Optimal Estimation Interval for Time-Varying V-BLAST Channels

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Abstract: In this paper, we study an effect of channel estimation error on time-varying V-BLAST system. A first-order auto regressive (AR) is introduced to model the time evolution characteristic. The receiver can obtain the current channel information by using a pilot symbol. The estimation accuracy depends on amount of pilot symbols. Based on the received signal, the receiver uses an estimated channel matrix to detect the transmit symbol where Maximum Likelihood (ML), Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) are applied. For given total amount of pilot symbol and interested time period, we propose to divide all channels into equal intervals or groups, which each group uses the same estimated channel. We also specify an optimal number of groups and show that the optimal number of groups mainly depends on time correlation parameter and training budget. Numerical results show that the system can achieve the performance closest to perfect channel estimation when operating on a suitable estimation interval.

1. Introduction

Using multiple antennas at both the transmitter and the receiver provides a multiple-input multiple-out (MIMO) propagation channel. Many works in the literatures have shown that communicating with multiple antennas system can achieve much higher spectral efficiencies than single antenna system in fading environments [1, see references therein]. Vertical Bell-Labs Layered Space-Time (V-BLAST) is an approach to utilizes multiple antenna systems for high data rate transmission. With V-BLAST system, data stream is encoded and transmitted independently for each transmit antenna [2]. Thus, the system capacity increases linearly with number of transmit antennas.

To decode the transmitted data in V-BLAST system, the receiver needs to know the instantaneous channel information. Practically, the receiver can estimate the current channel state by using training symbols [3]. Then, the receiver uses the estimated channel to detect the transmitted data. Performance of the system depends on the accuracy of channel detection, which relies on training symbols [3]. Effect of channel estimation error on V-BLAST system was studied in [4]. Actually, channel varies with time and its information is correlated over time. Correlation between time slots can be described by a first-order auto regressive model [5]. For a time-correlated channel, the receiver can use the estimated channel from previous time slot to detect the transmitted data in current time slot. Hence, the training symbols can be saved by this strategy. This motivates us to study an effect of channel estimation error on V-BLAST time-correlated system and to find a method to manage the number of training symbols, which consume the channel bandwidth. Utilization of time correlation in multiple antennas systems was investigated in [6].

In this paper, we study an effect of channel estimation error on the performance of V-BLAST system with the Maximum Likelihood (ML), Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) in time-correlated fading channels. To reduce the number of total pilot symbols required in an interested time period, we propose to divide channels into intervals or groups. The channel which is estimated from the first time slot is used to detect the transmitted symbols for all time slots in each group. For a given training budget, we try to find the optimal number of groups which minimizes average bit error rate (BER). We show by simulation that the optimal number of groups varies with time correlation coefficient and training budget. Furthermore, we also show that operating on the suitable number of groups can significantly increase the system performance.

This paper is organized as follows. Channel model, detection algorithms and model of estimation error are introduced in Section 2. In Section 3, we propose an idea to manage the training budget and determine the optimal number of groups. Numerical results are shown and discussed in Section 4. Some conclusions are given in Section 5.

2. Time-Varying V-BLAST

We consider a time-slotted frame structure multiple antennas system with M_T transmit antennas and M_R receive antennas. Assuming that $\boldsymbol{H}(n) = [h_{i,j}(n)]$ is an $M_R \times M_T$ channel matrix for the nth time slot, whose element $h_{i,j}(n)$ is the channel coefficient between the jth transmit and the ith receive antennas. For ideal scattering and Rayleigh fading, $h_{i,j}(n)$ is independent and complex Gaussian distributed with zero mean and unit variance. We also assume that adjacent antennas in antenna arrays at both the transmitter and receiver are placed sufficiently far apart so that elements of $\boldsymbol{H}(n)$

are independent, for all n. The channel matrix $\boldsymbol{H}(n)$ is assumed to be static within the nth time slot. To describe the temporal correlation between the nth and $(n-1)$th time slots, we apply a first-order auto regressive (AR) model as follows

