Source Reconstruction From Near Field Scan Data of Stripline Structures

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***Abstract*— Inverse problems have always been a major subject in engineering science. There are still no standard solution algorithms working reliably without adding considerable amount of expert knowledge. This class of problems is often described by integral equations. These equations can be interpreted as a convolution of a source with the pulse response of a linear time invariant system. This suggests that it is possible to solve them in spectral domain. As reconstructing the sources of a near field scan data constitutes an inverse problem, the aim of this paper is to show an algorithm to determine an approximate solution. This involves the usage of the spatial Fourier transform and the corresponding impulse response functions in spectral domain.**

I. INTRODUCTION

Estimating the far field of radiating devices by determination of equivalent sources is a well described problem [1]. However, reconstructing the physical sources constitutes an inverse problem. Often these problems can only be solved for simple geometrical structures due to their ill posed nature. A way to solve them is applying regularized Least Squares algorithms [2]. But these algorithms lack numerical stability. Therefore this paper tries to introduce a method without application of regularized Least Squares-Algorithms (LSQ) based on [3] but suitable for two-dimensional structures on a grounded substrate. Using this method, Calculation in spectral domain and the usage of Fast Fourier Transformation avoids an inverse convolution in the space domain.

In chapter II the basic principles of the method will be explained. This section gives a quick overview over the basic integral equation, the point spread functions of a stripline on a grounded dielectric substrate and the application of the Fast Fourier Transform to a near field scan data. In the next section this principle will be evaluated for two simple structures. The results will be compared to those of the Finite Difference Time Domain (FDTD) tool Microwave Studio by CST.

II. BASICS

A. Fundamental Principle

The calculation of the magnetic field of a two dimensional current distribution $\underline{K}_\nu$ is based on the superposition principle. The current is considered as a sum of dipoles and its field is described by the sum of the individual fields. The following integral expresses this principle.

$$\underline{H}_\mu(\vec{\mathbf{r}}_\mathbf{p}) = \iint_{A_q} \underline{G}_{\mu\nu}(\vec{\mathbf{r}}_\mathbf{p} - \vec{\mathbf{r}}_\mathbf{q}) \underline{K}_\nu(\vec{\mathbf{r}}_\mathbf{q}) \mathrm{d}\, A_q \tag{1}$$

The vector $\vec{\mathbf{r}}_\mathbf{p}$ denotes the observation point and $\vec{\mathbf{r}}_\mathrm{q}$ denotes the source point. $\underline{G}_{\mu\nu}$ is called Green's function of the system. It links the ν-component of a current element to the μ-component of the magnetic field. The structure of the integral above can be identified with the structure of a convolution integral. One can see that (1) is in fact the convolution of the current distribution $\underline{K}_\nu$ with Green's Function of the considered source. This Green's Function represents the pulse response of the underlying linear time invariant (LTI) system which is defined by the materials around the current element. As commonly known convolution in spatial domain corresponds to multiplication in spectral domain.

$$\underline{\tilde{H}}_\mu(k_\mu, k_\nu) = \underline{\tilde{G}}_{\mu\nu}(k_\mu, k_\nu)\, \underline{\tilde{K}}_\nu(k_\mu, k_\nu) \tag{2}$$

The tilde symbol in (2) denotes the Fourier transform of the corresponding function and k_μ and k_ν denote the propagation constants in μ and ν direction. This correlation can be used to solve the inverse problem of source reconstruction. For this it is necessary to determine the system's point spread function in Fourier domain as well as the Fourier transform of the near field data.

B. The Green's Function of dieletric substrate in spectral domain

The calculation of the point spread functions of a dielectric substrate with $\varepsilon_r > 1$ and $\mu_r = 1$ is given in [4]. Thus the following chapter will provide just a quick overview concerning this determination. It yields the spectral Green's Function of an arrangement of infinite extent in x and y direction with a dielectric material from z = 0 to z = h_{Subs}, a ground plane at z = 0 and air above z = h_{Subs}. In the following the dielectric region will be called region 1 and the air region will be called region 2. Starting point of the calculation is the set of Maxwell equations with time harmonic variation assumed and suppressed.

$$\nabla\times\underline{E} = -\mathrm{j}\omega\mu_0\underline{H}$$
$$\nabla\times\underline{H} = -\mathrm{j}\omega\varepsilon_r\varepsilon_0\underline{E} \qquad (3)$$

After evaluating the boundary conditions at the interface these equations yield the desired relations.

