# Optimum Tilt of a Phased Array Antenna for Elevation Scan 

Randy L. Haupt<br>Electrical Engineering and Computer Science<br>Colorado School of Mines<br>Golden, CO, USA


#### Abstract

Radars and communications systems must maintain contact with satellites from horizon to zenith. The tilt angle of a planar phased array in addition to its maximum scan angle. Previous studies derive an optimum tilt angle based on the element spacing and number of elements. This paper includes the satellite orbit and differentiates between a communications system and radar when finding the optimum tilt angle.


Keywords-antenna array; phased array; satellite communications; beam steering

## I. INTRODUCTION

Antenna arrays that maintain contact with satellites must be able to scan from horizon to zenith. If the array is conformal to a hemisphere, then selecting appropriate elements on the surface to activate and scanning the beam provides hemispherical coverage. Spherical arrays are expensive to build and complicated to control, so multi-faced planar arrays are more typically used [1]. The faces of the planar arrays tilt in order to optimize the coverage from horizon to zenith.

Radar systems that scan from horizon to 45 to 70 degrees above the horizon have optimum tilt angles between 20 and 30 degrees [2]. These angles are derived from simple analytical formulas based on the element spacing and maximum scan angle. Refinements included array size, type of element, and the amplitude taper [3]. In general, these additional factors only impact the optimum tilt angle by a few degrees.

This paper extends the previous work to radar and communications systems that scan from horizon to zenith in order to maintain contact with satellites in low earth orbit (LEO), medium earth orbit (MEO), and geostationary orbit (GEO). Section II shows how to derive the array gain using analytical equations. Section III explains how the satellite orbit impacts the tilt angle of the array and presents results for the array tilt angle based on the maximum scan angle, element spacing, and orbit.

## II. Antenna Gain in the Scan Region

References [2] and [3] provide a simple equation for the relative gain of a phased array.

$$
\begin{equation*}
G(\theta) \approx \frac{\cos \theta}{1+\sin \theta_{s \max }} \tag{1}
\end{equation*}
$$

where $\theta$ is measured from broadside. The numerator is due to the projected area of the array, while the denominator is due to
the element spacing derived from a grating lobe-free scan region.

Fig. 1 is a diagram of the array. The angle between the horizon and the boresight is the tilt angle ( $\theta_{\text {tilt }}$ ). The maximum scan angle is assumed to be $\theta_{s \text { max }} \leq 60^{\circ}$. If $\theta_{\text {tilt }} \geq 30^{\circ}$ then the maximum scan angle is given by $90^{\circ}-\theta_{\text {tilt }}$, and the array scans from the horizon to the zenith. The faces of the planar arrays have a tilt in order to optimize the gain over the desired scan region. The angle $\gamma$ is measured from the horizon.


Fig. 1. Diagram of the phased array.


Fig. 2. Relative gain as a function of $\gamma$ for various tilt angles.
The curves in Fig. 2 represent (1) for four different tilt angles. If the maximum scan angle is limited to less than or equal to $60^{\circ}$ then the tilt angle must be greater than or equal to $30^{\circ}$. The curves peak at broadside and decrease towards the horizon and zenth. A typical array tilt for a radar system is 20 degrees. A good example is the PAVE PAWS radar that has a

20 degree tilt for a scan region of 3 degrees above the horizon to 85 degrees above the horizon [4].

## III. Space Loss for Satellite Systems

The Friis transmission formula is the basis for the link budget calculations of a satellite communications systems. The power received is given by

$$
\begin{equation*}
P_{r}=\frac{P_{t} G_{t} G_{r} \lambda^{2}}{(4 \pi R)^{2}} \tag{2}
\end{equation*}
$$

where
$P_{r}=$ power transmitted
$G_{t}=$ gain of the transmit antenna
$G_{r}=$ gain of the receive antenna
$\lambda=$ wavelength
$R=$ distance between transmit and receive antennas

For a radar system, the radar range equation is used to determine the power received.

$$
\begin{equation*}
P_{r}=\frac{P_{t} G_{t} G_{r} \sigma \lambda^{2}}{(4 \pi)^{3} R^{4}} \tag{3}
\end{equation*}
$$

where $\sigma$ is the radar cross section. Assuming the power transmitted, satellite antenna gain, and frequency of operation are set in these equations, then to maintain a minimum receive power over the orbit means that the transmit gain (related to the tilt angle) and the orbit height (determines $R$ ) are the factors that influence the power received.