$$\boldsymbol{H}(n) = \alpha \boldsymbol{H}(n-1) + \sqrt{1-\alpha^2}\boldsymbol{u}(n) \tag{1}$$

where $\boldsymbol{u}(n)$ is an $M_R \times M_T$ noise matrix with independent zero-mean and unit-variance complex Gaussian entries and α, $(0 \leq \alpha \leq 1)$, is a correlation coefficient which depends on doppler frequency. With the Jakes's model [7], $\alpha = J_0(2\pi f_d \Delta n)$ where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, f_d is the maximum Doppler frequency and Δn is the time interval between consecutive channel blocks. We note that the correlation parameter is assumed to be perfectly available at both the transmitter and the receiver. Reference [5] has shown that the model in (1) can be used to predict the time evolution very well.

We assume that the parallel data streams are simultaneously transmitted through multiple antennas in the same frequency band. At the receiver, for the nth time slot, the $M_R \times 1$ received signal is given by

$$\boldsymbol{r}(n) = \boldsymbol{H}(n)\boldsymbol{x}(n) + \boldsymbol{z}(n) \tag{2}$$

where $\boldsymbol{x}(n) = [x_1(n), ..., x_{M_T}(n)]^T$ is a transmit vector which each element has zero mean and unit variance, and $\boldsymbol{z}(n)$ is $M_R \times 1$ AWGN vector with zero mean and covariance $\sigma_z^2 \boldsymbol{I}$ and $\boldsymbol{I}$ is an identity matrix. By observing (2) we see that each receive antenna receives a combination of all faded symbols. To detect the transmitted vector $\boldsymbol{x}$, we consider three detection algorithms as follows.

2.1 Maximum Likelihood (ML) Detection

The first detector we will consider is the Maximum Likelihood detector which is optimal in the probabilistic sense. With ML algorithm, the received signal is compared with all possible transmitted vectors which are multiplied by channel matrix $\boldsymbol{H}(n)$. The candidate vector with the minimum euclidean distance to the received signal is considered to be the estimated signal as follows

$$\hat{\boldsymbol{x}}(n) = \arg_{\boldsymbol{x}_i \in \mathcal{X}} \min \|\boldsymbol{r}(n) - \boldsymbol{H}(n)\boldsymbol{x}_i\|^2 \tag{3}$$

where $\mathcal{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_{2^{M_T}}\}$ is a set of all candidate vectors. As mentioned above, an ML algorithm offers the optimum detector. To perform the calculation in (3), the receiver needs to search for all possible choices in the set $\mathcal{X}$. Thus, the complexity depends on the number of members in $\mathcal{X}$ and increases exponentially with the number of transmit antennas.

2.2 Zero Forcing (ZF) Detection

The problem in search complexity of ML algorithm can be avoided by employing a linear receiver. The first linear receiver we are interested in is Zero-Forcing (ZF). With ZF algorithm, noises are treated to be zero and then the transmitted data streams are separated and decoded independently. The detected symbol can be found by

$$\hat{\boldsymbol{x}}(n) = \left(\boldsymbol{H}(n)^H \boldsymbol{H}(n)\right)^{-1} \boldsymbol{H}(n)^H \boldsymbol{r} \tag{4}$$

where $(\cdot)^H$ denotes a hermitian transpose operation. We note that the ZF decoder works well in high signal-to-noise ratio (SNR) regime. However, when the SNR decreases, the performance of the ZF detector drastically degrades. This is because this kind of detectors essentially ignores the presence of noises.

2.3 Minimum Mean Square Error (MMSE) Detection

To improve the performance of the Zero-Forcing algorithm, noise components should be taken into account in the detection process. With the MMSE receiver, the detected symbol is given by

$$\hat{\boldsymbol{x}}(n) = \left(\boldsymbol{H}(n)^H \boldsymbol{H}(n) + \sigma_z^2 \boldsymbol{I}\right)^{-1} \boldsymbol{H}(n)^H \boldsymbol{r}. \tag{5}$$

The solution in (5) can be obtained by using the orthogonality principle. It should be noted that the performance of the MMSE receiver approaches to that of the Zero-Forcing receiver as SNR increases.