$$\underline{\tilde{H}}_{x2} = \mathrm{e}^{-\mathrm{j}k_2(z-h_{Subs})}\,\underline{\tilde{K}}_y \frac{\mathrm{j}\sin(k_1 h)}{k_x^2+k_y^2}\left(k_x^2\frac{k_2}{T_e}+k_y^2\varepsilon_r\frac{k_1}{T_m}\right) = \underline{\tilde{G}}_{xy}(k_x,k_y)\underline{\tilde{K}}_y \qquad (4)$$

$$\underline{\tilde{H}}_{y2} = -\mathrm{e}^{-\mathrm{j}k_2(z-h_{Subs})}\,\underline{\tilde{K}}_y \frac{\mathrm{j}\sin(k_1 h)}{k_x^2+k_y^2}k_x k_y\left(\frac{k_2}{T_e}-\varepsilon_r\frac{k_1}{T_m}\right) = \underline{\tilde{G}}_{yy}(k_x,k_y)\underline{\tilde{K}}_y \qquad (5)$$

For x-directed currents interchanging x and y in (4) and (5) yields the corresponding point spread functions.

C. Fourier Transform of Field Data

The Fast Fourier Transform (FFT) is the weapon of choice for transforming numerical field data into the spectral domain. Nevertheless some postprocessing steps have to be made to yield the approximate continuous Fourier transform instead of a discrete line spectrum. To demonstrate this principle the following figures are used. Consider a general, dimensionless rectangular signal as shown in Fig. 1. The input for the FFT is the single pulse around x = 0. The FFT interprets this input as a periodic signal which is denoted by the dashed rectangles. As commonly known, periodic signals have a discontinuous spectrum. But here the continuous spectrum of the single pulse is needed. So the signal of Fig. 1 must be multiplied with a rectangular window. As the FFT interprets every input as one period of a periodic signal, the data windowing is not possible in spatial domain. However a multiplication in the spatial domain is equivalent to a convolution in the spectral domain.

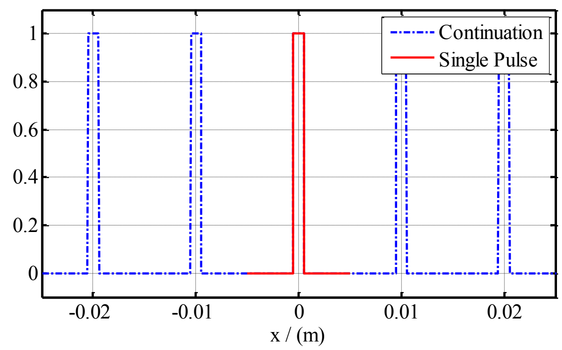

Fig. 1. Rectangular Signal

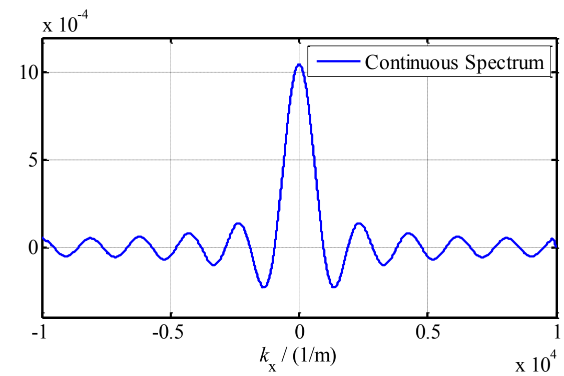

Fig. 2. Continuous spectrum

Thus convolving the line spectrum with the analytically calculated Fourier transform of the window function (a simple sin cardinalis function) yields the continuous spectrum of the single pulse around x = 0 as shown in Fig. 2. This principle holds for the twodimensional Fourier transformation as well. Thus it can be used for the calculation of the spectral distribution of twodimensional near field data.

III. Evaluation for Simple Geometries

The basic principles of chapter II will now be used to calculate the current density in simple stripline structures from their near field data. The data are generated by the FDTD tool CST Microwave Studio. Using a simulation tool allows to compare the current densities calculated by FDTD and the proposed approach.

A. Stripline Patch

The considered structure is a simple stripline patch on a lossless, grounded substrate with a relative permittivity 9.8. It is connected to an AC source at 3 GHz. With respect to the wavelength it cannot be considered electrically small. The FDTD tool allows determining the current in conductors either

