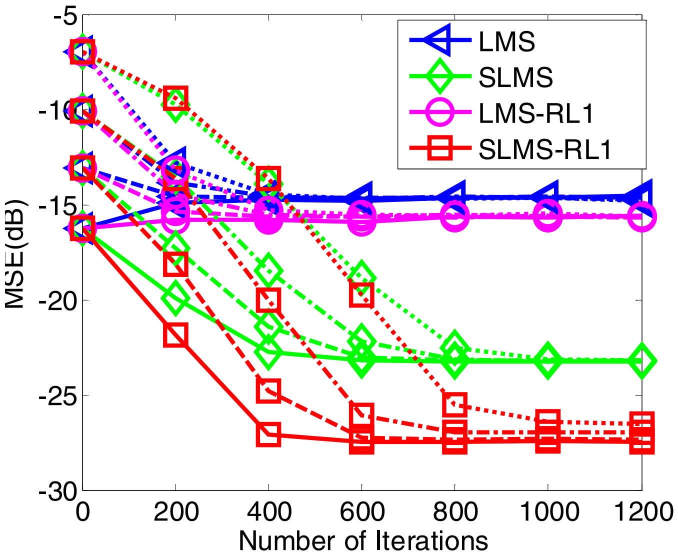

Fig. 5. Monte Carlo simulations averaging over 1000 runs for with mixture parameter $\phi = 0.1$, regularization paramter $\lambda = 8\times10^{-3}$, channl sparsity $K\in\{2,4,8,16\}$, $SNR = 10$dB in $T = 400$. Case 1 ($K = 2$): solid curves. Case 2 ($K = 4$): dashed curves. Case 3 ($K = 8$): dashed-dotted curves. . Case 4 ($K = 16$): dotted curves.

In the third example, average MSE curves of the proposed algorithm are depicted under different channel sparsity, i.e., $K\in\{2,4,8,16\}$ as shown in Fig. 5. Under certain circumstance, e.g., $\lambda = 8\times10^{-3}$, $SNR = 10$dB , GMM noise with impulsive-noise parameter $T = 400$ as well as mixture parameter $\phi = 0.1$, one can find that the proposed SLMS-RL1 is better than the state-of-the-art three algorithms under different channel sparsity K. In addition, one can find also that convergence speed of adaptive sparse algorithms (i,e, RL1-LMS and RL1-LAE) depends on K and steady-state MSE curves of corresponding algorithms are very close. For different channel sparsity, in other words, the adaptive sparse algorithms may differ from conventional compressive sensing based sparse channel estimation algorithms [13], [20]–[22] which depend highly on channel sparsity. Hence, the proposed algorithm is also stable for different channel sparsity.

V. Conclusions

Monte Carlo REPA selection based SLMS-RL1 algorithm was proposed to estimate sparse channels under GMM environments. Considered three kinds channel sparsity, without loss of generality, $\lambda = 8\times10^{-3}$ was selected for SLMS-RL1 algorithm to exploit channel sparisty dependably as to ensure convergence stably. Simulation results demonstrated that the proposed algorithm obtained at least 5dB performance gain than the conventional LMS-RL1 algorithm with respect to different GMM noise strength (T) and different channel sparsity (K), respectively.

This paper only considered a simple scenario of applying the proposed algorithm to estimation sparse channels. The unknown channel dimension is often up to a few of hundreds. It is very difficult to apply the proposed SLMS-RL1 algorithm directly in higher-order dimensional (e.g., thousands or even higher) system identification. Based on the existing stable algorithms, in future work, we plan to develop kernel adaptive filtering based SLMS-RL1 algorithms which can deal with high-dimensional signal processing under non-Gaussian noise environments.

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