To detect the transmitted data, the receiver needs to know the current channel matrix $\boldsymbol{H}(n)$. Practically, the receiver can obtain the knowledge of channel via pilot symbols. We assume that at the beginning of each time slot, T pilot symbols, which are known by both the transmitter and the receiver, are transmitted. The receiver uses these pilot symbols to estimate the current channel. With the assumption that the elements of $\boldsymbol{H}(n)$ are independent complex Gaussian random variables with zero mean and unit variance hence, we have

$$\boldsymbol{H}(n) = \hat{\boldsymbol{H}}(n) + \boldsymbol{w}(n) \tag{6}$$

where the estimated channel $\hat{\boldsymbol{H}}(n)$ and the error matrix $\boldsymbol{w}(n)$ are independent. Assuming the elements of $\boldsymbol{w}(n)$ are independent complex Gaussian random variables with zero mean and variance σ_w^2, as a result, the elements of $\hat{\boldsymbol{H}}(n)$ have zero mean and variance $(1-\sigma_w^2)$. Reference [3] has shown that, for the MMSE estimator, the relation between amount of training symbols T and σ_w^2 is given by

$$\sigma_w^2 = \frac{1}{1+\rho T} \tag{7}$$

where ρ is background signal-to-noise ratio (SNR) of the system which is uniformly allocated to each time slot. This is very clear to say that the estimation error will decrease if we increase the training budget ρ or T.

In the real system, the receiver uses an estimated channel matrix $\hat{\boldsymbol{H}}(n)$ to detect the transmitted data. Hence, for ML algorithm, solution of detected data becomes

$$\hat{\boldsymbol{x}}(n) = \arg_{\boldsymbol{x}_i \in \mathcal{X}} \min \|\boldsymbol{r}(n) - \hat{\boldsymbol{H}}(n)\boldsymbol{x}_i\|^2, \tag{8}$$

for ZF the solution is given by

$$\hat{\boldsymbol{x}}(n) = \left(\hat{\boldsymbol{H}}(n)^H \hat{\boldsymbol{H}}(n)\right)^{-1} \hat{\boldsymbol{H}}(n)^H \boldsymbol{r}, \tag{9}$$

and for MMSE

$$\hat{\boldsymbol{x}}(n) = \left(\hat{\boldsymbol{H}}(n)^H \hat{\boldsymbol{H}}(n) + \sigma_z^2 \boldsymbol{I}\right)^{-1} \hat{\boldsymbol{H}}(n)^H \boldsymbol{r}. \tag{10}$$

We note that the performance of all detection algorithms depends on the accuracy of estimation in (6).

3. Channel Estimation Strategy

Given a system with N time slots and T pilot symbols with constraint $T \leq N$, we propose that the transmitter divides all N time slots into G groups and each group contains $M = N/G$ time slots. At the beginning of each group, the transmitter sends $\bar{T} = T/G$ pilot symbols. The variance of an estimation error can be computed by replacing T with $\bar{T}$ into (7). The receiver estimates the channel matrix and uses it to detect the transmitted data for all M time slots in the group. With the proposed algorithm, the estimated channel matrix is most matched to the first time slot and regressively matched to subsequence time slots. Suppose that N is fixed. Consider a system with large G while M is small, the channels are estimated very often but with large estimation error. Oppositely, the estimated channel is accurate in the first time slot of the group but may be outdated in subsequence time slots for the system with small G and large M. On the other hand we can say that the performance of the system is a trade-off between training symbols and number of groups G, and hence there exists optimal number of groups for a given training budget.