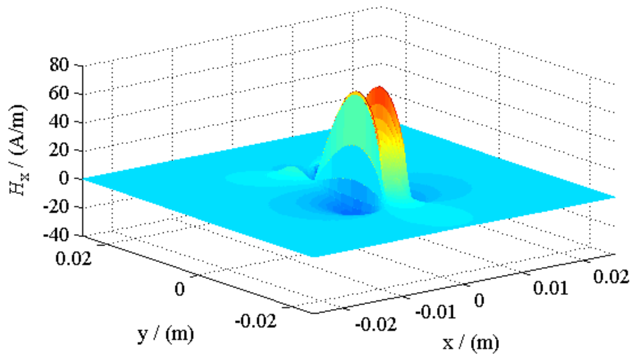

Fig. 3. Magnetic field above strip, real part of x-component

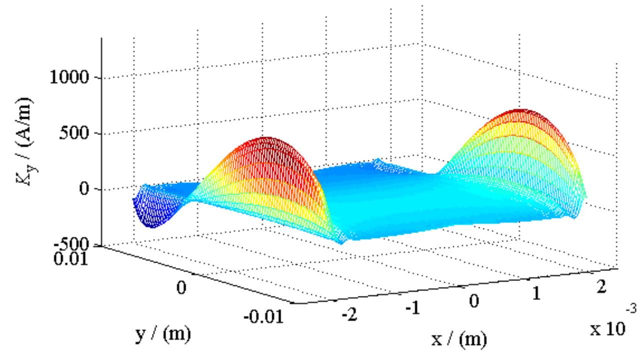

Fig. 4. Current density, y-component

by evaluating the tangential magnetic field component or the losses. The results of these two approaches will be used for verification. The observed near field is exported to Matlab. Fig. 3 shows the x-component of the magnetic field above the strip. This data is transformed to the spectral domain and convolved with the transform of a rectangular window function to obtain the continuous spectrum of the single pulse in Fig. 3. Now that the spectral distribution is known Green's Function of the y-directed current has to be taken into account. The assumption of an exclusively y-directed current density holds for the given geometry. Thus it is sufficient just to consider Green's Function $\underline{G}_{xy}$. It describes the x-component of the magnetic field caused by the y-component of the current. Problems caused by the inverse nature of the calculation become visible then. The larger the term

$$\beta = \sqrt{k_x^2 + k_y^2} \tag{6}$$

grows, the more $\underline{G}_{xy}$ tends to zero. If the Fourier transform of the near field data were known analytically it would tend towards zero just alike. But as the data is noisy, the signal to noise ratio decreases with a falling signal level. The division by small terms in

$$\underline{\tilde{K}}_y(k_x, k_y) = \frac{\underline{\tilde{H}}_x(k_x, k_y)}{\underline{\tilde{G}}_{xy}(k_x, k_y)} \tag{7}$$

amplifies the noise and causes large errors for high propagation constants. Thus a sufficient reconstruction of the source spectrum demands a window function to be applied to the field data. Here the Tukey window [5] with α = 0.3 was chosen. With this filtering applied, equation (7) delivers reasonable results. The inverse Fourier transform of (7) yields the current density on the stripline as shown in Fig. 4. The integral of the current density along the x-direction delivers the total current. In Fig. 5 it is compared to the results of the FDTD tool. The evaluation of the losses in the conducting material, the integral of the magnetic field and the proposed calculation method show sufficient accordance.

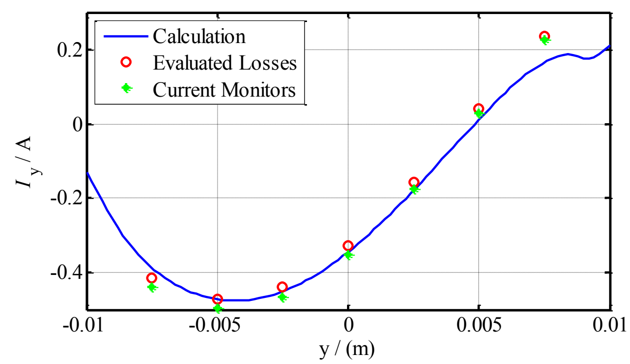

Fig. 5. Comparison of loss evaluation, current monitors and proposed calculation method

The largest difference can be seen at the ends of the strip around the feed lines. There, x- and z-directed currents occur and compromise the assumption of exclusively y-directed currents.

B. *Angled Stripline*

The case of a unidirectional current is very academic. There are currents in both transversal directions in planar conducting geometries. Therefore, the x- and the y-component of the magnetic field are caused by both transversal components of the current distribution.

$$\underline{\tilde{H}}_x = \underline{\tilde{G}}_{xy}\underline{\tilde{K}}_y + \underline{\tilde{G}}_{xx}\underline{\tilde{K}}_x$$
$$\underline{\tilde{H}}_y = \underline{\tilde{G}}_{yy}\underline{\tilde{K}}_y + \underline{\tilde{G}}_{yx}\underline{\tilde{K}}_x \tag{8}$$