As noted in [2], the satellite is farthest from the ground antenna at the horizon. The distance to the satellite at zenith is the least. The difference between the greatest and smallest distances depends upon the orbit. The distance from the ground antenna to the satellite is given by

$$
\begin{equation*}
R=-r_{e} \sin (\gamma)+0.5 \sqrt{\left[-2 r_{e} \sin (\gamma)\right]^{2}-4\left[r_{e}^{2}-\left(r_{e}+h\right)^{2}\right]} \tag{4}
\end{equation*}
$$

where

$$
\begin{aligned}
& r_{e}=\text { radius of earth } \\
& h=\text { height of orbit }
\end{aligned}
$$

Fig. 3 shows the $1 / R^{2}$ space loss for three different orbits: LEO $(1000 \mathrm{~km})$, MEO $(5000 \mathrm{~km})$, and GEO $(36,000 \mathrm{~km})$. The space loss for the GEO orbit is flat, so the difference in loss between the horizon and zenith is small. Fig. 4 shows the $1 / R^{4}$ space loss of a radar signal for the same three orbits. The curves are steeper for the radar case with the GEO curve being the flattest.

## I. Optimum Tilt Angle Based on Orbit

In order to have a fairly even value for the power received for either a communications or radar system over the orbit from horizon to the maximum scan angle, the tilt is adjusted until the power received is the same when the satellite is at the horizon and the maximum scan angle. In other words, one of the curves in Fig. 2 is added to the curves in Fig. 3 and Fig. 4,
the result is the same at both the horizon and the maximum scan angle. The Nelder Meade downhill simplex algorithm is used to find the optimal tilt.

For any tilt angle less than 30 degrees, the array does not scan to the zenith. If a communications satellite is below $17,000 \mathrm{~km}$ and the maximum scan angle is the zenith, then the tilt angle equals 30 degrees. A radar that must scan from horizon to zenith will have an optimal tilt angle of 30 degrees for all orbits.


Fig. 3. Space loss associated with a satellite communications link for a LEO, MEO, and GEO orbits.


Fig. 4. Space loss associated with a radar for a LEO, MEO, and GEO orbits.


Fig. 5. Optimum tilt angle for a satellite communications phased array for various satellite orbits.


Fig. 6. Optimum tilt angle for a satellite radar phased array for various satellite orbits.

## II. Conclusions

This paper presents s simple way to determine the optimum tilt of a phased array for a satellite communications or radar
system. As previously reported, a radar phased array should have a shallow tilt angle. This same rule of thumb does not apply to satellite communications phased arrays when the satellite has a high MEO or GEO orbit. Plots for the optimal tilt based on the orbit and a max scan angle of 60 degrees are given.

## References

[1] R.L Haupt, Antenna Arrays: A Computational Approach, Hoboken, NJ: Wiley, 2010.
[2] K. Solbach, "Optimum tilt for elevation-scanned phased arrays," IEEE AP-S Mag., vol.32, pp. 39-41, Apr 1990.
[3] M.J. Lee, I. Song, S.C. Kim, and H.M. Kim, "Evaluation of the optimum tilt angles for elevation-scanned phased-array radars," IEEE Trans. Antennas Propag., vol. 47, pp. 214-215, Jan 1999.
[4] Engineering Panel on the PAVE PAWS Radar System, USAF, Radiation Intensity of the PAVE PAWS Radar System, ADA 088323, National Academy of Sciences, 1979.