In this work, we use the bit error rate (BER) to measure the system performance. Let $\mathrm{BER}_{\mathrm{g,m}}$ be the bit error rate for the mth time slot of the gth group, the averaged bit error rate over all time slots can be determined as follows

$$\bar{\mathrm{BER}} = \frac{1}{N} \sum_{g=1}^{G} \sum_{m=1}^{M} \mathrm{BER}_{\mathrm{g,m}}. \tag{11}$$

We would like to determine the optimal number of groups that minimizes the average bit error rate as follows

$$G^* = \arg \min_{1 \leq G \leq N} \bar{\mathrm{BER}}. \tag{12}$$

We note that the solution in (12) can be found by an intensive Mote-Carlo simulation. In the next section, we will show that the optimal number of group G^* is very significant to improve the system performance.

4. Numerical Results

To illustrate and obtain some insight about the proposed method, we provide an intensive Mote-Carlo simulation over 1,000,000 channel and noise realizations. In Fig. 1, we show the bit error rate with number of groups G for 2×2 V-BLAST channels with 128 time slots, training budget is 1 symbol per time slot and SNR = 10 dB. Different plot corresponds to different correlation parameter. In the figure, we see that there exists an optimal number of groups for each correlation model. For time-invariant channel ($\alpha = 1$), using the first time slot estimated channel over all time periods provides 75% lower BER than that for training all time slot with 1 symbol. However, uniformly channel estimation is suitable in fast fading channel ($\alpha = 0.7$). For slow fading channel, selecting $G = 32$ gives the minimum BER.

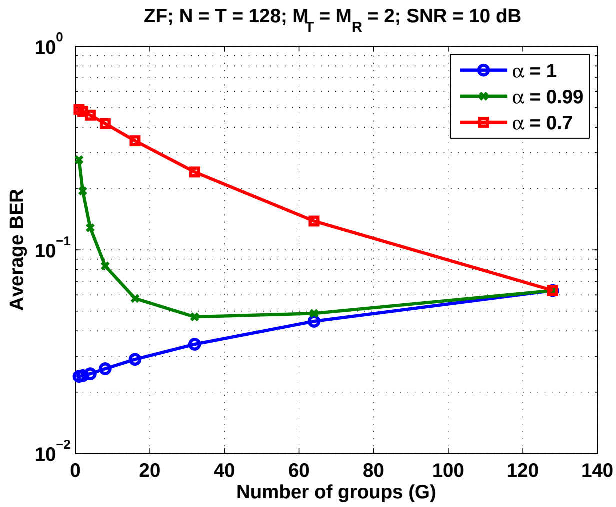

Figure 1. The averaged bit error rates from difference channel models are plotted with number of groups G with zero-forcing detection for $N = T = 128$, $M_T = M_R = 2$ and SNR = 10 dB.

Figs. 2 and 3 show the graphs of the optimal number of group G^* as functions of the time correlation parameter and of the SNR, respectively. In the both figures, the system parameters are set as $N = 128$, $T = 32$, and $M_T = M_R = 2$. In Fig. 2, we see that G^* decreases as correlation between time slots increases for every interested SNR. This implies that the estimation process should be occurred frequently in fast fading channel. However, as the channel becomes more static, the better strategy is to lessen the frequency of estimation but with longer training symbol.

Fig. 3 shows the optimal number of groups G^* with SNR. We can observe that, for sufficiently high SNR, estimating channel for all time slots is possible. However, using the optimal number of groups is more essential in low SNR regime, since it significantly improves the system performance.

In Fig. 4, we compare the systems operated with optimal number of groups G^* with perfect channel estimation for ML, ZF and MMSE detectors. We note that G^* can be obtained from (12). As expected, the ML algorithm provides the best performance while the ZF algorithm is comparable to the MMSE algorithm in a high SNR regime. In this figure, we can see that operating on G^* provides performance closest to the ideal channel estimation. Thus, the optimal G^* is very significant to improve