Four different point spread functions have to be taken into account and thus a 2x2 linear equation system has to be solved for each point in the $k_x k_y$–plane. The point spread functions in (8) feature the same behavior for growing propagation constants. So a window function has to be applied to the data again. The relation (8) is applied the angled stripline in Fig. 6. The frequency of the AC source is again 3 GHz and thus the structure cannot be considered electrically small with respect

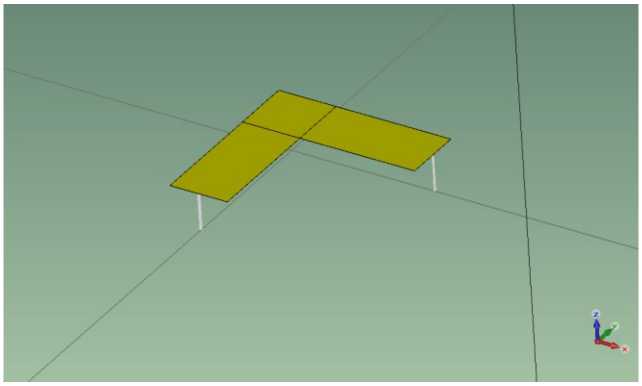

Fig. 6. Angled stripline

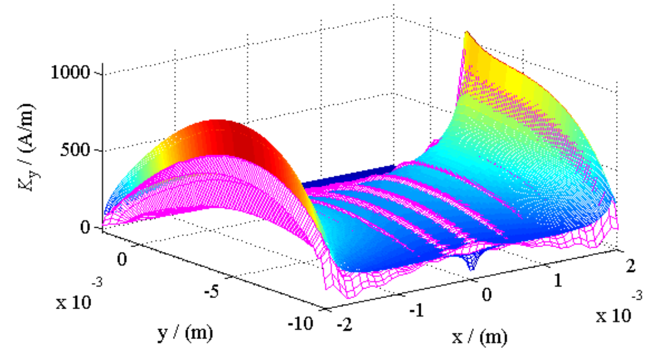

Fig. 8. K_y – component, FDTD and proposed method

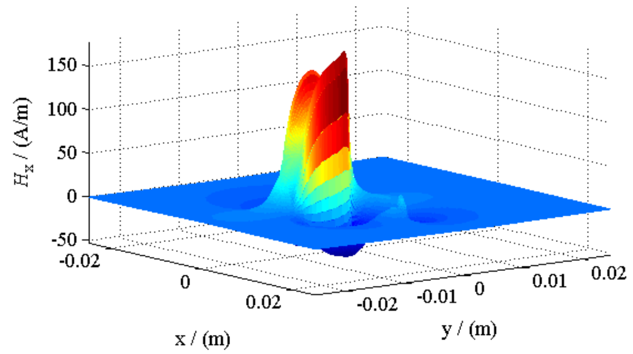

Fig. 7. H_x – component, real part

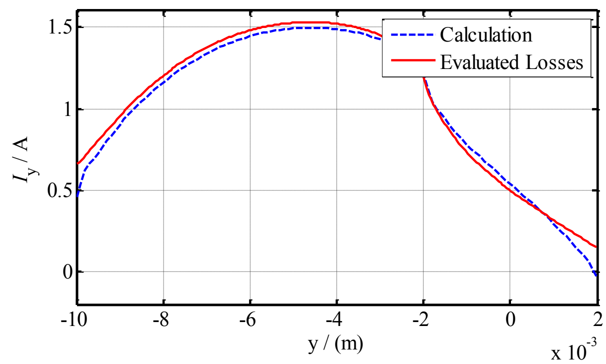

Fig. 9. I_y – component, FDTD and proposed method

to its dimensions. The x-component of the magnetic field above this conducting geometry is shown in Fig. 7. The field data are Fourier transformed and applied to the equation system (8). Inversion yields the Fourier transform of the x- and y-component of the current distribution. Evaluating the inverse Fourier transform yields the current distribution in spatial domain. Fig. 8 shows the y-directed current distribution in the y-directed part of the angled stripline. The comparison to the result of the FDTD tool shows sufficient accordance of the two different methods. In Fig. 9 the integral of the current density in x-direction is shown. The continuous line constitutes the result of the FDTD tool whereas the dashed line is the result of the introduced method. They show sufficient accordance. The same holds for the x-directed current densities.

IV. Conclusion

This paper focuses on solving an inverse problem in the spectral domain. It tries to avoid the inverse convolution in the space domain which is either difficult or in most cases not possible at all. By applying modern numerical tools like FFT the Fourier transform of the field data can be obtained in reasonable time. This allows the usage of spectral domain Green's functions. So a sufficient approximation of the sources' spectral distribution can be achieved. The inverse transform then yields the sources in the spatial domain. Although inevitable numerical difficulties occur, the results are sufficient to predict the currents in the examined structures.

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