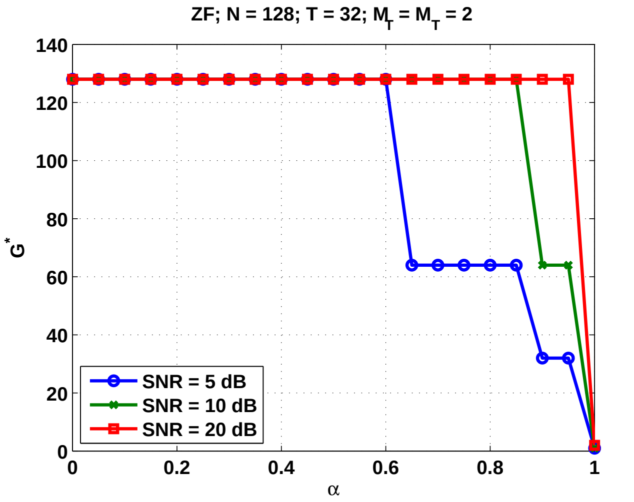

Figure 2. Optimal number of groups G^* is shown with time correlation parameter α with zero-forcing detection for $N = 128$, $T = 32$ and $M_T = M_R = 2$.

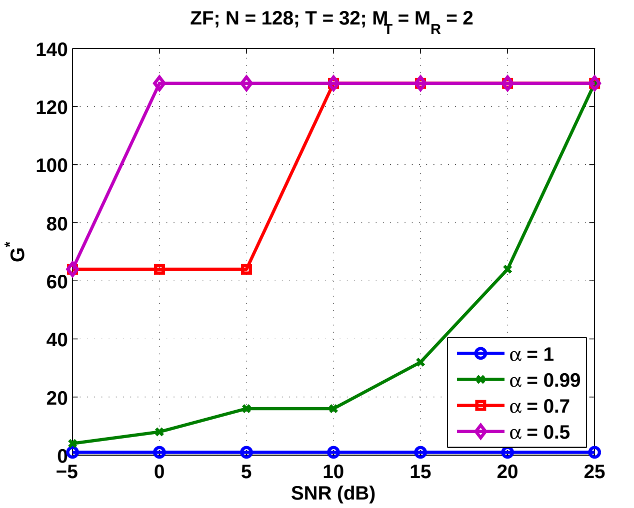

Figure 3. Optimal number of groups G^* is shown with SNR with zero-forcing detection for $N = 128$, $T = 32$ and $M_T = M_R = 2$.

the system performance in the scenario, which training budget is very limited.

5. Conclusions

We have studied an effect of channel estimation error on the performance of V-BLAST time-correlated channels. The system performance is degraded due to an estimation error especially in case of very limited training budget. Furthermore, we also proposed the method to manage the training budget and determined the optimal number of groups G^* which is mainly depended on time correlation parameter α and training budget. In the numerical example, a system can achieve performance closet to the ideal channel estimation when operating with the suitable number of groups, especially for

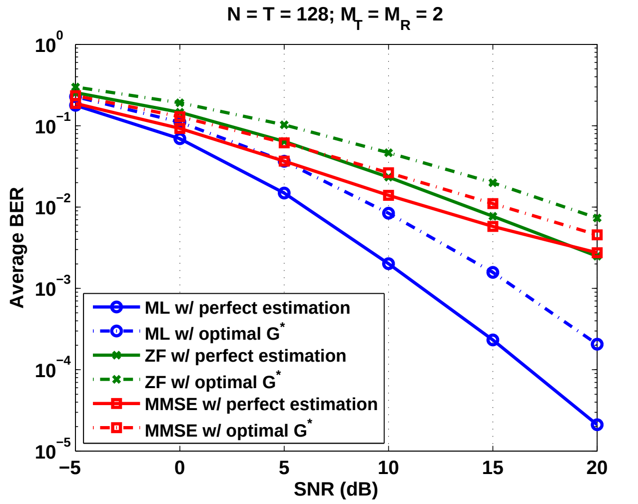

Figure 4. Comparison between optimal G^* and ideal channel estimation is shown for ML, ZF and MMSE detections with $N = T = 128$ and $M_T = M_R = 2$.

the ZF and MMSE detectors.

In the scope of this work, the system performance is only evaluated by numerical simulation. To obtain more insight about the system, we need to derive a closed-form expression of the average BER. This is very interesting and will be our work in the future.

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